A Parametric Estimation Method for Dynamic Factor Models of Large Dimensions*

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Abstract

The estimation of dynamic factor models for large sets of variables has attracted considerable attention recently, due to the increased availability of large datasets. In this paper we propose a new parametric methodology for estimating factors from large datasets based on state space models, discuss its theoretical properties and compare its performance with that of two alternative non-parametric estimation approaches based, respectively, on static and dynamic principal components. The new method appears to perform best in recovering the factors in a set of simulation experiments, with static principal components a close second best. Dynamic principal components appear to yield the best fit, but sometimes there are leakages across the common and idiosyncratic components of the series. A similar pattern emerges in an empirical application with a large dataset of US macroeconomic time series.

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1 Introduction

Recent work in the macroeconometric literature considers the problem of summarising efficiently a large set of variables and using this summary for a variety of purposes including forecasting. Work in this field has been carried out in a series of recent papers by Stock and Watson (2001, 2002) (SW) and Forni, Lippi, Hallin and Reichlin (1999,2000) (FHLR). Factor analysis has been the main tool used in summarising the large datasets.

The static version of the factor model was analyzed, among others, by Chamberlain and Rothschild (1983), Connor and Korajczyk (1986, 1993). Geweke (1977) and Sargent and Sims (1977) studied a dynamic factor model for a limited number of series. Further developments were due to Stock and Watson (1989, 1991), Quah and Sargent (1993) and Camba-Mendez et al (2001), but all these methods are not suited when the number of variables is very large due to the computational cost, even when a sophisticated EM algorithm is used for optimization, as in Quah and Sargent (1993).

For this reason, SW have suggested a non-parametric principal component based estimation approach in the time domain, and shown that principal components can estimate consistently the factor space asymptotically. FHLR have developed an alternative non-parametric procedure in the frequency domain, based on dynamic principal components (see Chapter 9 of Brillinger (1981)), that incorporates an explicitly dynamic element in the construction of the factors.

In this paper we suggest a third approach for factor estimation that retains the attractive framework of a parametric state space model but is computationally feasible for very large datasets because it does not use maximum likelihood but linear algebra methods, based on subspace algorithms used extensively in engineering, to estimate the state. To the best of our knowledge, this is the first time that these algorithms are used for factor estimation.

We analyze the asymptotic properties of the new estimators, first for a fixed number of series, N, and then allowing N to diverge. We show that as long as N grows less than $T^{1/3}$, where T is the number of observations, the subspace algorithm still yields consistent estimators for the space spanned by the factors. Moreover, we suggest a modified subspace algorithm that permits to analyze datasets with N larger than T, i.e., more series than observations, and evaluate its performance using Monte Carlo simulations. Finally, we develop an information criterion that leads to consistent selection of the number of factors to be included in the model, along the lines of Bai and Ng (2002) for the static principal component approach.

Our second contribution is an extensive simulation study of the relative performance of the three competing estimation methods. We evaluate the relationship between the true factors and their estimated counterparts, and we further examine the properties of the resulting idiosyncratic component of the data. We find that our state space based method performs better in a variety of experiments compared to the principal component based methods, also when $N > T$, with the static principle component estimator ranked second. Though these findings may depend on the experimental designs, they appear to be rather robust. In this paper we only report a subset of the results in order to save space, but many more are available upon request.

Our final contribution is the analysis of a large dataset of 146 US macroeconomic time series, the balanced panel used by SW. As in the simulation experiments, it turns out that the performance of static principal components and state space methods is overall comparable. Moreover, when the state space based factors are included in small scale monetary VARs, more reasonable responses of output gap and inflation to interest rate shocks are obtained.

The paper is organised as follows. Section 2 presents the state space model approach and derives the properties of the estimators for the fixed N case. Section 3 deals with the diverging N case, with correlation of the idiosyncratic components, and with a modified algorithm to analyze datasets with $N > T$. Section 4 compares the competing estimation methods using an extensive set of Monte Carlo simulations. Section 5 discusses the empirical example. Section 6 summarizes and concludes.

2 The state space factor estimator

In this section we present and discuss the basic state space representation for the factor model, discuss the subspace estimators, and derive their asymptotic properties when T diverges and N is fixed. In the following section we extend the framework to deal with the N going to infinity case, with the analysis of datasets with a larger cross-section than time-series dimension, and with cross-sectionally or serially correlated idiosyncratic errors.

2.1 The basic state space model

Following Deistler and Hannan (1988), we consider the following state space model.

$$x_{Nt} = Cf_t + D^* \epsilon_t, \quad t = 1, \ldots, T \tag{1}$$
$$f_t = Af_{t-1} + B^* v_{t-1},$$

where x_{Nt} is an N-dimensional vector of stationary zero-mean variables observed at time t, f_t is a k-dimensional vector of unobserved states (factors) at time t, and ϵ_t and v_t are multivariate, mutually uncorrelated, standard orthogonal white noise sequences of dimension, respectively, N and k. D^* is assumed to be nonsingular. The aim of the analysis is to obtain estimates of the states f_t, for $t = 1, \ldots, T$. We make the following assumption

Assumption 1 *(a) $|\lambda_{\max}(A)| < 1$ and $|\lambda_{\min}(A)| > 0$ where $|\lambda_{\max}(.)|$ and $|\lambda_{\min}(.)|$ denote, respectively, the maximum and minimum eigenvalue of a matrix in absolute value.*

(b) The elements of C are bounded

The first part of assumption 1-(a), combined with assumption 1-(b) ensures that x_{Nt} is stationary. The second part of assumption 1-(a) implies that each factor is correlated over time, which is important to distinguish it from the idiosyncratic white noise error terms. Notice also that the factors are driven by lagged errors, an important hypothesis for the methodology developed in this paper, as we will discuss below.

This model is quite general. Its aim is to use the states as a summary of the information available from the past on the future evolution of the system. To illustrate its generality we give an example where a factor model with factor lags in the measurment equation can be recast in the above form indicating the ability of the model to model dynamic relationships between x_{Nt} and f_t. Define the original model to be

$$\begin{aligned} x_{Nt} &= C_1 f_t + C_2 f_{t-1} + D^* \epsilon_t, \quad t = 1, \ldots, T \\ f_t &= A f_{t-1} + B^* v_{t-1}, \end{aligned} \tag{2}$$

This model can be written as

$$\begin{aligned} x_{Nt} &= (C_1, C_2)\tilde{f}_t + D^* \epsilon_t, \quad t = 1, \ldots, T \\ \tilde{f}_t &= \begin{pmatrix} f_t \\ f_{t-1} \end{pmatrix} = \begin{pmatrix} A & 0 \\ I & 0 \end{pmatrix} \begin{pmatrix} f_{t-1} \\ f_{t-2} \end{pmatrix} + \begin{pmatrix} B^* & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_{t-1} \\ 0 \end{pmatrix}, \end{aligned} \tag{3}$$

which is a special case of the specification in (1), even though by not taking into account the particular structure of the A matrix and the reduced rank of the error process we are losing in terms of efficiency.[1]

A large literature exists on the identification issues related with the state space representation given in (1). An extensive discussion may be found in Deistler and Hannan (1988). In particular, they show in Chapter 1 that (1) is equivalent to the prediction error representation of the state space model given by

$$\begin{aligned} x_{Nt} &= C f_t + D u_t, \quad t = 1, \ldots, T \\ f_t &= A f_{t-1} + B u_{t-1}. \end{aligned} \tag{4}$$

[1]These restrictions can be imposed but we prefer to work with the general unrestricted formulation and evaluate the loss of efficiency through Monte Carlo simulations, since in practice the exact parametric structure of the model is not known.

where u_t is an orthogonal white noise process. This form will be used for the derivation of our estimation algorithm. Note that as at this stage the number of series, N, is large but fixed we need to impose no conditions on the structure of C. Conditions on this matrix will be discussed later when we consider the case of N tending to infinity and possibly correlated idiosyncratic errors.

2.2 Subspace Estimators

As we have mentioned in the introduction, maximum likelihood techniques, possibly using the Kalman filter, may be used to estimate the parameters of the model under some identification scheme. Yet, for large datasets this is very computationally intensive. Quah and Sargent (1993) developed an EM algorithm that allows to consider up to 50-60 variables, but it is still so time-consuming that it is not feasible to evaluate its performance in a simulation experiment.

To address this issue, we exploit subspace algorithms, which avoid expensive iterative techniques by relying on matrix algebraic methods, and can be used to provide estimates for the factors as well as the parameters of the state space representation.

There are many subspace algorithms, and vary in many respects, but a unifying characteristic is their view of the state as the interface between the past and the future in the sense that the best linear prediction of the future of the observed series is a linear function of the state. A review of existing subspace algorithms is given by Bauer (1998) in an econometric context. Another review with an engineering perspective may be found in

Van Overschee and De Moor (1996). To the best of our knowledge, our paper is the first application of subspace algorithms for factor estimation.

The starting point of most subspace algorithms is the following representation of the system which follows from the state space representation in (4) and the assumed nonsingularity of D.

$$X_t^f = \mathcal{O}\mathcal{K}X_t^p + \mathcal{E}E_t^f \tag{5}$$

where $X_t^f = (x'_{Nt}, x'_{Nt+1}, x'_{Nt+2}, \ldots)'$, $X_t^p = (x'_{Nt-1}, x'_{Nt-2}, \ldots)'$, $E_t^f = (u'_t, u'_{t+1}, \ldots)'$, $\mathcal{O} = [C', A'C', (A^2)'C', \ldots]'$, $\mathcal{K} = [\bar{B}, (A - \bar{B}C)\bar{B}, (A - \bar{B}C)^2\bar{B}, \ldots]$, $\bar{B} = BD^{-1}$ and

$$\mathcal{E} = \begin{pmatrix} D & 0 & \ldots & 0 \\ CB & D & \ddots & \vdots \\ CAB & \ddots & \ddots & 0 \\ \vdots & & CB & D \end{pmatrix}.$$

The derivation of this representation is simple once we note that (i) $X_t^f = \mathcal{O}f_t + \mathcal{E}E_t^f$ and (ii) $f_t = \mathcal{K}X_t^p$. The best linear predictor of the future of the series at time t is given by $\mathcal{O}\mathcal{K}X_t^p$. The state is given in this context by $\mathcal{K}X_t^p$ at time t. The task is therefore to provide an estimate for $\mathcal{K}$.

The above representation involves infinite dimensional vectors. In practice, truncation is used to end up with finite sample approximations given by $X_{s,t}^f = (x'_{Nt}, x'_{Nt+1}, x'_{Nt+2}, \ldots, x'_{Nt+s-1})'$ and $X_{p,t}^p = (x'_{Nt-1}, x'_{Nt-2}, \ldots, x'_{Nt-p})'$. Then an estimate of $\mathcal{F} = \mathcal{O}\mathcal{K}$ may be obtained by regressing $X_{s,t}^f$ on $X_{p,t}^p$. Following that, the most popular subspace algorithms use a singular value decomposition (SVD) of an appropriately weighted version of the least squares estimate of $\mathcal{F}$, denoted by $\hat{\mathcal{F}}$. In particular the algorithm we will use, due to Larimore (1983), applies an SVD to $\hat{\Gamma}^f\hat{\mathcal{F}}\hat{\Gamma}^p$, where $\hat{\Gamma}^f$ and $\hat{\Gamma}^p$ are the sample covariances of $X^f_{s,t}$ and $X^p_{p,t}$ respectively. These weights are used to determine the importance of certain directions in $\hat{\mathcal{F}}$. Then, the estimate of $\mathcal{K}$ is given by

$$\hat{\mathcal{K}} = \hat{S}_k^{1/2}\hat{V}_k'\hat{\Gamma}^{p^{-1/2}}$$

where $\hat{U}\hat{S}\hat{V}'$ represents the SVD of $\hat{\Gamma}^{f^{-1/2}}\hat{\mathcal{F}}\hat{\Gamma}^{p^{1/2}}$, $\hat{V}_k$ denotes the matrix containing the first k columns of $\hat{V}$ and $\hat{S}_k$ denotes the heading $k \times k$ submatrix of $\hat{S}$. $\hat{S}$ contains the singular values of $\hat{\Gamma}^{f^{-1/2}}\hat{\mathcal{F}}\hat{\Gamma}^{p^{1/2}}$ in decreasing order. Then, the factor estimates are given by $\hat{\mathcal{K}}X^p_t$. We refer to this method as SSS.

For what follows it is important to note that the choice of the weighting matrices $\hat{\Gamma}^f$ and $\hat{\Gamma}^p$ is important but not crucial for the asymptotic properties of the estimation method. This is because the choice does not affect neither the consistency nor the rate of convergence of the factor estimator. For these properties, the weighting matrices are only required to be nonsingular. Therefore, for the sake of simplicity, in the theoretical analysis and in the Monte Carlo study,

Assumption 2 *We set* $\hat{\Gamma}^f = I_{sN}$ *and* $\hat{\Gamma}^p = I_{pN}$

A second point to note is that consistent estimation of the factor space requires the "lag" truncation parameter p to increase at a rate greater than $ln(T)^{\alpha}$, for some $\alpha > 1$ that depends on the maximum eigenvalue of A, but at a rate lower than $T^{1/3}$. A simplified condition for p is to set it to $T^{1/r}$ for any $r > 3$.

For consistency, the "lead" truncation parameter s is also required to be set so as to satisfy $sN \geq k$. As N is usually going to be very large for the applications we have in mind, this restriction is not binding and we can use $s = 1$. This is relevant in particular in a forecasting context because with $s = 1$ only contemporaneous and lagged values of the variables are used for factor estimation. Yet, it turns out that s in an important parameter in determining the small sample performance of the subspace estimator. Therefore, we will consider its choice in the Monte Carlo experiments in Section 4.

Once estimates of the factors have been obtained, if estimates of the parameters of the model (including the factor loadings) are subsequently required, least squares methods may be used with the estimated factors instead of the true ones. The resulting estimates have been proved to be $\sqrt{T}$-consistent and asymptotically normal in Bauer (1998). We note that the identification scheme underlying the above estimators of the parameters is implicit, and depends on the normalisation used in the computation of the SVD. In particular, the SVD used in the Monte Carlo simulations in Section 4 normalises the left and right singular value vectors by restricting them to have an identity second moment matrix.

