Final Midterm Exam

ECONOMICS 241A
SPRING 2005

May 9, 2005

Instructions: This is a 30 point exam, with weights given for each question; all subsections of each question have equal weight. The answers must be turned in no later than 25 hours after you pick up the exams, to Jim Powell (669 Evans). You may consult and cite any lecture notes and any of the references on the syllabus; you may not cite any other outside source, and under no circumstances should you discuss the exam with anyone other than the instructor before you submit your answers. Please make your answers elegant – that is, clear, concise, and, above all, correct.

1. (10 points) Consider the linear model

$$y_i = x_i' \beta_0 + \varepsilon_i,$$

and suppose that the unobservable error term ε_i satisfies *both* a conditional mean restriction

$$E[\varepsilon_i | x_i] = 0$$

and a conditional median restriction

$$E[sgn(\varepsilon_i) | x_i] = 0.$$

Assuming that ε_i is continuously distributed conditional on x_i, with a conditional density $f_{\varepsilon|x}(\varepsilon|x_i)$ that has lots of derivatives and moments, derive an (infeasible) efficient estimator of β_0 under these two restrictions, and give an expression for the form of its asymptotic covariance matrix. (Assume the relevant stochastic equicontinuity condition holds, so that the order of differentiation and expectation can be interchanged if necessary.)

2. (10 points) Consider the nonparametric regression model

$$y_t = g(x_t) + \varepsilon_t, \qquad t = 1, ..., T,$$

where x_t and y_t are scalar, jointly-continuous random variables with finite variances, joint density function $f_{x,y}(x, y)$, marginals $f_x(x)$ and $f_y(y)$, and with $E[\varepsilon_t | x_t] \equiv 0$ (that is, $g(x_t) \equiv E[y_t | x_t]$). An estimator for the value of $g(x)$ at a fixed value $x = x_0$ is the uniform kernel regression estimator

$$\widehat{g}(x_0) \equiv \left[\frac{1}{T} \sum_{t=1}^{T} w_{tT} \cdot y_t \right] \cdot \left[\frac{1}{T} \sum_{t=1}^{T} w_{tT} \right]^{-1},$$

where the "local weight" w_{tT} takes the form

$$w_{tT} \equiv \frac{1}{h_T} \cdot 1\{|x_t - x_0| \leq \frac{h_T}{2}\}$$

and $\{h_T\}$ is a nonrandom sequence of bandwidths. Assume that

1. **i.** the functions $g(x)$ and the marginal density $f_x(x)$ of x_t have lots of continuous derivatives at $x = x_0$ (as many as needed);

   ii. ε_t and x_s are statistically independent for all t and s;

   iii. x_t is an i.i.d. sequence with $f_x(x_0) > 0$; and

   iv. ε_t is a (weakly) stationary process with autocovariance sequence $\gamma_\varepsilon(s)$ that is absolutely summable, i.e.

$$\sum_{s=0}^{\infty} |\gamma_\varepsilon(s)| < \infty.$$

   (a) Consider the numerator of $\widehat{g}(x_0)$,

$$\hat{n}(x_0) \equiv \widehat{g}(x_0) \cdot \widehat{f}_x(x_0) \equiv \frac{1}{T}\sum_{t=1}^{T} w_{tT} \cdot y_t,$$

   where$\widehat{f}_x(x_0)$ is the kernel density estimator of $f_x(x_0)$,

$$\widehat{f}_x(x_0) \equiv \frac{1}{T}\sum_{t=1}^{T} w_{tT}.$$

   Give an expression for the variance of the numerator term $\widehat{n}(x_0)$, and show that, as $h_T \to 0$, the leading (largest) term in the expansion of the variance in powers of h_T does not depend upon the autocovariances $\gamma_\varepsilon(s)$ for $s \neq 0$.

   (b) Find conditions on the bandwidth sequence h_T under which $\hat{g}(x_0)$ is weakly consistent. Try to make your assumptions as weak (general) as possible.

3. (15 points) Suppose that economic theory suggests that a latent dependent variable y_i^* satisfies a classical linear model

$$y_i^* = x_i'\beta_0 + \varepsilon_i,$$

but that you do not observe y_i^* over its entire range. Instead, you observe a random sample of size n of y_i and x_i, where

$$\begin{aligned} y_i &\equiv \tau_i(y_i^*) \\ &= 0 \quad if \quad y_i^* \leq 0, \\ &= y_i^* \quad if \quad 0 < y_i^* \leq L_i, \\ &= L_i \quad if \quad L_i < y_i^* \leq U_i, \text{ and} \\ &= y_i^* - (U_i - L_i) \quad if \quad U_i < y_i^*. \end{aligned}$$

That is, the latent variable y_i^* is observed unless it is less than zero or in the interval (L_i, U_i), where the threshold variables L_i and $U_i > L_i > 0$ are assumed known for all i.

A. Assuming that ε_i is normally distributed with zero mean and unknown variance σ_0^2, and is independent of x_i, derive the form of the average log-likelihood function for the unknown parameters of this problem and the form of the asymptotic distribution of the corresponding maximum likelihood estimator.

B. Suppose that the parametric form of the error distribution is unknown. Find a $\sqrt{n}$-consistent estimator of β_0, imposing a suitable stochastic restriction on the conditional distribution of ε_i given x_i, and *without* imposing a scale normalization on β_0. If possible, give an expression for the asymptotic distribution of your estimator.

C. Now suppose that y_i^* is never observed, but only the range that it falls into is observed. More specifically, the dependent variable y_i is now defined as

$$\begin{aligned} y_i &\equiv t_i(y_i^*) \\ &= 0 \quad \textit{if} \quad y_i^* \leq 0, \\ &= 1 \quad \textit{if} \quad 0 < y_i^* \leq L_i, \\ &= 2 \quad \textit{if} \quad L_i < y_i^* \leq U_i, \text{ and} \\ &= 3 \quad \textit{if} \quad U_i < y_i^*. \end{aligned}$$

Describe an alternative consistent estimator of β_0 under a semiparametric restriction on the conditional distribution of the errors given the regressors. Is a scale normalization on β_0 needed, or are all the components of β_0 (including the scale) identifiable under your restriction?