**Problem Set #1**

ECONOMICS 240B
SPRING 2010

Due February 10

Turn in (correct) answers to the following exercises from *An Introduction to Classical Econometric Theory*, by Paul A. Ruud:

Chapter 13: Exercises 13.5, 13.6, 13.9, 13.11, 13.12.
Chapter 25: Exercises 25.11, 25.13.

**Additional Questions:**

1. Suppose the coefficients $\beta = (\beta_1, \beta_2)'$ in the linear model $y = \mathbf{X}\beta + \varepsilon$ are estimated by classical least squares, where it is assumed that the errors $\varepsilon$ are independent of the matrix $\mathbf{X}$ of regressors with scalar covariance matrix $\mathbf{V}(\varepsilon) = \mathbf{V}(\varepsilon|\mathbf{X}) = \boldsymbol{\sigma}^2\mathbf{I}$. An analysis of $N = 347$ obervations yields

$$\widehat{\beta} = \begin{pmatrix} 0.25 \\ -0.25 \end{pmatrix}, \qquad s^2 = 0.1, \qquad \mathbf{X}'\mathbf{X} = \begin{bmatrix} 40 & 10 \\ 10 & 5 \end{bmatrix}.$$

Construct an approximate 95% confidence interval for $\gamma \equiv \beta_1/\beta_2$, under the (possibly heroic) assumption that the sample size is large enough for the usual limit theorems and linear approximations to be applicable. Is $\gamma_0 = 0$ in this interval?

2. Consider the linear regression model

$$y = \beta x + \alpha z + u,$$

where $\beta$ and $\alpha$ are unknown scalar parameters, $x$ and $z$ are $n$-dimensional vectors of (jointly) i.i.d. random variables, and $u$ is an $n$-dimensional vector of unobservable i.i.d. random variables ("error terms") with zero mean and unit variance which is independent of $x$ and $z$. Suppose we are given a preliminary estimator $\hat{\alpha}$ of $\alpha$ that is independent of $u$ and has the asymptotic distribution

$$\sqrt{n}(\hat{\alpha} - \alpha) \xrightarrow{d} \mathcal{N}(0, 1).$$

Define a "second stage" estimator $\tilde{\beta}$ of $\beta$ as

$$\tilde{\beta} \equiv (x'x)^{-1}x'(y - \hat{\alpha}z).$$

Assuming *plim* $n^{-1}x'x \equiv c \neq 0$ and *plim* $n^{-1}x'z \equiv d \neq 0$ exist, and all random variables have finite fourth moments, obtain the asymptotic distribution of $\tilde{\beta}$.