It is worth pointing out that the estimated parameters can be used with the Kalman filter on the state space model to obtain both filtered and smoothed estimates of the factors. Since the SSS method produces factor estimates at time t conditional on data available at time $t-1$, it may be possible that smoothed estimates from the Kalman filter are superior to those obtained by the SSS method. However, the parameter estimates are conditional on the factor estimates obtained in the first step by the SSS method. Limited experimentation using the Monte Carlo setup reported below suggests that the loss in performance of the smoothed Kalman filter factor estimate because of the use of estimated factors from the SSS method, is roughly similar to the benefit of using all the data. Moreover, in general, factors estimated using the SSS method outperform filtered Kalman filter factor estimates.

Finally, we must note that the SSS method is also applicable in the case of unbalanced panels. In analogy to the work of SW, use of the EM algorithm, described there, can be made to provide estimates both of the factors and of the missing elements in the dataset.

2.3 Asymptotic properties

We now discuss the asymptotic properties of the SSS factor estimators and derive their standard errors.

Let us denote the true number of factors by k^0 and investigate in more detail OLS estimation of the multivariate regression model

$$X_{s,t}^f = \mathcal{F}X_{p,t}^p + \mathcal{E}E_{s,t}^f \tag{6}$$

where $E_t^f = (u_t', u_{t+1}', \dots, u_{t+s}')'$. Estimation of the above is equivalent to estimation of each equation separately. We make the following assumptions

Assumption 3 *u_t is an i.i.d. $(0, \Sigma_u)$ sequence with finite fourth moments.*

Assumption 4 *$p_1 \leq p \leq p_2$ where $p_1 = O(T^{1/r}), r > 3$ and $p_2 = o(T^{1/3})$*

Denote $X^p = (X_{p,1}^p, ..., X_{p,T}^p)'$. Then we have the following theorem:

Theorem 1 ***(Consistency).*** *If we define $\hat{f}_t = \hat{\mathcal{K}}X_{p,t}^p$, then, under assumptions 1-4, $\hat{f}_t$ converges, in probability, to the space spanned by the true factors.*

Proof. By (4) and (5) we can see that $\mathcal{K}X_{p,t}^{p}$ spans the space of the true factors. So we need to concentrate on the properties of $\hat{\mathcal{K}}$ as an estimator of $\mathcal{K}$. By Theorem 4 of Berk (1974), who provides a variety of results for parameter estimates in infinite autoregressions, we have that $\hat{\mathcal{F}}$ is consistent for $\mathcal{F}$ and that $\sqrt{T-Np}(\hat{\mathcal{F}}-\mathcal{F})$ has an asymptotic normal distribution with the standard OLS covariance matrix. This result follows straightforwardly from equation (2.17) of Berk (1974) once we note that the sum of the absolute values of the coefficients in each regression multiplied by $p^{1/2}$ tends to zero. This follows by the fact that the absolute value of the maximum eigenvalue of $\mathcal{F}=\mathcal{OK}$, denoted $|\lambda_{\max}(\mathcal{F})|$, is less than one implying exponentially declining coefficients with respect to p. This implies consistent estimation of the factors since $\hat{\mathcal{K}}$ is a continuous function of $\hat{\mathcal{F}}$ for large enough T. Since both T and p grow, by assumption 3 the rate of convergence of the factor estimates lies between $(T-Np)^{1/2-1/2r}$ and $(T-Np)^{1/3}$. This is because the factor is a linear combination of the elements of $\hat{\mathcal{K}}$. This rate of convergence follows if we note that the supremum norm of $E(X^{p\prime}X^{p}/T)^{-1}$ is of order p which follows from the absolute summability of the autocovariances of x_{Nt}. We will denote the square of the rate of convergence by T^{*}. ∎

It is important to mention that consistency is possible because in the model (1) the factors depend on lagged errors. Without this assumption, i.e., if f_t depends on v_t rathen than on v_{t-1}, the SSS estimator would be consistent for Af_{t-1} but not for the space spanned by f_t. The extent of the inconsistencty is evaluated in the Monte Carlo experiments in Section 4, and found to be minor.

Besides proving consistency, we have the following theorem on the asymptotic distribution of the factor estimator.

Theorem 2 ***(Asymptotic distribution).*** *Under assumptions 1-4, the asymptotic distribution of* $\sqrt{T^*}(vec(\hat{f}) - vec(H^k f))$ *with* $f = (f_1, \ldots, f_T)'$ *is* $N(0, V_f)$*, with*

$$V_f = E\left((I_{T-Np} \otimes X^p)\frac{\partial g}{\partial(A_1\mathcal{F}A_2)}(A_2' \otimes A_1)(\Gamma^{p^{-1}} \otimes \Sigma)(A_2 \otimes A_1')\frac{\partial g'}{\partial(A_1\mathcal{F}A_2)}(I_{T-Np} \otimes X^{p'})\right)$$

for $s = 1$ *and*

$$V_f = E\left((I_{T-Np} \otimes X^p)\frac{\partial g}{\partial(A_1\mathcal{F}A_2)}(A_2' \otimes A_1)\Phi(A_2 \otimes A_1')\frac{\partial g'}{\partial(A_1\mathcal{F}A_2)}(I_{T-Np} \otimes X^{p'})\right)$$

for $s > 1$ *where* H^k *is a square matrix of full rank and* Φ, g, A_1, A_2 *are defined in the proof of the Theorem.*

Proof. Asymptotic normality of the estimators follows from asymptotic normality of $\hat{\mathcal{K}}$ which follows from the asymptotic normality of $\sqrt{T-Np}(\hat{\mathcal{F}} - \mathcal{F})$ proved in Theorem 4 of Berk (1974). The normality of $\hat{\mathcal{K}}$ follows by using a simple Taylor expansion of the function implicitly defined by the SVD of $\hat{\mathcal{F}}$. Denote this function by g. The existence of the Taylor expansion follows from continuity and differentiability of g which follows from Theorems 5.6 and 5.8 of Chatelin (1983). The variance calculations will be carried out conditional on X_t^p, as when obtaining variances of regression coefficients conditional on the regressors. From $f = X^p\hat{\mathcal{K}}'$, simple manipulations indicate that

$$V\left(\sqrt{T^*}(vec(\hat{f}) - vec(H^k f))\right) = (I_{T-Np} \otimes X^p)V\left(\sqrt{T^*}\left(vec(\hat{\mathcal{K}}') - vec(\mathcal{K}')\right)\right)(I_{T-Np} \otimes X^{p'})$$

We need to derive the asymptotic variance of $V\left(\sqrt{T^*}\left(vec(\hat{\mathcal{K}}') - vec(\mathcal{K}')\right)\right)$. In general, $\hat{\mathcal{K}}'$ is a function of the SVD of $\hat{\Gamma}^f\hat{\mathcal{F}}\hat{\Gamma}^p$, where $\hat{\Gamma}^f$ and $\hat{\Gamma}^p$ are weighting matrices discussed before. To simplify matters we assume that the SVD is carried out on $\hat{\mathcal{F}}$. It is straightforward to modify what follows to accomodate the weighting matrices. Note the importance of $sN \geq k$ for the calculation of the SVD. Note that there is serial correlation in the error terms in (5) for $s > 1$. Nevertheless, the error term and X_t^p remain uncorrelated in this case.

We define formally the function $g(.)$ such that $vec(\hat{\mathcal{K}}') = g\left(vec(A_1\hat{\mathcal{F}}A_2)\right)$. This implicitly defines the matrices A_1, A_2 which define the tranformation from $\hat{\mathcal{F}}$ to $\hat{\mathcal{K}}'$ via the singular value decomposition. By a first order Taylor expansion of $g(vec(A_1\hat{\mathcal{F}}A_2))$ and $g(vec(A_1\mathcal{F}A_2))$ around $A_1\mathcal{F}^*A_2$, possible since $g(.) \in C^\infty$ and where each element of $\mathcal{F}^*$ lies between the respective elements of $\mathcal{F}$ and $\hat{\mathcal{F}}$, we have that

$$V\left(\sqrt{T^*}\left(vec(\hat{\mathcal{K}}') - vec(\mathcal{K}')\right)\right) = \frac{\partial g}{\partial(A_1\mathcal{F}A_2)}$$
$$V\left(\sqrt{T^*}\left(vec(A_1\hat{\mathcal{F}}A_2) - vec(A_1\mathcal{F}A_2)\right)\right)\frac{\partial g'}{\partial(A_1\mathcal{F}A_2)}$$

Consistency and a $\sqrt{T^*}$ rate of convergence of the parameter estimates $\hat{\mathcal{F}}$ to their true values implies that the remainder of the Taylor approximation is $o_p(1)$. So we need to derive the variance of $\sqrt{T^*}\left(vec(A_1\hat{\mathcal{F}}A_2) - vec(A_1\mathcal{F}A_2)\right)$. Again simple manipulations imply that

$$V\left(\sqrt{T^*}\left(vec(A_1\hat{\mathcal{F}}A_2) - vec(A_1\mathcal{F}A_2)\right)\right) = (A_2'\otimes A_1)V\left(\sqrt{T^*}\left(vec(\hat{\mathcal{F}}) - vec(\mathcal{F})\right)\right)(A_2\otimes A_1')$$

From multivariate regression analysis we know that for $s = 1$

$$V\left(\sqrt{T^*}\left(vec(\hat{\mathcal{F}}) - vec(\mathcal{F})\right)\right) = (\Gamma^{p^{-1}} \otimes \Sigma)$$

where Γ^p and Σ are the variance covariance matrices of X^p and of the regression error respectively, which yields the result for $s = 1$. For the general case $s > 1$ since the error terms have serial correlation we have

$$V\left(\sqrt{T^*}\left(vec(\hat{\mathcal{F}}) - vec(\mathcal{F})\right)\right) = (\Gamma^{p^{-1}} \otimes I_{sN})\Phi(\Gamma^{p^{-1}} \otimes I_{sN})$$

where Φ is equal to $(X^{p\prime} \otimes I_{sN})\Sigma_u(X^p \otimes I_{sN})$ and $\Sigma_u = E(e^f_s e^{f\prime}_s)$ where $e^f = vec(E^f)$ and $E^f = (E^f_1, ..., E^f_T)$. A consistent estimator for Σ_u may be easily obtained by calculating the autocovariances of the residuals of (6) up to order $s-1$ since the error term is autocorrelated only up to order $s-1$. ■

3 The case: $N \to \infty$

In this section we firstly investigate the conditions for consistency of the SSS method when N diverges. Second, we discuss correlation of the idiosyncratic errors. Third, we derive an information criterion for the selection of the number of factors. Finally, we develop a modified SSS algorithm for datasets with more time series than observations.

3.1 Consistency of the SSS estimator

To prove consistency of the SSS estimator, we need to add an assumption to those in the previous Section. In particular, we require

Assumption 5 *$Np = o(T^{1/3})$; $p = O(T^{1/r})$, $r > 3$;*

Then we have

Theorem 3 ***(Consistency when** $N \to \infty$**).*** *If N is $o(T^{1/3-1/r})$, then when N and T diverge, and under assumptions 1-6, $\hat{f}_t = \hat{\mathcal{K}}X^p_t$ converges to* *the space spanned by the true factors in probability.*

Proof. Consistent estimation of the coefficients of the model in (6) by OLS, and therefore of the factors, holds if the number of regressors in each of the Ns equations tends to infinity at a rate lower than $T^{1/3}$ but the number of lags, p, grows at a minimum rate of $T^{1/r}$ where $r > 0$. Since the number of regressors is Np we see that N can grow at rates of at most $T^{1/3-1/r}$. Under these conditions the estimates of the factors will be consistent at rate $(T/Np)^{1/2}$ as the results by Berk (1974) applied to every equation separately hold. ■

Thus, divergence of N requires to be accompanied by a faster divergence of T for the SSS factor estimators to remain consistent. Asymptotic normality of the factor estimators follows along the lines of Theorem 2.

3.2 Correlation in the idiosyncratic errors

In this subsection we discuss the case of cross-sectional and/or serial correlation of the idiosyncratic errors. This extension can be rather simply handled within the state space method. Basically, the idiosyncratic errors can be treated as additional pseudo-factors that enter only a few of the variables via restrictions on the matrix of loadings C. These pseudo-factors can be serially correlated processes or not depending on the matrix A in equation (1).

The problem becomes one of distinguishing common factors and pseudo-factors, i.e., cross-sectionally correlated idiosyncratic errors. This is virtually impossible for finite N, while when N diverges a common factor is one which enters an infinite number of series, i.e, the column of the, now infinite dimensional, matrix C associated with a common factor will have an infinity of non-zero entries, and likewise a pseudo-factor will only have a finite number of non-zero entries in the respective column of C. Let k_1 denote the number of common factors thus defined and k_2 the number of pseudo-factors. Note that k_2 may tend to infinity but not faster than N. Then, following Forni et al. (2000), we make the following assumption.

Assumption 6 *The matrix $\mathcal{OK}$ in (5) has k_1 singular values tending to infinity as N tends to infinity and k_2 non-zero finite singular values.*

For example, the condition in the assumption is satisfied if k_1 common factors enter a non zero fraction, bN, $0 < b < 1$, of the series x_{Nt}, in the state space model given by (1), while $k_2(N)$ pseudo-factors enter a vanishing proportion of the series x_{Nt}, i.e. each such factors enter $c(N)N$ of the series x_{Nt} where $lim_{N\to\infty}c(N)N = 0$ and $k_2(N)$ is at most $O(N)$.

3.3 Choice of the number of factors

The choice of the number of factors to be included in the model is a relevant issue, see e.g. Bai and Ng (2002). We will show that it is possible to obtain a consistent estimator of the number of factors even when N diverges or the idiosyncratic errors are correlated using an information criterion of the form

$$IC(k_1) = V(k_1, \hat{f}^{k_1}) + k_1 g(N, T) \tag{7}$$

where

$$V(k_1, \hat{f}^{k_1}) = (NT)^{-1} \sum_{t=1}^{T} tr[(x_{Nt} - \hat{C}\hat{f}_t^{k_1})(x_{Nt} - \hat{C}\hat{f}_t^{k_1})'], \tag{8}$$

$\hat{f}^{k_1} = (\hat{f}_1^{k_1}, ..., \hat{f}_T^{k_1})'$, $\hat{f}_t^{k_1}$ are the factor estimates for the k_1 first common factors (according to the singular values), $\hat{C}$ is the OLS estimate of C based on $\hat{f}_t^{k_1}$ and $g(N,T)$ is a penalty term.

