First Midterm Exam, Econ. 240B

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March 17, 2008

Instructions: You have 110 minutes to complete this exam. This is a 30 point exam; all subsections of all questions have equal weight (5 points each). This is a closed book exam, but one sheet of notes is permitted. All needed statistical tables are appended. Please make your answers elegant – that is, clear, concise, complete, and correct.

1. **True/False/Explain** (15 points): For **three** of the following four statements below, determine whether it is correct, and, if correct, explain why. If not, state precisely why it is incorrect and give a modification which is correct. **Answer only three questions;** if you answer more, only the **first three** answers will count in your score.

A. Suppose the Generalized Classical Regression model applies to $\mathbf{y}$ and $\mathbf{X}$, where $E(\mathbf{y}) = \mathbf{X}\boldsymbol{\beta}$ and $V(\mathbf{y}) = \boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is **not** proportional to an identity matrix. Suppose the first column of $\mathbf{X}$ is a vector of ones, and let $\hat{\mathbf{y}}$ and $\hat{\mathbf{e}}$ denote the fitted values and residuals of the LS regression of $\mathbf{y}$ on $\mathbf{X}$. Then the covariance of each element of $\hat{\mathbf{y}}$ with the corresponding element of $\hat{\mathbf{e}}$ may be nonzero, but the sum of those covariances is identically zero.

B. The Two-Stage Least Squares estimator $\hat{\beta}_{2SLS}$ is unchanged if the original $N \times L$ matrix of instrumental variables Z is replaced by a new matrix Z^* of instruments if $Z^* = ZH$, where H is an invertible $L \times L$ matrix.

C. In the ,linear model with a lagged dependent variable, $y_t = x_t'\beta + \gamma y_{t-1} + \varepsilon_t$, suppose it is possible that the error terms ε_t are serially correlated, i.e., $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$, where u_t is an i.i.d. sequence with zero mean and variance σ^2, and is independent of x_s for all t and s. Then the classical LS estimators $\hat{\beta}_{LS}$ and $\hat{\gamma}_{LS}$ will generally be inconsistent for β and γ, though they will generally be consistent if $\rho = 0$ or if $\gamma = 0$ (or both).

D.. As the proof of the Weak Law of Large Numbers illustrates, the variance of a consistent estimator $\hat{\theta}$ of a parameter θ converges to zero with the sample size N, and, if the estimator is $\sqrt{N}$-consistent and asymptotically-normal, the variance shrinks to zero at speed $1/N$.

2. (5 points) The coefficients $\boldsymbol{\beta} = (\beta_1, \beta_2)'$ in a linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ were estimated by generalized least squares, where it was assumed that the errors ε were independent of the matrix $\mathbf{Z}$ of instruments with scalar covariance matrix $\mathbf{V}(\boldsymbol{\varepsilon}) = \mathbf{V}(\boldsymbol{\varepsilon}|\mathbf{X}) = \sigma^2\boldsymbol{\Omega}$. Analysis of $N = 400$ obervations yielded

$$\hat{\boldsymbol{\beta}}_{GLS} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \qquad (\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{GLS})'\hat{\boldsymbol{\Omega}}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{GLS}) = 1600, \qquad (\mathbf{X}'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{X}) = \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix}.$$

Construct an approximate 95% confidence interval for $\gamma \equiv \beta_1 \cdot \beta_2$, under the assumption that the sample size is large enough for the usual limit theorems and linear approximations to be applicable. Is $\gamma_0 = 0$ in this interval?

3. (10 points) Suppose you have a panel of observations on a dependent variable y_{it} and a K-dimensional vector of explanatory variables $\mathbf{x}_{it}$ for a panel of observations, for $i = 1, ..., N$ and $t = 1, ..., T$, that satisfies the following linear model:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_i + \varepsilon_{it}, \qquad i = 1, ..., N \text{ and } t = 1, ..., T.$$

That is, all coefficients (intercept and slope coefficients) can vary across individuals. Also suppose that N is considered **fixed**, and is small relative to T, which is "tending to infinity." Finally, suppose that the observation are independent across i, but that the errors may be heteroskedastic and/or autocorrelated across t for each i, with the normalized averages of $\mathbf{x}_{it}\varepsilon_{it}$ satisfying a central limit theorem for each i:

$$\frac{1}{\sqrt{T}}\sum_{i=1}^{T} \mathbf{x}_{it}\varepsilon_{it} \xrightarrow{d} N(\mathbf{0}, \mathbf{C}_i).$$

A. Assume that the coefficient vectors $\boldsymbol{\beta}_i$ are i.i.d. across i , and are independent of the regressors $\mathbf{x}_{it}$ and errors ε_{it} for all i and t. If you are interested in the average value of $\boldsymbol{\beta}_i$,

$$\boldsymbol{\beta} \equiv E[\boldsymbol{\beta}_i],$$

can you use observations on

$$\mathbf{y}_i \equiv \begin{pmatrix} y_{i1} \\ \cdots \\ y_{it} \\ \cdots \\ y_{iT} \end{pmatrix} \qquad \text{and} \qquad \mathbf{X}_i \equiv \begin{bmatrix} \mathbf{x}'_{i1} \\ \cdots \\ \mathbf{x}'_{it} \\ \cdots \\ \mathbf{x}'_{iT} \end{bmatrix}$$

for $i = 1, ..., N$ to consistently estimate $\boldsymbol{\beta}$ as $T \to \infty$? If so, give a consistent estimator, and briefly justify its consistency; if not, explain why not..

B. Now consider the special case $\boldsymbol{\beta}_i \equiv \boldsymbol{\beta}$, i.e., all the coefficients are constant across individuals. If you are given least-squares estimators $\hat{\boldsymbol{\beta}}_i = (\mathbf{X}'_i\mathbf{X}_i)^{-1}\mathbf{X}'_i\mathbf{y}_i$ for each i, and given consistent (Newey-West) estimators $\hat{\mathbf{C}}_i$ of the asymptotic covariance matrices $\mathbf{C}_i$, how would you combine the values of $\mathbf{X}_i$, $\hat{\boldsymbol{\beta}}_i$, and $\hat{\mathbf{C}}_i$ to construct an efficient estimator of $\boldsymbol{\beta}$? Be as explicit as possible about the form of your estimator, and explain the sense in which it is efficient.