

Problem Set #1

ECONOMICS 240B
SPRING 2006

Due February 6

Turn in (correct) answers to the following exercises from Ruud's text:

Chapter 13: Exercises 13.5, 13.6, 13.9, 13.11, 13.12.

Chapter 25: Exercises 25.11, 25.13.

Additional Questions:

1. Suppose the coefficients $\beta = (\beta_1, \beta_2)'$ in the linear model $y = \mathbf{X}\beta + \varepsilon$ are estimated by classical least squares, where it is assumed that the errors ε are independent of the matrix \mathbf{X} of regressors with scalar covariance matrix $\mathbf{V}(\varepsilon) = \mathbf{V}(\varepsilon|\mathbf{X}) = \sigma^2\mathbf{I}$. An analysis of $N = 347$ observations yields

$$\hat{\beta} = \begin{pmatrix} 0.25 \\ -0.25 \end{pmatrix}, \quad s^2 = 0.1, \quad \mathbf{X}'\mathbf{X} = \begin{bmatrix} 40 & 10 \\ 10 & 5 \end{bmatrix}.$$

Construct an approximate 95% confidence interval for $\gamma \equiv \beta_1/\beta_2$, under the (possibly heroic) assumption that the sample size is large enough for the usual limit theorems and linear approximations to be applicable. Is $\gamma_0 = 0$ in this interval?

2. Let $\{X_i, i = 1, \dots, n\}$ be an i.i.d. sample of scalar random variables with $E[X_i] = \mu$, $Var(X_i) = \sigma^2$, $E(X_i - \mu)^3 = 0$, and $E(X_i - \mu)^4 = \tau$, all finite.

- A.** Define $T_n \equiv (\bar{X}/s)$, where, as usual, $\bar{X} \equiv n^{-1} \sum_i X_i$ and $s^2 \equiv [n^{-1} \sum_i X_i^2] - \bar{X}^2$. Derive the limiting distribution of $\sqrt{n}T_n$ under the assumption that $\mu = 0$.
- B.** Now suppose it is not assumed that $\mu = 0$. Derive the limiting distribution of $\sqrt{n}(T_n - T_0)$, where $T_0 \equiv p \lim T_n$. Be sure your answer reduces to the result of part A. when $\mu = 0$.
- C.** Define $R_n \equiv (\bar{X}/\hat{\sigma})$, where $\hat{\sigma}^2 \equiv [n^{-1} \sum_i X_i^2]$ is the constrained estimator of σ^2 under the (possibly incorrect) assumption that $\mu = 0$. Derive the limiting distribution of $\sqrt{n}(R_n - R_0)$, for $R_0 = p \lim R_n$. Under what conditions on μ and σ^2 will this asymptotic distribution be the same as in part B.?