Before examining the properties of this criterion, note that, since the factors are orthogonal, any set of up to k_1^0 factor estimators are consistent for the respective set of true factors up to a nonsingular transformation determined by the normalisation used in the SVD carried out during the estimation and the identification of the state space model, see SW for a similar point . Thus, denoting the $T \times k_1$ matrix of the k_1 first true factors by f^{0,k_1}, we have that

$$(T/Np)^{1/2}||f_t^{k_1} - H^{k_1'} f_t^{0,k_1}|| = O_p(1)$$

for some nonsingular matrix H^{k_1}. This follows from Theorem 3. Then, strengthening assumption 3 with

Assumption 7 *u_t is an i.i.d. $(0, \Sigma_u)$ sequence with finite eighth moments.*

the following theorem holds

Theorem 4 *Let the factors be estimated by the SSS method and denote the true number of common factors k_1^0. Let $\hat{k}_1 = argmin_{1 \le k \le kmax} IC(k_1)$. Then, $\lim_{T\to\infty} Pr(\hat{k}_1 = k_1^0) = 1$ if i) $g(N,T) \to 0$ and ii) $Ng(N,T) \to \infty$ as $N, T \to \infty$.*

Proof. The proof builds upon a set of results by Bai and Ng (2002). Therefore, to start with, we examine whether our parametric setting in terms of the representation 1 satisfies their assumptions. Assumption A of Bai and

Ng (2002) is satisfied if $|\lambda_{max}(A)| < 1$, where $|\lambda_{max}(A)|$ denotes the maximum eigenvalue of A in absolute value and the fourth moments of u_t exist. These conditions are satisfied by our assumptions 1 and 3. Their Assumption B on factor loadings is straightforwardly satisfied by assuming boundedness of the elements of the C matrix. Their assumption C is satisfied by assuming that the eighth moments of u_t exist combined with our cross correlation structure in Assumption 6. Finally, their Assumption D is trivially satisfied because we assume that factors and idiosyncratic errors are uncorrelated.

We must now prove that $lim_{N(T),T\to\infty} Pr(IC(k_1) < IC(k_1^0)) = 0$ for all $k_1 \neq k_1^0$, $k_1 < k^{max}$. Denoting the $T \times k_2$ matrix of the first k_2 true idiosyncratic pseudo factors by $f^{0,2,k_2}$, we examine

$$V(k_1, (f^{0,k_1}, f^{0,2,k_2})) - V(k_1, (f^{0,k_1}))$$

for any finite k_2. We know that, for all elements of x_{Nt} in which $f^{0,2,k_2}$ does not enter, it is

$$1/T \sum_{t=1}^{T} (x_{i,Nt} - \hat{C}'_{i,1,2}(f_t^{0,k_1'}, f_t^{0,2,k_2'})')^2 - 1/T \sum_{t=1}^{T} (x_{i,Nt} - \hat{C}'_{i,1} f_t^{0,k_1})^2 = O_p(T^{-1})$$

For a finite number of elements of x_{Nt}

$$1/T \sum_{t=1}^{T} (x_{i,Nt} - \hat{C}'_{i,1,2}(f_t^{0,k_1'}, f_t^{0,2,k_2'})')^2 - 1/T \sum_{t=1}^{T} (x_{i,Nt} - \hat{C}'_{i,1} f_t^{0,k_1})^2 = O_p(1)$$

Therefore, overall

$$V(k_1, (f^{0,k_1}, f^{0,2,k_2})) - V(k_1, (f^{0,k_1})) = O_p(N^{-1}) \qquad (9)$$

First consider $k_1 < k_1^0$. Then

$$IC(k_1) - IC(k_1^0) = V(k_1, \hat{f}^{k_1}) - V(k_1^0, \hat{f}^{k_1^0}) - (k_1^0 - k_1) g(N, T)$$

and the required condition for the result is

$$Pr[V(k_1, \hat{f}^{k_1}) - V(k_1^0, \hat{f}^{k_1^0}) < (k_1^0 - k_1)g(N,T)] = 0$$

as $N(T), T \to \infty$. Now

$$V(k_1, \hat{f}^{k_1}) - V(k_1^0, \hat{f}^{k_1^0}) = [V(k_1, \hat{f}^{k_1}) - V(k_1, f^{k_1}H^{k_1})] + [V(k_1, f^{k_1}H^{k_1}) - V(k_1^0, f^{k_1^0}H^{k_1^0})] + [V(k_1^0, f^{k_1^0}H^{k_1^0}) - V(k_1^0, \hat{f}^{k_1^0})]$$

By the rate of convergence of the factor estimators and Lemma 2 of Bai and Ng (2002) we have

$$V(k_1, \hat{f}^{k_1}) - V(k_1, f^{k_1}H^{k_1}) = O_p((T/Np)^{-1})$$

and

$$V(k_1^0, \hat{f}^{k_1^0}) - V(k_1^0, f^{k_1^0}H^{k_1^0}) = O_p((T/Np)^{-1})$$

Note that Lemma 2 of Bai and Ng (2002) stands independently from the factor estimation method discussed in that paper and only uses the rate of convergence of the factor estimators derived in their Theorem 1. Then $V(k_1, f^{k_1}H^{k_1}) - V(k_1^0, f^{k_1^0}H^{k_1^0})$ can be written as $V(k_1, f^{k_1}H^{k_1}) - V(k_1^0, f^{k_1^0})$ which has positive limit by Lemma 3 of Bai and Ng (2002). Thus, as long as $g(N,T) \to 0$, $Pr(IC(k_1) < IC(k_1^0)) = 0$ for all $k_1 < k_1^0$.

Then, to prove $Pr(IC(k_1) < IC(k_1^0)) = 0$ for all $k_1 > k_1^0$ we have to prove that

$$Pr[V(k_1^0, \hat{f}^{k_1^0}) - V(k_1, \hat{f}^{k_1}) < (k_1 - k_1^0)g(N,T)] \to 0$$

By (9) we know that asymptotically the analysis of the state space model will be equivalent to the case of a model where there are no idiosyncratic pseudo factors up to an order of probability of N^{-1}. Then

$$|V(k_1^0, \hat{f}^{k_1^0}) - V(k_1, \hat{f}^{k_1})| \leq 2max_{k_1^0 < k_1 \leq kmax}|V(k_1, \hat{f}^{k_1}) - V(k_1, f^{0,k_1^0})|.$$

By following the analysis of Lemma 4 of Bai and Ng (2002) we know that

$$max_{k_1^0<k_1\leq kmax}|V(k_1,\hat{f}^{k_1})-V(k_1,f^{0,k_1^0})|=O_p((T/Np)^{-1})$$

Combining this expression with (9) and the fact that Np grows slower than $T^{1/3}$, gives the required result since then $(T/Np)^{-1}<N^{-1}$. Note again that Lemma 4 of Bai and Ng (2002) stands independently from the factor estimation method discussed in that paper and only uses the rate of convergence of the factor estimators derived in their Theorem 1. ■

3.4 Dealing with Large Datasets

Up to now we have outlined a method for estimating factors which requires the number of observations to be larger than the number of elements in X_t^p, while SW and FHLR do not require this condition. We therefore suggest a modification of our methodology to let the number of series be larger than the number of observations.

The problem arises because the least squares estimator of $\mathcal{F}$ in (6) is not uniquely defined due to the rank deficiency of $X^{p'}X^p$. As we mentioned in section 2, we do not neccesarily want an estimator of $\mathcal{F}$ but an estimator of the states $X^p\mathcal{K}'$. That could be obtained if we had an estimator of $X^p\mathcal{F}'$ and used an SVD of that.

It is well known (see e.g. Magnus and Neudecker (1988)) that although $\mathcal{F}$ may not be estimable, $X^p\mathcal{F}'$ always is using least squares methods. In particular, the least squares estimator of $X^p\mathcal{F}'$ is given by

$$\widehat{X^p\mathcal{F}'}=X^p(X^{p'}X^p)^+X^{p'}X^f \qquad (10)$$

where $X^f = (X_1^f, \ldots, X_T^f)$ and A^+ denotes the unique Moore-Penrose inverse of matrix A. However, when the row dimension of X^p is smaller than its column dimension, $X^p(X^{p'}X^p)^+X^{p'} = I$ implying that $\widehat{X^p\mathcal{F}'} = X^f$. A decomposition of X^f is then easily seen to be similar, but not identical, to the eigenvalue decomposition of the covariance matrix of X^f which is the SW principle component method. We will refer to this method as SSS0. This method is static, abstracting from the fact that s may be larger than 1, thereby leading to a decomposition involving leads of x_{Nt}.

Alternative solutions exist to this problem. In particular, note that we are after a subspace decomposition of the estimator of the fitted value $X^p\mathcal{F}'$. Essentially, we are after a reduced rank approximation of $X^p\mathcal{F}'$, and several possibilities exist. The main requirement is that, as the assumed rank (number of factors) tends to the full rank of the estimate of the fitted value, the approximation should tend to the estimated fitted value $\widehat{X^p\mathcal{F}'} = X^p(X^{p'}X^p)^+X^{p'}X^f = X^f$. The alternative decomposition we suggest is a SVD on $X^{f'}X^p(X^{p'}X^p)^+ = \hat{U}\hat{S}\hat{V}'$. Then the estimated factors are given by $\hat{\mathcal{K}}X_t^p$ where $\hat{\mathcal{K}}$ is obtained as before but using the SVD of $X^{f'}X^p(X^{p'}X^p)^+$.

This approach, compared to SSS0, has the advantage that the estimated factors are combinations of lags and contemporaneous values of the variables (and also of leads when $s > 1$). We choose to set both weighting matrices to the identity matrix in this case. We also refer to this decomposition as SSS, because it is simply a generalisation of the method in section 2 and if $Np < T$ it reduces to that method. As k tends to $\min(Ns, Np)$ the set of factor estimators tends to the OLS estimated fitted value X^f.

This method needs to be judged in terms of its small sample properties in approximating (linear combinations of) the true factors, and the simulations in the next section indicate that it performs very well, similar to the proper method of section 2 (and in general better than SSS0 on the basis of other experiments that are not reported to save space).

4 A comparison of the estimation methods

In this section we summarize the results of an extensive set of simulation experiments to investigate the small sample properties of the three competing factor extraction methods, i.e. static principal components (PCA, SW), dynamic principal components (DPCA, FHLR), and our state space approach (SSS). The first subsection describes the simulation set-up; the second one the results.

4.1 Monte Carlo experiments, set-up

The basic data generating process (DGP) we use is:

$$\begin{aligned} x_{Nt} &= Cf_t + \epsilon_t, \quad t = 1, \ldots, T \\ A(L)f_t &= B(L)u_t \end{aligned} \tag{11}$$

where $A(L) = I - A_1(L) - \ldots - A_p(L)$, $B(L) = I + B_1(L) + \ldots + B_q(L)$.

An important comment is in order for this model. We have developed our theory for predetermined factors, i.e. factors that are determined at time $t-1$. This is reflected by (1) where the error term of the factor equation is dated at time $t-1$. This assumption is not considered restrictive in the state space model literature, see e.g. Deistler and Hannan (1988). Yet, the specification we use for the simulations allows for factors that are determined at time t. This brings us in line with the nonparametric context of SW and FHLR. However, as the simulations will show, this choice still leaves the new estimation method performing comparably and, in a majority of cases, better than either PCA or DPCA. The rationale underlying this results is that the SSS estimator, when contemporaneous errors drive the factors, is consistent for the expected value of the factors conditional on information up to period $t-1$. Of course, the performance of the SSS estimator further improves when u_{t-1} is used in (11) rather than u_t.

For the SSS method, the "lag" truncation parameter is set at $p = \ln(T)^{\alpha}$. We have found that a range of α between 1.05 and 1.5 provides a satisfactory performance, and we have used the value $\alpha = 1.25$ in the reported results.

The "lead" truncation parameter s is set equal to the assumed number of factors for SSS, which typically coincides with the true number of factors, i.e. $s = k$. For robustness, and since it is relevant for forecasting, we will present selected result for the case $s = 1$ as well.[2] For the DPCA method we use 3 leads and 3 lags.

With the exceptions noted below, the C matrix is generated using standard normal variates as elements and the error terms are generated as uncorrelated standard normal pseudo-random variables. We have considered

[2]We have also experimented with other values of s but $s = 1$ or $s = k$ appear to be the preferable choices. To select the value of s we can either include this parameter as a variable in the information criterion search or, perhaps more straightforwardly, we can choose the value that maximises the proportion of the variance of each series explained by the factors, averaged over all series.

several combinations of N, T and report results for the following N, T pairs: $(50, 50)$, $(50, 100)$, $(100, 50)$, $(100, 100)$, $(50, 500)$, $(100, 500)$ and $(200, 50)$.

To provide a comprehensive evaluation of the relative performance of the three factor estimation methods, we consider several types of experiments. They differ for the number of factors (one or several), the choice of s ($s = k$ or $s = 1$), the factor loadings (static or dynamic), the choice of the number of factors (true number or misspecified), the properties of the idiosyncratic errors (uncorrelated or serially correlated), and the way the C matrix is generated (standard normal or uniform with non-zero mean). Each experiment is replicated 500 times. Depending on these characteristics, the experiments can be divided into five groups.

In the first group, we assume that we have a single VARMA factor with 8 specifications that differ for the extent of serial correlation and the AR and MA order:

(1) $a_1 = 0.2, b_1 = 0.4$;

(2) $a_1 = 0.7, b_1 = 0.2$;

(3) $a_1 = 0.3, a_2 = 0.1, b_1 = 0.15, b_2 = 0.15$;

(4) $a_1 = 0.5, a_2 = 0.3, b_1 = 0.2, b_2 = 0.2$;

(5) $a_1 = 0.2, b_1 = -0.4$;

(6) $a_1 = 0.7, b_1 = -0.2$;

(7) $a_1 = 0.3, a_2 = 0.1, b_1 = -0.15, b_2 = -0.15$;

(8) $a_1 = 0.5, a_2 = 0.3, b_1 = -0.2, b_2 = -0.2$.

Experiment 9 is as experiment 1 but both the ARMA factor and its lag enter the measurement equation, i.e., the C matrix is $C(L) = C_0 + C_1 L$ where L is the lag operator. We fix a priori the number of factors to $p + q$, which is the true number in the state space representation. It is larger than the true number in the FHLR setup, and it should provide a reasonable approximation for SW too. As a robustness check, we consider the case where the factor is generated as in Experiment 1 but only one factor is assumed to exist rather than $p + q$. We refer to this experiment as Experiment 10. In the case of experiments 9 and 10, qualitatively similar results are obtained when the mentioned modifications are applied to the parameter specifications 2-8 (results available upon request).

