Problem Set #4

ECONOMICS 240B
SPRING 2010

Due April 14

Turn in (correct) answers to the following exercises from Ruud's text:

Chapter 14: Exercises 14.4, 14.9
Chapter 16: Exercises 16.4, 16.8
Chapter 17: Exercise 17.9

Extra Theoretical Question: A random variable U is said to have a *Pareto* distribution with parameter λ, denoted $U \sim Pareto(\lambda)$, if it is continuously distributed on the interval $(1, \infty)$ with density

$$f(u; \lambda) = \lambda \cdot u^{-(\lambda+1)}.$$

Suppose you have a random sample $\{(y_i, x_i')\}_{i=1}^n$ where the conditional distribution of y_i given the vector x_i is Pareto with parameter $\exp\{x_i'\beta_0\}$, i.e.,

$$y_i|x_i \sim Pareto(\exp\{x_i'\beta_0\}).$$

Also, suppose the marginal distribution of the K-dimensional regressors x_i is unspecified and, as usual, β_0 is unknown.

(i) Derive the average log-likelihood function $L(\beta)$ for this problem, and show that the first-order condition for the MLE $\hat{\beta}$ can be rewritten in the form

$$0 = \frac{1}{n}\sum_{i=1}^{n} u_i(\hat{\beta}) \cdot x_i$$

for some "pseudo-residual" function $u_i(\beta)$ which satisfies $E[u_i(\beta_0)|x_i] = 0$.

(ii) Derive an expression for the asymptotic distribution of the ML estimator $\hat{\beta}$, including an explicit expression for its asymptotic covariance matrix, and give a consistent estimator for that matrix. Also, assuming $K = 1$ (that is, β is a scalar), give an expression for an approximate 95% confidence interval for β_0.

(iii) For general K, use the ML $\hat{\beta}$ to estimate the probability that $y_i > y_0$ conditional on $x_i = x_0$, for some fixed values of y_0 (in the interval $(1, \infty)$) and x_0, and derive the large-sample distribution of this estimator and an estimator of its asymptotic variance.

(iv) Derive the algebraic form of the Wald, likelihood ratio, and score ("LM") tests of the null hypothesis $H_0 : \beta_0 = 0$, and describe the critical region for the test.

(v) Now, assuming the first component of the regressors is a constant, $x_{i1} \equiv 1$, and the true "slope coefficients" on the remaining regressors are denoted $\beta_0^{(2)}$, derive the Wald, LR, and score tests and critical regions for the null hypothesis $H_0 : \beta_0^{(2)} = 0$.