

First Midterm Exam, Econ. 240B  
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**Instructions:** You have 110 minutes to complete this exam. This is a 30 point exam; all subsections of all questions have equal weight (5 points each). This is a closed book exam, but one sheet of notes is permitted. All needed statistical tables are appended. Please make your answers elegant - that is, clear, concise, and correct.

1. **True/False/Explain** (15 points): For each of the following three statements, determine whether it is correct, and, if correct, explain why. If not, state precisely why it is incorrect and give a modification which is correct.

A. In the two-equation Seemingly Unrelated Regression model, even if the regressors are identical for the two equations, the corresponding coefficient estimators are generally not uncorrelated across equations.

B. To test for heteroskedastic errors in a linear model, it is useful to regress functions of the absolute values of least-squares residuals (e.g., the squared residuals) on functions of the regressors. If the R-squared from this second stage regression exceeds the upper 5% critical value for a chi-squared random variable (with degrees of freedom equal to the number of non-constant functions of the regressors in the second-stage regression), we would reject the null hypothesis of no heteroskedasticity at an approximate 5% level.

C. In the regression model with serially correlated errors and nonrandom regressors,  $E(y_t) = x_t'\beta$ ,  $Var(y_t) = \sigma^2/(1 - \rho^2)$ , and  $Cov(y_t, y_{t-1}) = \rho\sigma^2/(1 - \rho^2)$ . So if the sample correlation of the dependent variable  $y_t$  with its lagged value  $y_{t-1}$  exceeds  $1.96/\sqrt{T}$  in magnitude, we should reject the null hypothesis of no serial correlation, and should either estimate  $\beta$  and its asymptotic covariance matrix by FGLS or some other efficient method or replace the usual estimator of the LS covariance matrix by the Newey-West estimator (or some variant of it).

2. (5 points) Suppose the coefficients  $\beta = (\beta_1, \beta_2)'$  in the linear model  $y = \mathbf{X}\beta + \varepsilon$  are estimated by classical least squares, where it is assumed that the errors  $\varepsilon$  are independent of the matrix  $\mathbf{X}$  of regressors with scalar covariance matrix  $\mathbf{V}(\varepsilon) = \mathbf{V}(\varepsilon|\mathbf{X}) = \sigma^2\mathbf{I}$ . An analysis of  $N = 347$  observations yields

$$\hat{\beta} = \begin{pmatrix} 0.25 \\ -0.25 \end{pmatrix}, \quad s^2 = 0.1, \quad \mathbf{X}'\mathbf{X} = \begin{bmatrix} 40 & 10 \\ 10 & 5 \end{bmatrix}.$$

Construct an approximate 95% confidence interval for  $\gamma \equiv \beta_1/\beta_2$ , under the (possibly heroic) assumption that the sample size is large enough for the usual limit theorems and linear approximations to be applicable. Is  $\gamma_0 = 0$  in this interval?

3. (10 points) Suppose that, for the simple linear model with no intercept term,

$$y_i = \beta x_i + \varepsilon_i,$$

that both  $z_{i1} \equiv 1$  and  $z_{i2} \equiv x_i$  are valid instrumental variables for  $x_i$ ; that is

$$\begin{aligned} E(z_{i1}\varepsilon_i) &= E(\varepsilon_i) = 0, \\ E(z_{i2}\varepsilon_i) &= E(x_i\varepsilon_i) = 0, \end{aligned}$$

and

$$\begin{aligned} E(z_{i1}x_i) &= E(x_i) \equiv \mu \neq 0, \\ E(z_{i2}x_i) &= E(x_i^2) \equiv \tau^2 \neq 0. \end{aligned}$$

A. Under the assumption that  $\varepsilon_i$  and  $x_i$  are jointly i.i.d. and  $\varepsilon_i$  is independent of  $x_i$  with  $E(\varepsilon_i^2) = \sigma^2$ , derive the asymptotic distribution of the IV estimators  $\hat{\beta}_1$  and  $\hat{\beta}_2$  which use  $z_{i1} = 1$  or  $z_{i2} = x_i$ , respectively, as an instrument for  $x_i$ , and compare the asymptotic variances of these two estimators.

B. Under the same assumptions as in part A., explicitly derive the asymptotic variance for the GMM estimator  $\hat{\beta}_{GMM}$  which optimally uses both  $z_{i1} = 1$  and  $z_{i2} = x_i$  as instrumental variables, and show that this variance reduces to the asymptotic variance of one of the estimators in part A. [HINT: the relevant matrices  $\mathbf{M}_{XZ}$  and  $\mathbf{V}_0$  can be written in terms of the parameters given above.]