In the second group of experiments, we investigate the case of serially correlated idiosyncratic errors. The DGP for that is specified as in experiments 1-10 but with each idiosyncratic error being an AR(1) process with coefficient 0.2 rather than an i.i.d. process. These experiments are labelled 11-20. The results are rather robust to higher values of serial correlation but 0.2 is a reasonable value in practice since usually the common component captures most of the persistence of the series. We have also investigated the case of cross-correlated errors by assuming that the contemporaneous covariance matrix of the idiosyncratic errors is tridiagonal with diagonal elements equal to 1 and off-diagonal elements equal to 0.2. These experiments produced the same ranking of methods as in the case of serial correlation and virtually no deterioration of performance with respect to the idiosyncratic errors case (results available upon request).

In the third group of experiments, we use a 3 dimensional VAR(1) as the data generation process for the factors as opposed to an ARMA process. We report results for the case where the A matrix is diagonal with elements equal to 0.5. This is labelled experiment 21.

In the fourth group of experiments, we consider the DGPs in experiments 1-21 but generate the C matrix using standard uniform variates, thereby allowing for the factor loadings to have a non zero mean. To save space, we only report results for $(N, T) =$ $(50, 50)$ for this case.

Finally, we consider again experiments 1-21 but using $s = 1$ instead of $s = k$. We present results for the (N, T) pairs $(50, 50)$ and $(100, 100)$.

We concentrate on the relationship between the true and estimated common components (Cf_t and $\widehat{C}\widehat{f_t}$), measured by their correlation, and on the properties of the estimated idiosyncratic components ($\widehat{\epsilon}_t$), using an LM(4) test to evaluate whether they are white noise as in the DGP, and presenting the rejection probabilities of the test. These are the most common evaluation criteria used in the literature. Throughout, we report the average values of the different evaluation criteria (averaging over all variables for each replication and then over all replications), and the standard errors of the averages over replications.

4.2 Monte Carlo experiments, results

The results are summarized in Tables 1 to 7 for different combinations of N and T, while Table 8 presents the outcome for the uniform factor loadings C and $(N, T) = (50, 50)$. Finally, Tables 9-11 present results for the case $s = 1$.

Starting with the $(N, T) = (50, 50)$ case in Table 1, and the single ARMA factor experiments (1-8), the SSS method clearly outperforms the other two. The gains with respect to PCA are rather limited, in the range 5-10%, but systematic across experiments. The gains are larger with respect to DPCA, about 20%, and again systematic across experiments. For all the three methods the correlation is higher the higher the persistence of the factor. There is little evidence that the idiosyncratic component is serially correlated on the basis of the LM(4) test for any of the methods, but the DPCA yields systematically larger rejection probabilities.

The presence of serially correlated idiosyncratic errors (experiments 11-18) does not affect significantly the results. The values for each method, the ranking of the methods and the relative gains are virtually the same as in the basic case. Non correlation of the errors is rejected more often, but still in a very low number of cases. This is related to the low power of the LM test in small (T) samples, for larger values of T the rejection rate increases substantially, see Tables 2 and 3.

Allowing for a lagged effect of the factor on the variables, instead, leads to a serious deterioration of the SSS performance, with a drop of about 25% in the correlation values, compare experiments 1 and 9, and 11 and 19. The performance of DPCA, which is particularly suited for this generating process from a theoretical point of view, does improve, but it is still beaten by PCA even though the difference shrinks. The choice of a lower value for s improves substantially the performance of SSS in this case, making it comparable with PCA, compare the relevant lines of Table 9 for $s = 1$. This finding, combined with the fact that DPCA is still beaten by PCA, suggests that the use of leads of the variables for factor estimation is complicated when the factors can have a dynamic impact on the variables.

When a lower number of factors than true is assumed for SSS, one instead of two in experiments 10 and 20, the performance does not deteriorate. Actually, comparing experiments 1 and 10, and 11 and 20, there is a slight increase in correlation. A similar improvement can be observed for PCA and DPCA, and it is likely due to the fact that a single factor can do most of the work of capturing the true common component, while estimation uncertainty is reduced.

The presence of three autoregressive factors, experiment 21, reduces the gap PCA-DPCA. The correlation values are higher than in the single factor case, reflecting in general the higher persistence of the factors. Yet, the performance of SSS deteriorates substantially. The latter improves and becomes comparable to PCA with $s = 1$, see table 11.

The next three issues we consider are the effects of larger temporal dimension, cross-sectional dimension, and uniform rather than standard normal loading matrix.

Tables 2 and 3 report results for $N = 50$ and, respectively, $T = 100$ and $T = 500$. The correlation between the true and estimated common component increases monotonically for all the three methods, but neither the ranking of methods nor the performance across experiments are affected. The performance of the LM tests in detecting serial correlation in the error process gets also closer and closer to the theoretical one.

When N increase to 100 while T remains equal to 50 (Table 4), the figures for SSS are basically unchanged in all experiments, while the performance of PCA and DPCA improves systematically. Yet, the gains are not sufficient to match the SSS approach, which still yields the highest correlation in all cases, except with a dynamic effect of the factors of the variables (experiments 9 and 19), and with three autoregressive factors (experiment 21). This pattern continues if we further increase N to 200 (Table 7).

When both N and T increase, $N = 100, T = 100$ in Table 5 while $N = 100, T = 500$ in Table 6, the performance of all methods improves with respect to Table 1, proportionally more so for PCA and DPCA that benefit more for the larger value of N, as mentioned before. But also in these cases SSS is in general the best in terms of correlation.

The final issue we consider is the choice of s. This is examined through Tables 9-11 where we set $s = 1$. For this case PCA and SSS perform very similarly. The advantage SSS had for the ARMA experiments shrinks substantially, SSS is still better but only marginally so. On the other hand, the large disadvantage SSS had for VAR experiments and experiments with factor lags disappears, as mentioned above, with SSS and PCA performing equally well.

In summary, the DPCA method shows consistently lower correlation between true and estimated common components than SSS and PCA. It shows, in general, more evidence of serial correlation, although not to any significant extent. Additionally, from results we are not presenting here the DPCA method has the lowest variance for the idiosyncratic component or, in other words, has the highest explanatory power of the series in terms of the common components. These results seem to indicate that i) part of the idiosyncratic component seems to leak into the estimated common component in the DPCA case, thus reducing the correlation between true and estimated common components and the variance of the idiosyncratic component and ii) some (smaller in terms of variance) part of the common component leaks into the estimated idiosyncratic component thus increasing the serial correlation of the idiosyncratic component. The conclusion from these results is that if one cares about isolating common components as summaries of underlying common features of the data, then a high R^2 may not always be the appropriate guide. When instead the factors have a dynamic effect on the variables, the performance of DPCA improves, but it is still beaten by PCA. This experiment and the one with three autoregressive factors are the only cases where PCA beats SSS, but the difference can be annihilated by means of a proper choice of the s parameter. In all other experiments SSS leads to gains in terms of higher correlation in the range 5-10%.

5 An empirical example

We now use a dynamic factor model estimated with the three methods to analyze a large balanced dataset of 146 US macroeconomic variables, over the period 1959:1-1998:12, taken from SW to whom we refer for additional details. To start with, we estimate the common component of each variable according to the three methods (with $s = 1$ for SSS), and then compute the resulting (adjusted) R^2 and the correlation among the three common components. SW showed that the first two SW factors are the most relevant for forecasting several variables in the dataset, while Favero, Marcellino and Neglia (2002) found that 3 or 4 FHLR factors are sufficient. Since it is better to overestimate the number of factors rather than underestimate it, we have chosen to use six factors.

Focusing on the R^2 first, the performance of SSS and PCA is comparable, the latter is slightly better than the former on average over all variables (0.44 versus 0.39), while DPCA is ranked first, with an average R^2 of about

0.52, see Table 12. A similar pattern emerges from a more disaggregate analysis, DPCA yields a higher R^2 for most variables. The better fit of DPCA could be explained by the longer sample available, which improves substantially the multivariate spectrum estimation underlying this method, and by the use of future information in the computation of the spectrum On the other hand, as the Monte Carlo results show, the better fit may be an artefact of the tendency of the DPCA method to soak up part of the idiosyncratic component in the data. The correlation among the estimated common components is highest for SSS-PCA, with an average value of 0.93, slightly lower but still considerable for PCA-DPCA, 0.76, and SSS-DPCA, 0.73. Overall, these values are in line with the Monte Carlo simulations, which showed a higher similarity of PCA and SSS.

The second exercise we consider is the inclusion of the estimated factors in a monetary VAR to evaluate the response of inflation and the output gap to unexpected monetary shocks. The standard VARs in the literature consider the output gap (USGAP), inflation (USINFL), a commodity price index, the effective exchange rate, and the federal fund rate (USPR), to which we add six factors treated as exogenous regressors. Four lags are included for each endogenous variable and the VAR is estimated over the sample 1980:1-1998:12 to cover a relatively homogenous period from the monetary policy point of view but long enough to obtain reliable estimates of the parameters. Impulse response functions are obtained with a Choleski decomposition with the variables ordered as listed above.

The responses of USGAP, USINFL and USPR to a one standard deviation shock in USPR are graphed in Figure 1 for the cases where the factors are excluded from the VAR (base), and when they are included as exogenous regressors and estimated according to each of the three methods. To use a comparable information set, the DPCA are lagged three periods, since two future quarters are used to compute the spectrum, while the PCA and SSS only once. Favero et al. (2002) performed a similar exercise using modified DPCA derived from one-sided estimation in order not to use future information, see Forni et al. (2003) for details on the method, but found similar results as for DPCA.

The base case shows a positive (though not significant) response of USINFL for about 3 years, what is commonly named price puzzle since inflation should instead decrease. The positive reaction of USGAP is also not in line with standard economic theory. The inclusion of the dynamic principal components does not change sensibly the pattern of response; with static principal components the USGAP decreases; but only with the SSS factors also the price puzzle is eliminated. To obtain such a result with PCA or DPCA a larger number of factors has to be included in the VAR, up to 12.

6 Conclusion

In this paper we have developed a parametric estimation method for dynamic factor models of large dimension based on a subspace algorithm applied to the state space representation of the model (SSS). We have derived the asymptotic properties of the estimators, formulae for their standards errors, and information criteria for a consistent selection of the number of factors.

Then we have undertaken a comparative analysis of the performance of alternative factor estimation methods using Monte Carlo experiments. Our main conclusion is that the SSS method, which takes explicit account of the dynamic nature of the data generating process, performs better than alternative approaches for a number of experimental setups. Static principal components seem to perform satisfactorily overall, while dynamic principal components appear slightly less able to distinguish between common and idiosyncratic factors, in the particular setup we have considered which is, nevertheless, quite general.

Finally, we have provided an empirical application with a large dataset for the US, that further confirms the good empirical performance of the SSS method and, more generally, the usefulness of the dynamic factor model as a modelling tool for datasets of large dimension.

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Table 1: Results for case: N=50, T=50

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.821_{(0.052)}$	$0.860_{(0.054)}$	$0.727_{(0.053)}$	$0.067_{(0.033)}$	$0.066_{(0.035)}$	$0.097_{(0.042)}$
Exp 2	$0.859_{(0.049)}$	$0.890_{(0.050)}$	$0.780_{(0.056)}$	$0.072_{(0.040)}$	$0.075_{(0.039)}$	$0.103_{(0.045)}$
Exp 3	$0.740_{(0.054)}$	$0.805_{(0.054)}$	$0.634_{(0.056)}$	$0.073_{(0.036)}$	$0.081_{(0.040)}$	$0.137_{(0.052)}$
Exp 4	$0.803_{(0.058)}$	$0.855_{(0.054)}$	$0.713_{(0.068)}$	$0.076_{(0.040)}$	$0.086_{(0.038)}$	$0.143_{(0.054)}$
Exp 5	$0.806_{(0.053)}$	$0.848_{(0.055)}$	$0.703_{(0.052)}$	$0.067_{(0.034)}$	$0.066_{(0.034)}$	$0.094_{(0.042)}$
Exp 6	$0.823_{(0.053)}$	$0.861_{(0.053)}$	$0.731_{(0.055)}$	$0.068_{(0.035)}$	$0.070_{(0.038)}$	$0.103_{(0.042)}$
Exp 7	$0.717_{(0.053)}$	$0.787_{(0.054)}$	$0.604_{(0.052)}$	$0.064_{(0.034)}$	$0.076_{(0.038)}$	$0.135_{(0.049)}$
Exp 8	$0.724_{(0.057)}$	$0.791_{(0.058)}$	$0.616_{(0.057)}$	$0.067_{(0.035)}$	$0.080_{(0.038)}$	$0.137_{(0.053)}$
Exp 9	$0.898_{(0.028)}$	$0.693_{(0.061)}$	$0.823_{(0.036)}$	$0.071_{(0.036)}$	$0.039_{(0.030)}$	$0.123_{(0.049)}$
Exp 10	$0.904_{(0.061)}$	$0.904_{(0.060)}$	$0.848_{(0.050)}$	$0.068_{(0.037)}$	$0.068_{(0.036)}$	$0.079_{(0.039)}$
Exp 11	$0.813_{(0.055)}$	$0.855_{(0.055)}$	$0.721_{(0.052)}$	$0.102_{(0.043)}$	$0.116_{(0.045)}$	$0.132_{(0.050)}$
Exp 12	$0.848_{(0.051)}$	$0.881_{(0.052)}$	$0.772_{(0.056)}$	$0.100_{(0.042)}$	$0.112_{(0.045)}$	$0.132_{(0.050)}$
Exp 13	$0.722_{(0.058)}$	$0.789_{(0.058)}$	$0.620_{(0.059)}$	$0.084_{(0.037)}$	$0.123_{(0.045)}$	$0.155_{(0.053)}$
Exp 14	$0.791_{(0.060)}$	$0.846_{(0.055)}$	$0.704_{(0.068)}$	$0.089_{(0.040)}$	$0.123_{(0.049)}$	$0.162_{(0.056)}$
Exp 15	$0.798_{(0.055)}$	$0.845_{(0.057)}$	$0.697_{(0.053)}$	$0.113_{(0.045)}$	$0.130_{(0.049)}$	$0.150_{(0.051)}$
Exp 16	$0.813_{(0.055)}$	$0.854_{(0.056)}$	$0.724_{(0.055)}$	$0.105_{(0.043)}$	$0.118_{(0.046)}$	$0.143_{(0.050)}$
Exp 17	$0.703_{(0.055)}$	$0.776_{(0.058)}$	$0.596_{(0.053)}$	$0.082_{(0.039)}$	$0.125_{(0.047)}$	$0.157_{(0.056)}$
Exp 18	$0.715_{(0.057)}$	$0.785_{(0.059)}$	$0.610_{(0.057)}$	$0.082_{(0.039)}$	$0.127_{(0.048)}$	$0.165_{(0.058)}$
Exp 19	$0.889_{(0.031)}$	$0.685_{(0.063)}$	$0.814_{(0.037)}$	$0.086_{(0.039)}$	$0.052_{(0.032)}$	$0.138_{(0.049)}$
Exp 20	$0.892_{(0.064)}$	$0.893_{(0.063)}$	$0.840_{(0.053)}$	$0.119_{(0.047)}$	$0.120_{(0.047)}$	$0.128_{(0.050)}$
Exp 21	$0.974_{(0.009)}$	$0.692_{(0.051)}$	$0.947_{(0.014)}$	$0.078_{(0.038)}$	$0.111_{(0.068)}$	$0.125_{(0.046)}$

