Final Midterm Exam

Economics 241A
Spring 2004

May 10, 2004

Instructions: This is a 30 point exam, with weights given for each question; all subsections of each question have equal weight. The answers must be turned in no later than 25 hours after you pick up the exams, to Jim Powell (669 Evans). You may consult and cite any lecture notes and any of the references on the syllabus; you may not cite any other outside source, and under no circumstances should you discuss the exam with anyone other than the instructor before you submit your answers. Please make your answers elegant – that is, clear, concise, and, above all, correct.

1. (10 points) Suppose a scalar dependent variable y_{ij} for n_j individuals in J groups ($i = 1, ..., n_j$ and $j = 1, ..., J$) is assumed to satisfy a linear model

$$y_{ij} = x_j'\beta_0 + \varepsilon_{ij}$$

for some group-specific regressors x_j with error terms ε_{ij} that are independent across i and j and satisfy a conditional quantile restriction

$$\Pr\{\varepsilon_{ij} < 0|x_j\} = \pi \qquad ((*))$$

for some π between zero and one. The ε_{ij} are assumed to be continuously distributed conditional on x_j, with conditional densities that are strictly positive everywhere (with probability one).

Define the π^{th} sample quantile $\hat{q}_j$ of y_{ij} for the j^{th} group as

$$\hat{q}_j = \arg\min_c \sum_{i=1}^{n_j} |\pi - 1\{y_{ij} < c\}| \cdot |y_{ij} - c|, \qquad j = 1, ..., J.$$

Under the assumption that $N = \sum_j n_j \to \infty$ with $\lim(n_j/N) \equiv p_j > 0$ for all j, find the form of the optimal weights for a weighted least-squares regression of $\hat{q}_j$ on x_j. These weights should be "optimal" in the sense that they minimize the asymptotic covariance matrix of the resulting estimator, which you should derive explicitly using the well-known form of the asymptotic distribution of the sample quantile $\hat{q}_j$. You should also show that this estimator achieves the relevant efficiency bound for the quantile restriction defined by (*).

In addition, propose a "feasible" version of this efficient estimator (using consistent estimators of the optimal weights). Finally, calculate the probability limit of the weighted least-squares estimator of β_0 when the linear regression function is misspecified – i.e., when

$$y_{ij} = g(x_j) + \varepsilon_{ij}$$

with $g(x)$ being nonlinear in x – and discuss the asymptotic behavior of the feasible estimator under this misspecification.

2. (20 points) For the censored regression model with a single (scalar) regressor,

$$y_i = \max\{0, x_i \cdot \beta_0 + u_i\}, \qquad i = 1, ..., N,$$

suppose that the error terms u_i are symmetrically distributed about zero conditionally, not on x_i, but on some q-dimensional vector of "instrumental variables" z_i. The regressors x_i are assumed to be related to the instruments z_i by a linear reduced form:

$$x_i = z_i'\pi_0 + v_i,$$

where the error terms u_i and v_i are jointly continuous and symmetrically distributed given z_i – more precisely, for any fixed numbers α and λ, the linear combination $\alpha u_i + \lambda v_i$ is symmetric about zero given z_i.

A. Consider the following two-stage procedure:first, estimate π_0 by least squares, then estimate β_0 by symmetrically-censored least squares (SCLS) estimation, after replacing the "endogenous" regressors x_i by their fitted values $\hat{x}_i \equiv z_i'\hat{\pi}$. Thus, the second-stage estimator $\hat{\beta}$ will be the (consistent) solution to the equation

$$\begin{aligned} 0 &= \frac{1}{N}\sum_{i=1}^{n} 1\{\hat{x}_i \cdot \hat{\beta} > 0\} \cdot \min\{y_i - \hat{x}_i \cdot \hat{\beta}, \hat{x}_i \cdot \hat{\beta}\} \cdot \hat{x}_i \\ &\equiv \frac{1}{N}\sum_{i=1}^{n} \psi(y_i, z_i, \hat{\pi}, \hat{\beta}), \end{aligned}$$

where

$$\hat{\pi} \equiv \left[\frac{1}{N}\sum_{i=1}^{n} z_i z_i'\right]^{-1}\left[\frac{1}{N}\sum_{i=1}^{n} z_i x_i'\right].$$

Assuming this estimator is consistent, and assuming i.i.d. sampling, all needed moments exist, etc., derive the asymptotic distribution of the second-stage estimator $\hat{\beta}$. (Don't check regularity conditions, stochastic equicontinuity, etc. – just do the calculations.)

B. Suppose instead that the reduced form for x_i' was substituted into the model for the dependent variable y_i, and the reduced-form parameter $\delta_0 \equiv \pi_0\beta_0$ for the resulting censored regression model for y_i and z_i was estimated using SCLS. Given the SCLS estimator $\hat{\delta}$ and the least-squares estimator $\hat{\pi}$ from the first stage, propose an efficient way to combine these two estimators to obtain an estimator of β_0, and derive its asymptotic distribution. Discuss the sense in which this estimator is efficient. (As above, don't bother listing or verifying regularity conditions.)