# First Midterm Exam, Econ. 240B

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**Instructions:** You have 110 minutes to complete this exam. This is a 30 point exam; all subsections of all questions have equal weight (5 points each). This is a closed book exam, but one sheet of notes is permitted. All needed statistical tables are appended. Please make your answers elegant – that is, clear, concise, complete, and correct.

1. **True/False/Explain** (20 points): For **four** of the following five statements below, determine whether it is correct, and, if correct, explain why. If not, state precisely why it is incorrect and give a modification which is correct. **Answer only three questions;** if you answer more, only the **first four** answers will count in your score.

A. For either the stationary first-order autoregressive process (that is, $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$, with $\varepsilon_t$ a white noise process with variance $\sigma^2$) or for the (stationary) first-order moving average process (i.e., $y_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$, again with $\varepsilon_t$ a white noise process with variance $\sigma^2$), the correlation between $y_t$ and $y_{t-1}$ can be any value strictly between $-1$ and 1, .as long as $|\beta| < 1$ and $|\theta| < 1$.

B. In the linear model with a lagged dependent variable, $y_t = x_t'\beta + \gamma y_{t-1} + \varepsilon_t$, suppose the error terms have first-order serial correlation, i.e., $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$, where $u_t$ is an i.i.d. sequence with zero mean, variance $\sigma^2$, and is independent of $x_s$ for all $t$ and $s$. For this model, the classical LS estimator will be inconsistent for $\beta$ and $\gamma$, but Aitken's GLS estimator (for a known $\Omega$ matrix) will consistently estimate these parameters.

C. In the two-equation Seemingly Unrelated Regression model, if the explanatory variables in the two equations are orthogonal (i.e., $X_1'X_2 = 0$), then the LS coefficient estimators for the two equations are uncorrelated with each other, and GLS reduces to LS for each equation.

D. By the Continuous Mapping theorem, if $\hat{\theta}$ is root-$n$ consistent and asymptotically normal for the scalar parameter $\theta_0$, then its squared value, when multiplied by an appropriate function of the sample size $n$, will have a limiting chi-square distribution.

E. For a balanced panel data regression model with random individual effects, $y_{it} = x_{it}'\beta + \alpha_i + \varepsilon_{it}$ (where the $\alpha_i$ are are independent of $\varepsilon_{it}$ and $x_{it}$, and all error terms have mean zero, constant variance, and are serially independent across $i$ and $t$), suppose that only the number of time periods $T$ tends to infinity, while the number of individuals $N$ stays fixed. Then the "fixed effect" estimator for $\beta$ will be consistent as $T \to \infty$, but the "random effects" GLS estimator is infeasible, since the joint covariance matrix of the error terms is not consistently estimable.

2. (5 points) Suppose $\hat{\boldsymbol{\theta}}$ is an asymptotically normal estimator of a 3-dimensional parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)'$, which has the asymptotic distribution

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \mathbf{V}).$$

Suppose that $\hat{\boldsymbol{\theta}} = (1, -1, -1)'$ is the realized value of this estimator, and that a consistent estimator $\hat{\mathbf{V}}$ of $\mathbf{V}$ has the realized value

$$\hat{\mathbf{V}} = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix},$$

where it is assumed that the sample size $N = 400$ is large enough so that the normal approximation is accurate for this problem.

Use these results to test the joint null hypothesis $H_0 : \theta_1^2 + \theta_3^2 = 1$ and $\theta_2 = 0$, against the alternative that one or both of these restrictions fail, at an asymptotic 5% level.

3. (5 points) Suppose that, for the simple linear model with no intercept term,

$$y_i = \beta x_i + \varepsilon_i,$$

that both $z_{i1} \equiv 1$ and $z_{i2} \equiv w_i$ are valid instrumental variables for $x_{i;}$ that is

$$\begin{aligned} E(z_{i1}\varepsilon_i) &= E(\varepsilon_i) = 0, \\ E(z_{i2}\varepsilon_i) &= E(w_i\varepsilon_i) = 0, \end{aligned}$$

and

$$\begin{aligned} E(z_{i1}x_i) &= E(x_i) \equiv \mu \neq 0, \\ E(z_{i2}x_i) &= E(w_i x_i) \equiv \gamma \neq 0. \end{aligned}$$

Under the assumption that $\varepsilon_i$, $x_i$, and $w_i$ are jointly i.i.d. and $\varepsilon_i$ is independent of $w_i$ with $E(\varepsilon_i^2) = \sigma^2 > 0$ and $E(z_{i2}^2) = E(w_i^2) \equiv \tau^2 > 0$, derive the asymptotic distribution of the IV estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ which use either $z_{i1} = 1$ or $z_{i2} = w_i$, respectively, as an instrument for $x_i$, and compare the asymptotic variances of these two estimators. For what parameter values will $\hat{\beta}_1$ be more efficient than $\hat{\beta}_2$, and vice versa?