[a] PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b] Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c] Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 2: Results for case: N=50, T=100

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.856_{(0.044)}$	$0.903_{(0.045)}$	$0.781_{(0.044)}$	$0.057_{(0.033)}$	$0.057_{(0.032)}$	$0.068_{(0.036)}$
Exp 2	$0.890_{(0.041)}$	$0.928_{(0.039)}$	$0.830_{(0.045)}$	$0.060_{(0.034)}$	$0.061_{(0.033)}$	$0.073_{(0.036)}$
Exp 3	$0.777_{(0.044)}$	$0.862_{(0.042)}$	$0.689_{(0.045)}$	$0.057_{(0.034)}$	$0.064_{(0.036)}$	$0.086_{(0.040)}$
Exp 4	$0.844_{(0.044)}$	$0.906_{(0.038)}$	$0.776_{(0.052)}$	$0.061_{(0.034)}$	$0.068_{(0.035)}$	$0.086_{(0.040)}$
Exp 5	$0.839_{(0.043)}$	$0.891_{(0.045)}$	$0.754_{(0.043)}$	$0.056_{(0.034)}$	$0.056_{(0.033)}$	$0.069_{(0.038)}$
Exp 6	$0.859_{(0.043)}$	$0.904_{(0.044)}$	$0.785_{(0.044)}$	$0.057_{(0.033)}$	$0.058_{(0.035)}$	$0.070_{(0.036)}$
Exp 7	$0.752_{(0.044)}$	$0.847_{(0.044)}$	$0.658_{(0.045)}$	$0.056_{(0.032)}$	$0.061_{(0.033)}$	$0.084_{(0.039)}$
Exp 8	$0.767_{(0.046)}$	$0.855_{(0.045)}$	$0.677_{(0.049)}$	$0.057_{(0.032)}$	$0.064_{(0.034)}$	$0.088_{(0.041)}$
Exp 9	$0.923_{(0.021)}$	$0.703_{(0.055)}$	$0.869_{(0.026)}$	$0.061_{(0.034)}$	$0.028_{(0.025)}$	$0.081_{(0.039)}$
Exp 10	$0.935_{(0.047)}$	$0.935_{(0.047)}$	$0.894_{(0.040)}$	$0.056_{(0.032)}$	$0.057_{(0.032)}$	$0.061_{(0.033)}$
Exp 11	$0.849_{(0.043)}$	$0.898_{(0.043)}$	$0.776_{(0.043)}$	$0.212_{(0.060)}$	$0.242_{(0.061)}$	$0.235_{(0.061)}$
Exp 12	$0.888_{(0.039)}$	$0.926_{(0.038)}$	$0.830_{(0.041)}$	$0.204_{(0.057)}$	$0.229_{(0.058)}$	$0.226_{(0.059)}$
Exp 13	$0.770_{(0.045)}$	$0.859_{(0.043)}$	$0.686_{(0.048)}$	$0.157_{(0.051)}$	$0.240_{(0.062)}$	$0.228_{(0.059)}$
Exp 14	$0.836_{(0.042)}$	$0.902_{(0.037)}$	$0.771_{(0.050)}$	$0.157_{(0.050)}$	$0.233_{(0.060)}$	$0.221_{(0.058)}$
Exp 15	$0.836_{(0.041)}$	$0.890_{(0.042)}$	$0.753_{(0.041)}$	$0.232_{(0.061)}$	$0.263_{(0.064)}$	$0.263_{(0.062)}$
Exp 16	$0.853_{(0.043)}$	$0.900_{(0.045)}$	$0.782_{(0.044)}$	$0.208_{(0.060)}$	$0.239_{(0.064)}$	$0.239_{(0.064)}$
Exp 17	$0.743_{(0.043)}$	$0.840_{(0.042)}$	$0.652_{(0.044)}$	$0.167_{(0.053)}$	$0.245_{(0.062)}$	$0.229_{(0.064)}$
Exp 18	$0.764_{(0.046)}$	$0.853_{(0.045)}$	$0.677_{(0.049)}$	$0.162_{(0.054)}$	$0.246_{(0.062)}$	$0.230_{(0.061)}$
Exp 19	$0.916_{(0.022)}$	$0.695_{(0.050)}$	$0.862_{(0.027)}$	$0.183_{(0.055)}$	$0.097_{(0.042)}$	$0.220_{(0.058)}$
Exp 20	$0.931_{(0.049)}$	$0.932_{(0.049)}$	$0.889_{(0.041)}$	$0.244_{(0.062)}$	$0.245_{(0.061)}$	$0.250_{(0.062)}$
Exp 21	$0.984_{(0.005)}$	$0.686_{(0.040)}$	$0.970_{(0.007)}$	$0.062_{(0.033)}$	$0.205_{(0.100)}$	$0.083_{(0.038)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 3: Results for case: N=50, T=500

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.899_{(0.028)}$	$0.939_{(0.042)}$	$0.855_{(0.031)}$	$0.052_{(0.030)}$	$0.062_{(0.043)}$	$0.058_{(0.032)}$
Exp 2	$0.922_{(0.027)}$	$0.951_{(0.036)}$	$0.889_{(0.030)}$	$0.050_{(0.031)}$	$0.064_{(0.044)}$	$0.056_{(0.034)}$
Exp 3	$0.822_{(0.033)}$	$0.907_{(0.049)}$	$0.773_{(0.036)}$	$0.056_{(0.033)}$	$0.088_{(0.075)}$	$0.066_{(0.033)}$
Exp 4	$0.885_{(0.030)}$	$0.946_{(0.026)}$	$0.851_{(0.034)}$	$0.051_{(0.031)}$	$0.083_{(0.059)}$	$0.064_{(0.036)}$
Exp 5	$0.881_{(0.033)}$	$0.937_{(0.039)}$	$0.830_{(0.035)}$	$0.050_{(0.031)}$	$0.055_{(0.036)}$	$0.056_{(0.032)}$
Exp 6	$0.900_{(0.030)}$	$0.943_{(0.039)}$	$0.857_{(0.033)}$	$0.052_{(0.031)}$	$0.059_{(0.043)}$	$0.056_{(0.031)}$
Exp 7	$0.803_{(0.036)}$	$0.904_{(0.055)}$	$0.749_{(0.039)}$	$0.051_{(0.029)}$	$0.071_{(0.067)}$	$0.062_{(0.035)}$
Exp 8	$0.822_{(0.037)}$	$0.914_{(0.049)}$	$0.773_{(0.039)}$	$0.052_{(0.033)}$	$0.077_{(0.070)}$	$0.065_{(0.035)}$
Exp 9	$0.946_{(0.014)}$	$0.718_{(0.055)}$	$0.924_{(0.017)}$	$0.050_{(0.031)}$	$0.122_{(0.143)}$	$0.058_{(0.033)}$
Exp 10	$0.967_{(0.031)}$	$0.966_{(0.032)}$	$0.948_{(0.026)}$	$0.052_{(0.031)}$	$0.052_{(0.031)}$	$0.053_{(0.031)}$
Exp 11	$0.893_{(0.030)}$	$0.941_{(0.044)}$	$0.851_{(0.033)}$	$0.945_{(0.032)}$	$0.945_{(0.040)}$	$0.950_{(0.030)}$
Exp 12	$0.920_{(0.026)}$	$0.954_{(0.032)}$	$0.889_{(0.028)}$	$0.944_{(0.032)}$	$0.937_{(0.043)}$	$0.949_{(0.030)}$
Exp 13	$0.820_{(0.037)}$	$0.914_{(0.043)}$	$0.772_{(0.040)}$	$0.924_{(0.038)}$	$0.933_{(0.054)}$	$0.941_{(0.034)}$
Exp 14	$0.879_{(0.031)}$	$0.944_{(0.030)}$	$0.846_{(0.034)}$	$0.922_{(0.038)}$	$0.920_{(0.062)}$	$0.940_{(0.036)}$
Exp 15	$0.883_{(0.031)}$	$0.937_{(0.042)}$	$0.834_{(0.034)}$	$0.950_{(0.031)}$	$0.954_{(0.031)}$	$0.956_{(0.030)}$
Exp 16	$0.897_{(0.029)}$	$0.943_{(0.048)}$	$0.856_{(0.031)}$	$0.942_{(0.034)}$	$0.940_{(0.052)}$	$0.950_{(0.031)}$
Exp 17	$0.793_{(0.036)}$	$0.901_{(0.051)}$	$0.740_{(0.038)}$	$0.925_{(0.038)}$	$0.943_{(0.046)}$	$0.943_{(0.033)}$
Exp 18	$0.817_{(0.035)}$	$0.911_{(0.049)}$	$0.769_{(0.038)}$	$0.926_{(0.037)}$	$0.940_{(0.053)}$	$0.942_{(0.032)}$
Exp 19	$0.945_{(0.015)}$	$0.721_{(0.052)}$	$0.922_{(0.018)}$	$0.932_{(0.036)}$	$0.662_{(0.176)}$	$0.940_{(0.034)}$
Exp 20	$0.965_{(0.036)}$	$0.961_{(0.051)}$	$0.945_{(0.030)}$	$0.956_{(0.029)}$	$0.956_{(0.029)}$	$0.955_{(0.029)}$
Exp 21	$0.991_{(0.001)}$	$0.609_{(0.030)}$	$0.988_{(0.002)}$	$0.053_{(0.031)}$	$0.569_{(0.117)}$	$0.058_{(0.033)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 4: Results for case: N=100, T=50

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.841_{(0.038)}$	$0.868_{(0.038)}$	$0.740_{(0.040)}$	$0.069_{(0.026)}$	$0.069_{(0.026)}$	$0.102_{(0.032)}$
Exp 2	$0.871_{(0.036)}$	$0.895_{(0.034)}$	$0.790_{(0.041)}$	$0.072_{(0.026)}$	$0.073_{(0.027)}$	$0.108_{(0.031)}$
Exp 3	$0.758_{(0.044)}$	$0.806_{(0.042)}$	$0.639_{(0.048)}$	$0.070_{(0.027)}$	$0.079_{(0.027)}$	$0.156_{(0.041)}$
Exp 4	$0.818_{(0.052)}$	$0.856_{(0.047)}$	$0.721_{(0.063)}$	$0.078_{(0.029)}$	$0.088_{(0.027)}$	$0.163_{(0.042)}$
Exp 5	$0.821_{(0.038)}$	$0.852_{(0.039)}$	$0.713_{(0.039)}$	$0.063_{(0.025)}$	$0.068_{(0.025)}$	$0.096_{(0.030)}$
Exp 6	$0.836_{(0.041)}$	$0.863_{(0.040)}$	$0.736_{(0.044)}$	$0.072_{(0.026)}$	$0.073_{(0.026)}$	$0.108_{(0.032)}$
Exp 7	$0.734_{(0.040)}$	$0.786_{(0.040)}$	$0.609_{(0.041)}$	$0.068_{(0.025)}$	$0.077_{(0.029)}$	$0.149_{(0.039)}$
Exp 8	$0.749_{(0.042)}$	$0.798_{(0.041)}$	$0.629_{(0.045)}$	$0.069_{(0.025)}$	$0.081_{(0.028)}$	$0.156_{(0.040)}$
Exp 9	$0.912_{(0.022)}$	$0.696_{(0.058)}$	$0.833_{(0.032)}$	$0.071_{(0.026)}$	$0.036_{(0.021)}$	$0.130_{(0.036)}$
Exp 10	$0.904_{(0.043)}$	$0.904_{(0.043)}$	$0.852_{(0.037)}$	$0.065_{(0.026)}$	$0.065_{(0.026)}$	$0.075_{(0.027)}$
Exp 11	$0.829_{(0.039)}$	$0.859_{(0.039)}$	$0.736_{(0.041)}$	$0.102_{(0.031)}$	$0.115_{(0.034)}$	$0.135_{(0.035)}$
Exp 12	$0.855_{(0.042)}$	$0.880_{(0.041)}$	$0.776_{(0.047)}$	$0.104_{(0.030)}$	$0.112_{(0.033)}$	$0.137_{(0.035)}$
Exp 13	$0.746_{(0.044)}$	$0.800_{(0.042)}$	$0.634_{(0.046)}$	$0.084_{(0.028)}$	$0.119_{(0.034)}$	$0.172_{(0.044)}$
Exp 14	$0.805_{(0.049)}$	$0.847_{(0.044)}$	$0.712_{(0.060)}$	$0.093_{(0.029)}$	$0.124_{(0.034)}$	$0.179_{(0.043)}$
Exp 15	$0.817_{(0.039)}$	$0.853_{(0.040)}$	$0.713_{(0.039)}$	$0.109_{(0.032)}$	$0.128_{(0.034)}$	$0.152_{(0.038)}$
Exp 16	$0.825_{(0.043)}$	$0.857_{(0.043)}$	$0.731_{(0.046)}$	$0.101_{(0.031)}$	$0.118_{(0.032)}$	$0.146_{(0.037)}$
Exp 17	$0.721_{(0.043)}$	$0.780_{(0.043)}$	$0.602_{(0.044)}$	$0.085_{(0.029)}$	$0.122_{(0.034)}$	$0.171_{(0.043)}$
Exp 18	$0.735_{(0.045)}$	$0.790_{(0.045)}$	$0.620_{(0.048)}$	$0.088_{(0.030)}$	$0.124_{(0.032)}$	$0.176_{(0.044)}$
Exp 19	$0.904_{(0.024)}$	$0.686_{(0.055)}$	$0.826_{(0.032)}$	$0.088_{(0.030)}$	$0.050_{(0.023)}$	$0.148_{(0.039)}$
Exp 20	$0.902_{(0.046)}$	$0.902_{(0.047)}$	$0.847_{(0.039)}$	$0.117_{(0.034)}$	$0.117_{(0.034)}$	$0.125_{(0.036)}$
Exp 21	$0.979_{(0.006)}$	$0.696_{(0.048)}$	$0.952_{(0.010)}$	$0.076_{(0.028)}$	$0.109_{(0.063)}$	$0.123_{(0.037)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 5: Results for case: N=100, T=100

