Second Midterm Exam

Economics 241A

Spring 2009

Instructions: This is a 30 point exam, with given weights for each question; all subsections have equal weight. No books, notes, tables, or calculating devices are permitted. You have one hour and 20 minutes to answer the following two questions.. Please make your answers elegant – that is, clear, concise, and, above all, correct.

1. (15 points) Consider the linear model

$$y_i = \beta_0 + \varepsilon_i,$$

and suppose that the unobservable error term ε_i satisfies both a zero mean restriction

$$E[\varepsilon_i] = 0$$

and a conditional median restriction

$$E[sgn(\varepsilon_i)] = 0.$$

Assume that ε_i is continuously distributed, with a density $f(\varepsilon)$ that has lots of derivatives and moments.

(a) Suppose you are given the sample mean $\bar{y}_N$ and sample median $\tilde{y}_N \equiv y_{[\frac{N+1}{2}]}$ for these data, along with a consistent estimator $\hat{V}$ of their joint asymptotic covariance matrix. How could you use these statistics to construct a more efficient estimator of β_0?

(b) Given a random sample of size N from this model, derive the asymptotic variance of the (infeasible) efficient GMM estimator of β_0 under these two restrictions. (Assume the relevant stochastic equicontinuity condition holds, so that the order of differentiation and expectation can be interchanged in the calculations.)

(c) Does the estimator you proposed in (a) attain the GMM efficiency bound you derived in (b)? [Hint: you can justify your answer with explicit derivations or a general argument.]

2. (15 points) Suppose you have a sample from a censored regression model with both sample selectivity and endogenous regressors:

$$\begin{aligned} d_i &= 1\{w_i'\delta_0 + u_i > 0\}, \\ x_i &= z_i'\pi_0 + v_i, \qquad \text{and} \\ y_i &= d_i \cdot \max\{0, x_i'\beta_0 + \varepsilon_i\}, \end{aligned}$$

where d_i, y_i, x_i, w_i, and z_i are observable random variables and vectors. It is assumed that the unobservable errors u_i, v_i, ε_i are jointly independent of the exogenous variables w_i and z_i with a smooth but unknown joint density function with lots of moments; the exogenous variables w_i and z_i are also jointly continuously distributed with well-behaved densities and moments.

Discuss identification of the unknown parameters (a) δ_0, (b) π_0, and (c) β_0 under the assumption that the joint distribution of w_i and z_i and the functions

$$\begin{aligned} p(w_i) &= E[d_i|w_i] \\ g(z_i) &= E[x_i|z_i], \qquad \text{and} \\ m(x_i, w_i, z_i, d_i) &= median[y_i|x_i, w_i, z_i, d_i] \end{aligned}$$

are known for all w_i and z_i. Be explicit about any normalizations you need to impose. [Hint: for part (c), consider the value of $m(\cdot)$ when $d_i = 1$.]