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.875_{(0.029)}$	$0.910_{(0.029)}$	$0.798_{(0.030)}$	$0.058_{(0.022)}$	$0.058_{(0.023)}$	$0.070_{(0.025)}$
Exp 2	$0.904_{(0.028)}$	$0.931_{(0.028)}$	$0.843_{(0.032)}$	$0.061_{(0.024)}$	$0.061_{(0.024)}$	$0.075_{(0.026)}$
Exp 3	$0.807_{(0.033)}$	$0.870_{(0.031)}$	$0.711_{(0.036)}$	$0.058_{(0.024)}$	$0.062_{(0.024)}$	$0.091_{(0.028)}$
Exp 4	$0.865_{(0.033)}$	$0.910_{(0.029)}$	$0.793_{(0.041)}$	$0.062_{(0.024)}$	$0.066_{(0.025)}$	$0.093_{(0.030)}$
Exp 5	$0.860_{(0.032)}$	$0.897_{(0.032)}$	$0.773_{(0.033)}$	$0.058_{(0.023)}$	$0.059_{(0.023)}$	$0.072_{(0.026)}$
Exp 6	$0.876_{(0.030)}$	$0.910_{(0.030)}$	$0.798_{(0.032)}$	$0.060_{(0.024)}$	$0.060_{(0.025)}$	$0.072_{(0.027)}$
Exp 7	$0.783_{(0.032)}$	$0.852_{(0.031)}$	$0.679_{(0.034)}$	$0.055_{(0.024)}$	$0.060_{(0.025)}$	$0.090_{(0.029)}$
Exp 8	$0.796_{(0.035)}$	$0.860_{(0.033)}$	$0.696_{(0.037)}$	$0.061_{(0.026)}$	$0.063_{(0.025)}$	$0.093_{(0.030)}$
Exp 9	$0.938_{(0.015)}$	$0.702_{(0.042)}$	$0.883_{(0.021)}$	$0.058_{(0.024)}$	$0.024_{(0.016)}$	$0.081_{(0.028)}$
Exp 10	$0.938_{(0.034)}$	$0.938_{(0.034)}$	$0.898_{(0.028)}$	$0.057_{(0.023)}$	$0.057_{(0.022)}$	$0.063_{(0.024)}$
Exp 11	$0.867_{(0.030)}$	$0.902_{(0.030)}$	$0.792_{(0.031)}$	$0.213_{(0.040)}$	$0.238_{(0.042)}$	$0.236_{(0.044)}$
Exp 12	$0.896_{(0.031)}$	$0.923_{(0.030)}$	$0.837_{(0.034)}$	$0.210_{(0.040)}$	$0.233_{(0.043)}$	$0.229_{(0.045)}$
Exp 13	$0.797_{(0.034)}$	$0.864_{(0.032)}$	$0.704_{(0.037)}$	$0.161_{(0.036)}$	$0.236_{(0.045)}$	$0.230_{(0.044)}$
Exp 14	$0.857_{(0.034)}$	$0.905_{(0.029)}$	$0.786_{(0.040)}$	$0.161_{(0.036)}$	$0.230_{(0.044)}$	$0.228_{(0.044)}$
Exp 15	$0.858_{(0.030)}$	$0.899_{(0.029)}$	$0.772_{(0.032)}$	$0.227_{(0.043)}$	$0.260_{(0.044)}$	$0.264_{(0.045)}$
Exp 16	$0.870_{(0.032)}$	$0.905_{(0.033)}$	$0.798_{(0.033)}$	$0.210_{(0.041)}$	$0.241_{(0.042)}$	$0.245_{(0.044)}$
Exp 17	$0.773_{(0.033)}$	$0.848_{(0.032)}$	$0.672_{(0.035)}$	$0.167_{(0.037)}$	$0.245_{(0.042)}$	$0.235_{(0.044)}$
Exp 18	$0.790_{(0.034)}$	$0.859_{(0.032)}$	$0.694_{(0.038)}$	$0.164_{(0.037)}$	$0.242_{(0.044)}$	$0.238_{(0.041)}$
Exp 19	$0.934_{(0.015)}$	$0.694_{(0.040)}$	$0.879_{(0.020)}$	$0.179_{(0.039)}$	$0.091_{(0.030)}$	$0.228_{(0.043)}$
Exp 20	$0.933_{(0.036)}$	$0.933_{(0.036)}$	$0.891_{(0.032)}$	$0.247_{(0.043)}$	$0.247_{(0.043)}$	$0.251_{(0.043)}$
Exp 21	$0.988_{(0.003)}$	$0.688_{(0.037)}$	$0.974_{(0.005)}$	$0.062_{(0.023)}$	$0.215_{(0.104)}$	$0.082_{(0.026)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 6: Results for case: N=100, T=500

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.918_{(0.020)}$	$0.958_{(0.019)}$	$0.874_{(0.021)}$	$0.051_{(0.022)}$	$0.052_{(0.022)}$	$0.054_{(0.022)}$
Exp 2	$0.939_{(0.017)}$	$0.970_{(0.016)}$	$0.908_{(0.020)}$	$0.050_{(0.022)}$	$0.051_{(0.022)}$	$0.054_{(0.023)}$
Exp 3	$0.859_{(0.024)}$	$0.939_{(0.019)}$	$0.806_{(0.026)}$	$0.052_{(0.022)}$	$0.053_{(0.023)}$	$0.056_{(0.024)}$
Exp 4	$0.910_{(0.019)}$	$0.963_{(0.015)}$	$0.876_{(0.022)}$	$0.054_{(0.023)}$	$0.053_{(0.022)}$	$0.058_{(0.023)}$
Exp 5	$0.906_{(0.022)}$	$0.951_{(0.021)}$	$0.857_{(0.023)}$	$0.052_{(0.022)}$	$0.051_{(0.022)}$	$0.054_{(0.024)}$
Exp 6	$0.920_{(0.021)}$	$0.960_{(0.019)}$	$0.878_{(0.024)}$	$0.052_{(0.021)}$	$0.052_{(0.021)}$	$0.053_{(0.021)}$
Exp 7	$0.841_{(0.024)}$	$0.931_{(0.021)}$	$0.782_{(0.026)}$	$0.051_{(0.021)}$	$0.052_{(0.022)}$	$0.055_{(0.022)}$
Exp 8	$0.856_{(0.023)}$	$0.939_{(0.019)}$	$0.802_{(0.026)}$	$0.051_{(0.023)}$	$0.051_{(0.022)}$	$0.057_{(0.022)}$
Exp 9	$0.963_{(0.008)}$	$0.709_{(0.035)}$	$0.941_{(0.010)}$	$0.053_{(0.021)}$	$0.021_{(0.016)}$	$0.055_{(0.023)}$
Exp 10	$0.971_{(0.022)}$	$0.971_{(0.022)}$	$0.952_{(0.019)}$	$0.051_{(0.022)}$	$0.052_{(0.022)}$	$0.052_{(0.022)}$
Exp 11	$0.913_{(0.021)}$	$0.954_{(0.021)}$	$0.871_{(0.022)}$	$0.945_{(0.022)}$	$0.952_{(0.022)}$	$0.948_{(0.022)}$
Exp 12	$0.934_{(0.019)}$	$0.965_{(0.018)}$	$0.903_{(0.021)}$	$0.944_{(0.023)}$	$0.949_{(0.021)}$	$0.946_{(0.022)}$
Exp 13	$0.854_{(0.024)}$	$0.937_{(0.019)}$	$0.803_{(0.026)}$	$0.929_{(0.025)}$	$0.950_{(0.022)}$	$0.943_{(0.023)}$
Exp 14	$0.907_{(0.020)}$	$0.962_{(0.016)}$	$0.872_{(0.023)}$	$0.927_{(0.027)}$	$0.950_{(0.023)}$	$0.941_{(0.024)}$
Exp 15	$0.905_{(0.021)}$	$0.953_{(0.020)}$	$0.856_{(0.023)}$	$0.950_{(0.022)}$	$0.956_{(0.021)}$	$0.954_{(0.021)}$
Exp 16	$0.916_{(0.022)}$	$0.957_{(0.021)}$	$0.875_{(0.023)}$	$0.944_{(0.022)}$	$0.952_{(0.021)}$	$0.949_{(0.021)}$
Exp 17	$0.834_{(0.024)}$	$0.929_{(0.020)}$	$0.777_{(0.026)}$	$0.933_{(0.024)}$	$0.954_{(0.020)}$	$0.945_{(0.023)}$
Exp 18	$0.852_{(0.024)}$	$0.937_{(0.020)}$	$0.799_{(0.027)}$	$0.929_{(0.025)}$	$0.952_{(0.022)}$	$0.945_{(0.023)}$
Exp 19	$0.963_{(0.008)}$	$0.712_{(0.034)}$	$0.940_{(0.011)}$	$0.935_{(0.025)}$	$0.533_{(0.088)}$	$0.943_{(0.025)}$
Exp 20	$0.968_{(0.025)}$	$0.968_{(0.025)}$	$0.947_{(0.021)}$	$0.952_{(0.020)}$	$0.952_{(0.021)}$	$0.951_{(0.020)}$
Exp 21	$0.995_{(0.001)}$	$0.675_{(0.021)}$	$0.992_{(0.002)}$	$0.053_{(0.022)}$	$0.810_{(0.076)}$	$0.057_{(0.023)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 7: Results for case: N=200, T=50

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.849_{(0.030)}$	$0.869_{(0.029)}$	$0.748_{(0.035)}$	$0.067_{(0.018)}$	$0.069_{(0.018)}$	$0.108_{(0.024)}$
Exp 2	$0.881_{(0.029)}$	$0.897_{(0.028)}$	$0.797_{(0.038)}$	$0.074_{(0.019)}$	$0.075_{(0.020)}$	$0.112_{(0.027)}$
Exp 3	$0.775_{(0.035)}$	$0.810_{(0.033)}$	$0.648_{(0.040)}$	$0.069_{(0.018)}$	$0.078_{(0.020)}$	$0.179_{(0.033)}$
Exp 4	$0.830_{(0.041)}$	$0.857_{(0.038)}$	$0.726_{(0.054)}$	$0.077_{(0.020)}$	$0.088_{(0.022)}$	$0.181_{(0.035)}$
Exp 5	$0.833_{(0.031)}$	$0.855_{(0.031)}$	$0.721_{(0.032)}$	$0.066_{(0.018)}$	$0.066_{(0.018)}$	$0.103_{(0.024)}$
Exp 6	$0.849_{(0.031)}$	$0.869_{(0.031)}$	$0.748_{(0.037)}$	$0.070_{(0.017)}$	$0.071_{(0.018)}$	$0.112_{(0.025)}$
Exp 7	$0.753_{(0.031)}$	$0.791_{(0.031)}$	$0.618_{(0.034)}$	$0.067_{(0.018)}$	$0.077_{(0.019)}$	$0.169_{(0.033)}$
Exp 8	$0.765_{(0.036)}$	$0.801_{(0.034)}$	$0.635_{(0.040)}$	$0.071_{(0.019)}$	$0.080_{(0.020)}$	$0.176_{(0.035)}$
Exp 9	$0.921_{(0.017)}$	$0.689_{(0.053)}$	$0.838_{(0.027)}$	$0.069_{(0.018)}$	$0.035_{(0.014)}$	$0.144_{(0.030)}$
Exp 10	$0.912_{(0.030)}$	$0.912_{(0.030)}$	$0.857_{(0.028)}$	$0.067_{(0.018)}$	$0.067_{(0.018)}$	$0.079_{(0.021)}$
Exp 11	$0.840_{(0.031)}$	$0.862_{(0.030)}$	$0.743_{(0.035)}$	$0.102_{(0.021)}$	$0.114_{(0.024)}$	$0.139_{(0.029)}$
Exp 12	$0.866_{(0.032)}$	$0.885_{(0.030)}$	$0.788_{(0.038)}$	$0.105_{(0.022)}$	$0.110_{(0.024)}$	$0.141_{(0.029)}$
Exp 13	$0.764_{(0.034)}$	$0.805_{(0.033)}$	$0.645_{(0.039)}$	$0.092_{(0.022)}$	$0.119_{(0.024)}$	$0.195_{(0.041)}$
Exp 14	$0.814_{(0.045)}$	$0.848_{(0.040)}$	$0.714_{(0.057)}$	$0.098_{(0.023)}$	$0.125_{(0.026)}$	$0.201_{(0.039)}$
Exp 15	$0.831_{(0.031)}$	$0.858_{(0.031)}$	$0.722_{(0.033)}$	$0.111_{(0.022)}$	$0.130_{(0.024)}$	$0.160_{(0.027)}$
Exp 16	$0.839_{(0.031)}$	$0.863_{(0.030)}$	$0.743_{(0.037)}$	$0.105_{(0.022)}$	$0.118_{(0.023)}$	$0.152_{(0.028)}$
Exp 17	$0.742_{(0.032)}$	$0.787_{(0.031)}$	$0.614_{(0.034)}$	$0.089_{(0.021)}$	$0.123_{(0.023)}$	$0.190_{(0.038)}$
Exp 18	$0.752_{(0.037)}$	$0.795_{(0.035)}$	$0.629_{(0.041)}$	$0.091_{(0.023)}$	$0.124_{(0.027)}$	$0.200_{(0.041)}$
Exp 19	$0.913_{(0.019)}$	$0.687_{(0.050)}$	$0.833_{(0.028)}$	$0.089_{(0.022)}$	$0.049_{(0.017)}$	$0.161_{(0.032)}$
Exp 20	$0.902_{(0.033)}$	$0.902_{(0.033)}$	$0.848_{(0.030)}$	$0.118_{(0.023)}$	$0.118_{(0.022)}$	$0.126_{(0.024)}$
Exp 21	$0.981_{(0.005)}$	$0.694_{(0.046)}$	$0.954_{(0.009)}$	$0.077_{(0.019)}$	$0.111_{(0.057)}$	$0.126_{(0.029)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 8: Results for case: N=50, T=50 and non zero mean factor loadings C

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.881_{(0.039)}$	$0.916_{(0.039)}$	$0.815_{(0.040)}$	$0.070_{(0.037)}$	$0.071_{(0.038)}$	$0.101_{(0.045)}$
Exp 2	$0.904_{(0.036)}$	$0.932_{(0.035)}$	$0.852_{(0.039)}$	$0.073_{(0.037)}$	$0.074_{(0.038)}$	$0.105_{(0.047)}$
Exp 3	$0.817_{(0.045)}$	$0.873_{(0.042)}$	$0.734_{(0.049)}$	$0.068_{(0.035)}$	$0.081_{(0.040)}$	$0.135_{(0.051)}$
Exp 4	$0.865_{(0.046)}$	$0.908_{(0.040)}$	$0.799_{(0.055)}$	$0.075_{(0.038)}$	$0.090_{(0.041)}$	$0.143_{(0.053)}$
Exp 5	$0.867_{(0.042)}$	$0.905_{(0.042)}$	$0.794_{(0.042)}$	$0.064_{(0.033)}$	$0.068_{(0.035)}$	$0.091_{(0.040)}$
Exp 6	$0.881_{(0.042)}$	$0.915_{(0.040)}$	$0.817_{(0.045)}$	$0.070_{(0.036)}$	$0.070_{(0.037)}$	$0.101_{(0.045)}$
Exp 7	$0.798_{(0.046)}$	$0.860_{(0.043)}$	$0.712_{(0.048)}$	$0.065_{(0.035)}$	$0.074_{(0.037)}$	$0.131_{(0.049)}$
Exp 8	$0.807_{(0.047)}$	$0.867_{(0.044)}$	$0.722_{(0.051)}$	$0.070_{(0.036)}$	$0.082_{(0.038)}$	$0.143_{(0.052)}$
Exp 9	$0.921_{(0.023)}$	$0.757_{(0.048)}$	$0.863_{(0.031)}$	$0.071_{(0.036)}$	$0.034_{(0.026)}$	$0.126_{(0.048)}$
Exp 10	$0.938_{(0.048)}$	$0.945_{(0.049)}$	$0.907_{(0.041)}$	$0.071_{(0.036)}$	$0.071_{(0.036)}$	$0.081_{(0.040)}$
Exp 11	$0.878_{(0.042)}$	$0.913_{(0.040)}$	$0.815_{(0.043)}$	$0.101_{(0.043)}$	$0.114_{(0.044)}$	$0.133_{(0.048)}$
Exp 12	$0.900_{(0.041)}$	$0.927_{(0.040)}$	$0.849_{(0.043)}$	$0.103_{(0.042)}$	$0.111_{(0.043)}$	$0.129_{(0.050)}$
Exp 13	$0.808_{(0.046)}$	$0.870_{(0.042)}$	$0.728_{(0.049)}$	$0.085_{(0.042)}$	$0.122_{(0.048)}$	$0.161_{(0.056)}$
Exp 14	$0.860_{(0.048)}$	$0.903_{(0.043)}$	$0.796_{(0.055)}$	$0.091_{(0.043)}$	$0.125_{(0.049)}$	$0.162_{(0.057)}$
Exp 15	$0.863_{(0.041)}$	$0.905_{(0.041)}$	$0.792_{(0.043)}$	$0.105_{(0.043)}$	$0.129_{(0.046)}$	$0.147_{(0.051)}$
Exp 16	$0.875_{(0.045)}$	$0.910_{(0.043)}$	$0.813_{(0.046)}$	$0.104_{(0.046)}$	$0.121_{(0.046)}$	$0.144_{(0.050)}$
Exp 17	$0.790_{(0.044)}$	$0.857_{(0.040)}$	$0.704_{(0.047)}$	$0.083_{(0.039)}$	$0.126_{(0.046)}$	$0.153_{(0.054)}$
Exp 18	$0.797_{(0.046)}$	$0.861_{(0.043)}$	$0.714_{(0.050)}$	$0.085_{(0.040)}$	$0.127_{(0.047)}$	$0.159_{(0.051)}$
Exp 19	$0.919_{(0.025)}$	$0.755_{(0.049)}$	$0.864_{(0.032)}$	$0.086_{(0.040)}$	$0.048_{(0.032)}$	$0.140_{(0.052)}$
Exp 20	$0.932_{(0.048)}$	$0.937_{(0.047)}$	$0.900_{(0.040)}$	$0.121_{(0.049)}$	$0.121_{(0.047)}$	$0.129_{(0.047)}$
Exp 21	$0.983_{(0.006)}$	$0.777_{(0.052)}$	$0.975_{(0.007)}$	$0.075_{(0.036)}$	$0.154_{(0.096)}$	$0.122_{(0.049)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components; Exp. 21: three AR factors (non correlated), no correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 9: Results for case: N=50, T=50, $s = 1$

Exp.[a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.827_{(0.051)}$	$0.829_{(0.050)}$	$0.733_{(0.049)}$	$0.066_{(0.035)}$	$0.066_{(0.035)}$	$0.096_{(0.039)}$
Exp 2	$0.858_{(0.047)}$	$0.860_{(0.048)}$	$0.779_{(0.052)}$	$0.069_{(0.035)}$	$0.073_{(0.036)}$	$0.103_{(0.046)}$
Exp 3	$0.737_{(0.052)}$	$0.741_{(0.052)}$	$0.631_{(0.054)}$	$0.067_{(0.035)}$	$0.071_{(0.038)}$	$0.147_{(0.051)}$
Exp 4	$0.803_{(0.057)}$	$0.806_{(0.057)}$	$0.713_{(0.067)}$	$0.074_{(0.039)}$	$0.079_{(0.039)}$	$0.149_{(0.053)}$
Exp 5	$0.810_{(0.052)}$	$0.814_{(0.052)}$	$0.708_{(0.050)}$	$0.064_{(0.037)}$	$0.069_{(0.037)}$	$0.094_{(0.041)}$
Exp 6	$0.823_{(0.055)}$	$0.825_{(0.055)}$	$0.728_{(0.056)}$	$0.068_{(0.036)}$	$0.070_{(0.035)}$	$0.099_{(0.041)}$
Exp 7	$0.713_{(0.053)}$	$0.717_{(0.053)}$	$0.602_{(0.050)}$	$0.066_{(0.035)}$	$0.070_{(0.037)}$	$0.134_{(0.048)}$
Exp 8	$0.725_{(0.055)}$	$0.728_{(0.055)}$	$0.617_{(0.056)}$	$0.072_{(0.037)}$	$0.072_{(0.039)}$	$0.147_{(0.051)}$
Exp 9	$0.897_{(0.027)}$	$0.897_{(0.028)}$	$0.822_{(0.037)}$	$0.066_{(0.037)}$	$0.071_{(0.037)}$	$0.123_{(0.050)}$
Exp 10	$0.907_{(0.060)}$	$0.908_{(0.060)}$	$0.853_{(0.049)}$	$0.068_{(0.036)}$	$0.069_{(0.036)}$	$0.078_{(0.037)}$
Exp 11	$0.815_{(0.054)}$	$0.820_{(0.055)}$	$0.724_{(0.053)}$	$0.101_{(0.043)}$	$0.111_{(0.044)}$	$0.129_{(0.047)}$
Exp 12	$0.852_{(0.051)}$	$0.856_{(0.051)}$	$0.777_{(0.055)}$	$0.103_{(0.044)}$	$0.114_{(0.045)}$	$0.136_{(0.047)}$
Exp 13	$0.727_{(0.058)}$	$0.733_{(0.056)}$	$0.625_{(0.059)}$	$0.084_{(0.042)}$	$0.105_{(0.044)}$	$0.170_{(0.058)}$
Exp 14	$0.795_{(0.055)}$	$0.800_{(0.056)}$	$0.709_{(0.064)}$	$0.093_{(0.043)}$	$0.113_{(0.044)}$	$0.173_{(0.057)}$
Exp 15	$0.801_{(0.056)}$	$0.805_{(0.056)}$	$0.701_{(0.053)}$	$0.110_{(0.042)}$	$0.124_{(0.045)}$	$0.149_{(0.052)}$
Exp 16	$0.813_{(0.056)}$	$0.818_{(0.055)}$	$0.726_{(0.055)}$	$0.104_{(0.045)}$	$0.116_{(0.048)}$	$0.143_{(0.052)}$
Exp 17	$0.707_{(0.050)}$	$0.713_{(0.050)}$	$0.598_{(0.048)}$	$0.087_{(0.039)}$	$0.109_{(0.044)}$	$0.168_{(0.059)}$
Exp 18	$0.723_{(0.055)}$	$0.729_{(0.055)}$	$0.617_{(0.056)}$	$0.083_{(0.038)}$	$0.106_{(0.043)}$	$0.171_{(0.059)}$
Exp 19	$0.895_{(0.028)}$	$0.896_{(0.028)}$	$0.821_{(0.034)}$	$0.087_{(0.039)}$	$0.107_{(0.041)}$	$0.148_{(0.053)}$
Exp 20	$0.893_{(0.063)}$	$0.894_{(0.062)}$	$0.839_{(0.052)}$	$0.120_{(0.047)}$	$0.119_{(0.047)}$	$0.129_{(0.046)}$

[a] PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components.

[b] Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c] Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 10: Results for case: N=100, T=100, $s = 1$

Exp. [a]	Corr. with True[b]			Serial Correlation[c]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
Exp 1	$0.874_{(0.030)}$	$0.877_{(0.030)}$	$0.794_{(0.032)}$	$0.058_{(0.022)}$	$0.059_{(0.023)}$	$0.073_{(0.026)}$
Exp 2	$0.905_{(0.028)}$	$0.906_{(0.028)}$	$0.844_{(0.031)}$	$0.061_{(0.025)}$	$0.062_{(0.024)}$	$0.075_{(0.026)}$
Exp 3	$0.806_{(0.032)}$	$0.810_{(0.032)}$	$0.709_{(0.035)}$	$0.058_{(0.024)}$	$0.059_{(0.025)}$	$0.092_{(0.028)}$
Exp 4	$0.865_{(0.032)}$	$0.868_{(0.032)}$	$0.792_{(0.041)}$	$0.061_{(0.024)}$	$0.064_{(0.024)}$	$0.095_{(0.029)}$
Exp 5	$0.859_{(0.031)}$	$0.861_{(0.030)}$	$0.771_{(0.032)}$	$0.056_{(0.023)}$	$0.055_{(0.024)}$	$0.067_{(0.025)}$
Exp 6	$0.877_{(0.031)}$	$0.880_{(0.031)}$	$0.800_{(0.033)}$	$0.059_{(0.025)}$	$0.060_{(0.024)}$	$0.074_{(0.026)}$
Exp 7	$0.784_{(0.033)}$	$0.789_{(0.033)}$	$0.680_{(0.034)}$	$0.056_{(0.023)}$	$0.058_{(0.023)}$	$0.089_{(0.028)}$
Exp 8	$0.800_{(0.033)}$	$0.804_{(0.033)}$	$0.701_{(0.037)}$	$0.058_{(0.023)}$	$0.059_{(0.023)}$	$0.094_{(0.029)}$
Exp 9	$0.939_{(0.014)}$	$0.940_{(0.013)}$	$0.884_{(0.019)}$	$0.058_{(0.023)}$	$0.059_{(0.024)}$	$0.085_{(0.029)}$
Exp 10	$0.938_{(0.036)}$	$0.938_{(0.035)}$	$0.896_{(0.029)}$	$0.057_{(0.022)}$	$0.057_{(0.023)}$	$0.062_{(0.025)}$
Exp 11	$0.868_{(0.031)}$	$0.872_{(0.032)}$	$0.792_{(0.032)}$	$0.217_{(0.043)}$	$0.238_{(0.044)}$	$0.244_{(0.044)}$
Exp 12	$0.897_{(0.029)}$	$0.901_{(0.029)}$	$0.839_{(0.032)}$	$0.209_{(0.040)}$	$0.228_{(0.043)}$	$0.231_{(0.044)}$
Exp 13	$0.796_{(0.033)}$	$0.802_{(0.033)}$	$0.703_{(0.036)}$	$0.171_{(0.037)}$	$0.218_{(0.044)}$	$0.238_{(0.044)}$
Exp 14	$0.859_{(0.034)}$	$0.864_{(0.033)}$	$0.790_{(0.041)}$	$0.167_{(0.038)}$	$0.213_{(0.044)}$	$0.232_{(0.043)}$
Exp 15	$0.859_{(0.031)}$	$0.863_{(0.031)}$	$0.773_{(0.033)}$	$0.232_{(0.044)}$	$0.255_{(0.046)}$	$0.261_{(0.044)}$
Exp 16	$0.872_{(0.032)}$	$0.876_{(0.032)}$	$0.798_{(0.034)}$	$0.215_{(0.040)}$	$0.234_{(0.042)}$	$0.245_{(0.046)}$
Exp 17	$0.775_{(0.032)}$	$0.783_{(0.032)}$	$0.673_{(0.034)}$	$0.174_{(0.037)}$	$0.223_{(0.045)}$	$0.243_{(0.044)}$
Exp 18	$0.794_{(0.033)}$	$0.801_{(0.032)}$	$0.698_{(0.036)}$	$0.171_{(0.040)}$	$0.218_{(0.043)}$	$0.247_{(0.046)}$
Exp 19	$0.935_{(0.014)}$	$0.937_{(0.013)}$	$0.880_{(0.019)}$	$0.187_{(0.039)}$	$0.226_{(0.044)}$	$0.235_{(0.041)}$
Exp 20	$0.934_{(0.038)}$	$0.934_{(0.037)}$	$0.893_{(0.032)}$	$0.241_{(0.045)}$	$0.241_{(0.045)}$	$0.245_{(0.045)}$

[a]PCA: Principal Component Estimation Method; DPCA: Dynamic Principal Component Estimation Method; SSS: Subspace algorithm on state space form. Exp. 1-8 : one factor, different ARMA DGP, no correlation among idiosyncratic components; Exp 9: as Exp. 1 but dynamic impact on variables; Exp 10: as Exp. 1 but one factor imposed in estimation rather than p+q; Exp. 11-20: as 1-10 but temporal correlation among idiosyncratic components.

[b]Mean Correlation between true and estimated common component, with MC st.dev. in ().

[c]Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 11: Results for Experiment 21 (3 AR factors (non correlated), no correlation among idiosyncratic components) and $s = 1$

N/T	Corr. with True[a]			Serial Correlation[b]		
	PCA	SSS	DPCA	PCA	SSS	DPCA
$N = 50, T = 50$	$0.9754_{(0.008)}$	$0.9751_{(0.008)}$	$0.9478_{(0.013)}$	$0.076_{(0.040)}$	$0.074_{(0.038)}$	$0.125_{(0.048)}$
$N = 50, T = 100$	$0.9844_{(0.004)}$	$0.9843_{(0.004)}$	$0.9703_{(0.007)}$	$0.062_{(0.033)}$	$0.060_{(0.033)}$	$0.082_{(0.038)}$
$N = 100, T = 50$	$0.9792_{(0.006)}$	$0.9789_{(0.006)}$	$0.9520_{(0.011)}$	$0.076_{(0.028)}$	$0.076_{(0.027)}$	$0.124_{(0.037)}$
$N = 100, T = 100$	$0.9880_{(0.004)}$	$0.9879_{(0.004)}$	$0.9745_{(0.006)}$	$0.063_{(0.025)}$	$0.063_{(0.025)}$	$0.084_{(0.028)}$
$N = 500, T = 50$	$0.9827_{(0.003)}$	$0.9825_{(0.003)}$	$0.9554_{(0.007)}$	$0.076_{(0.013)}$	$0.075_{(0.012)}$	$0.126_{(0.021)}$
$N = 100, T = 500$	$0.9914_{(0.002)}$	$0.9913_{(0.002)}$	$0.9777_{(0.003)}$	$0.061_{(0.010)}$	$0.061_{(0.010)}$	$0.082_{(0.012)}$
$N = 200, T = 50$	$0.9835_{(0.006)}$	$0.9878_{(0.005)}$	$0.9741_{(0.008)}$	$0.074_{(0.039)}$	$0.074_{(0.038)}$	$0.127_{(0.050)}$

[a] Mean Correlation between true and estimated common component, with MC st.dev. in ().

[b] Mean rejection rate of LM serial correlation test of idiosyncratic component, with MC st.dev. in ().

Table 12: US dataset – Fit of factor model and correlations of permanent components

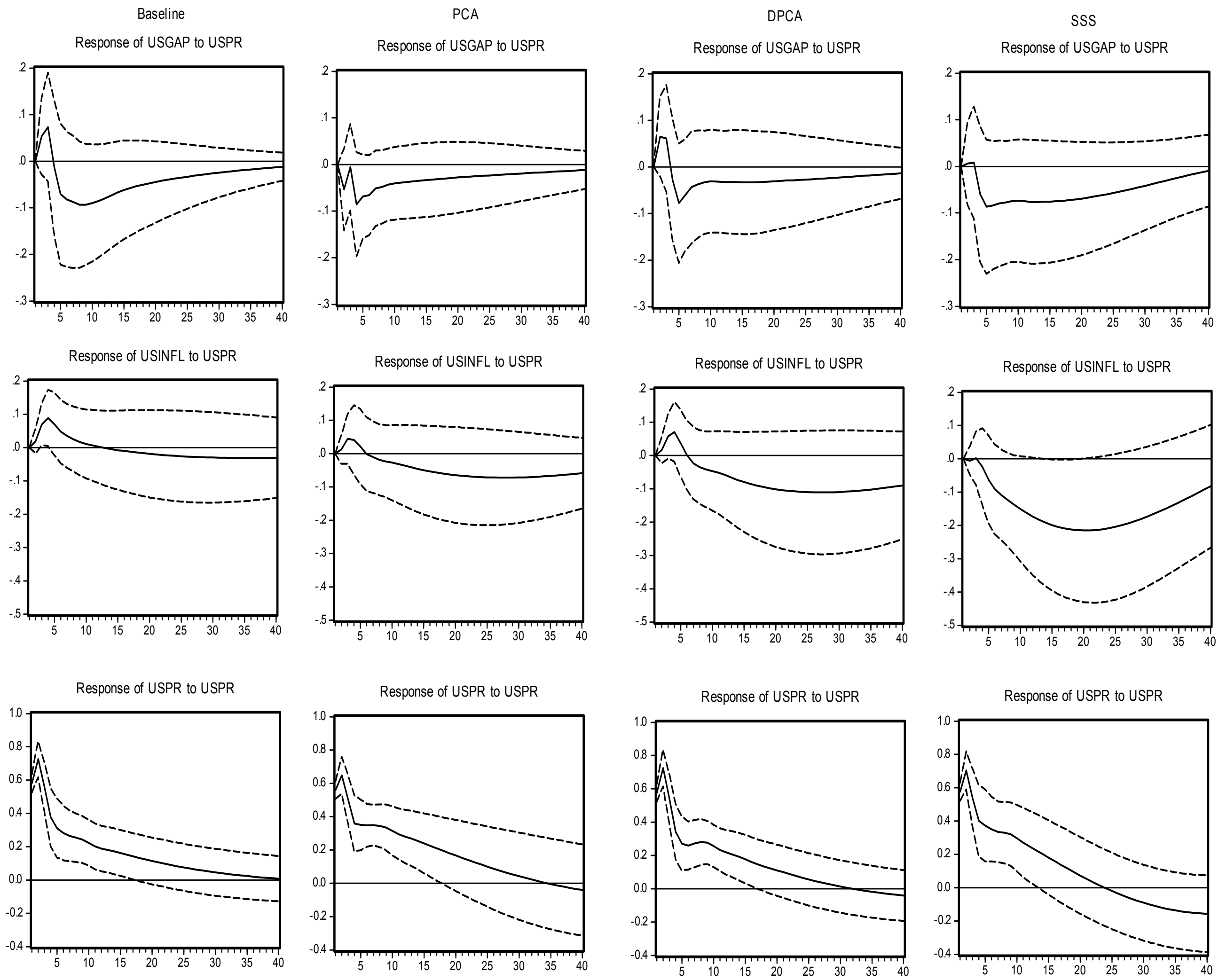
	Adjusted-R^2			Correlations		
Var.	SSS	PCA	DPCA	PCA-DPCA	PCA-SSS	SSS-DPCA
1	0.6814	0.7699	0.829	0.9329	0.9375	0.8917
2	0.6761	0.7129	0.8129	0.9004	0.9454	0.871
3	0.6305	0.6523	0.758	0.8853	0.9541	0.8634
4	0.5283	0.5195	0.7094	0.8214	0.9486	0.8212
5	0.4465	0.4743	0.6009	0.8522	0.9524	0.8309
6	0.2114	0.1758	0.3822	0.6444	0.9379	0.6742
7	0.4345	0.4805	0.4896	0.912	0.9426	0.8652
8	0.3726	0.4408	0.496	0.8939	0.9031	0.8326
9	0.4658	0.5665	0.6118	0.9312	0.9278	0.8785
10	0.3188	0.3673	0.4421	0.8646	0.9158	0.8139
11	0.6825	0.7838	0.8472	0.9326	0.9316	0.8862
12	0.6071	0.7032	0.7573	0.9241	0.9328	0.8742
13	0.4208	0.452	0.5672	0.8653	0.9163	0.8375
14	0.0329	0.0322	0.0646	0.5548	0.9292	0.5701
15	0.0387	0.0631	0.0915	0.4253	0.8912	0.3895
16	0.6165	0.7973	0.8476	0.9427	0.9017	0.8279
17	0.2577	0.3234	0.4394	0.8328	0.8695	0.7011
18	0.634	0.7441	0.8345	0.9348	0.894	0.8326
19	0.239	0.2603	0.3577	0.784	0.9049	0.733
20	0.2608	0.2972	0.3569	0.8385	0.9057	0.7743
21	0.5834	0.8001	0.9311	0.9004	0.8785	0.7624
22	0.669	0.6364	0.8275	0.8727	0.8244	0.8456
23	0.7052	0.8005	0.8323	0.9164	0.8914	0.8877
24	0.7946	0.8686	0.9241	0.9263	0.8816	0.8962
25	0.758	0.8185	0.9228	0.9103	0.8575	0.8709
26	0.7738	0.8629	0.9146	0.9309	0.8761	0.8826
27	0.5916	0.7607	0.7532	0.9388	0.902	0.8422
28	0.6228	0.7719	0.7692	0.9365	0.8983	0.8435
29	0.6263	0.7738	0.7673	0.9495	0.9058	0.8601
30	0.2442	0.2805	0.3475	0.7501	0.8559	0.6864
31	0.5725	0.7312	0.7513	0.9447	0.916	0.8468
32	0.5314	0.673	0.6878	0.9442	0.921	0.8534
33	0.3525	0.4671	0.532	0.8795	0.888	0.7646
34	0.3113	0.4196	0.4553	0.8605	0.9055	0.7614
35	0.3166	0.4155	0.4809	0.867	0.8857	0.7529
36	0.1767	0.2019	0.3267	0.7337	0.8858	0.7101
37	0.1837	0.2109	0.2597	0.8029	0.9155	0.7393
38	0.179	0.1859	0.1657	0.796	0.858	0.7797
39	0.3901	0.2453	0.8319	0.5377	0.7654	0.6717
40	0.3541	0.1466	0.8492	0.4288	0.7255	0.6556
41	0.0134	0.0151	0.1761	0.32	0.8822	0.3223
42	0.0113	0.0103	0.1243	0.3305	0.928	0.3022
43	0.5367	0.6399	0.8634	0.8325	0.8705	0.771
44	0.3702	0.3897	0.6201	0.7503	0.8831	0.7532
45	0.4422	0.4908	0.5396	0.911	0.9119	0.8671
46	0.5508	0.6057	0.7857	0.8327	0.8641	0.7868
47	0.3253	0.4393	0.6302	0.7968	0.8302	0.7102
48	0.5487	0.5976	0.8732	0.8053	0.8541	0.7778

49	0.3917	0.3029	0.564	0.6836	0.8812	0.7745
50	0.7778	0.7765	0.8364	0.9283	0.981	0.9221
51	0.6335	0.6354	0.7501	0.8992	0.9787	0.8885
52	0.5694	0.5818	0.6858	0.9052	0.9766	0.8894
53	0.285	0.2709	0.3764	0.7855	0.9718	0.7875
54	0.297	0.31	0.3867	0.7735	0.9806	0.7679
55	0.3232	0.3262	0.4428	0.8221	0.9649	0.8081
56	0.1686	0.1761	0.175	0.6944	0.9894	0.6878
57	0.3444	0.3476	0.5994	0.6969	0.9797	0.6861
58	0.154	0.1604	0.4729	0.5593	0.972	0.5403
59	0.3281	0.4199	0.4618	0.8007	0.9194	0.7203
60	0.3017	0.408	0.3864	0.8422	0.9052	0.7473
61	0.2657	0.3732	0.3544	0.8636	0.9079	0.7856
62	0.1351	0.1558	0.1316	0.6577	0.9228	0.5539
63	0.1047	0.1085	0.108	0.7662	0.9244	0.7121
64	0.0877	0.1388	0.3009	0.5727	0.9037	0.4793
65	0.7791	0.7885	0.8375	0.9234	0.9798	0.918
66	0.6465	0.6664	0.7411	0.9114	0.9742	0.8984
67	0.293	0.3	0.3778	0.7703	0.9852	0.7703
68	0.2321	0.2513	0.551	0.5616	0.9784	0.5478
69	0.6014	0.7659	0.8804	0.9329	0.9073	0.838
70	0.4828	0.7023	0.8258	0.9167	0.8953	0.7923
71	0.4903	0.7081	0.8179	0.9328	0.8882	0.8041
72	0.5319	0.5553	0.6797	0.8897	0.9108	0.8541
73	0.539	0.6093	0.6724	0.9241	0.9177	0.8666
74	0.6329	0.7839	0.8631	0.9348	0.9211	0.8475
75	0.4916	0.5131	0.6241	0.8864	0.9243	0.8533
76	0.5661	0.5849	0.6218	0.9348	0.9748	0.9136
77	0.5717	0.6329	0.8017	0.8488	0.9704	0.8122
78	0.2186	0.244	0.4698	0.627	0.9437	0.596
79	0.6991	0.7512	0.8295	0.8987	0.9819	0.8716
80	0.5231	0.5734	0.7408	0.8193	0.9743	0.7883
81	0.5658	0.6267	0.7947	0.8458	0.9732	0.8114
82	0.4699	0.5224	0.7255	0.7895	0.9685	0.7563
83	0.4102	0.4062	0.4811	0.8065	0.9888	0.8133
84	0.2102	0.2173	0.3042	0.7549	0.9823	0.7533
85	0.4266	0.4769	0.5993	0.8394	0.9431	0.8024
86	0.4131	0.4635	0.5853	0.8357	0.94	0.7987
87	0.146	0.2514	0.3088	0.7986	0.8595	0.6641
88	0.1631	0.1878	0.4318	0.6002	0.9298	0.5773
89	0.1549	0.1815	0.4328	0.5939	0.9268	0.5687
90	0.0694	0.0812	0.2015	0.5179	0.9669	0.492
91	0.058	0.0626	0.1771	0.4467	0.9338	0.4074
92	0.0335	0.0361	0.1034	0.3441	0.9027	0.3414
93	0.2214	0.2675	0.4387	0.6479	0.9247	0.5903
94	0.0137	0.02	0.1346	0.2883	0.9453	0.2732
95	0.014	0.0157	0.1845	0.2313	0.9432	0.2251
96	0.0517	0.0668	0.1584	0.4858	0.9656	0.4595
97	0.3773	0.4453	0.4041	0.7251	0.9554	0.6749
98	0.384	0.4497	0.401	0.7304	0.9579	0.6828
99	0.3673	0.43	0.3856	0.7272	0.9579	0.6797
100	0.3121	0.3641	0.3231	0.7255	0.9566	0.6743
101	0.313	0.3582	0.286	0.7649	0.9599	0.7398
102	0.4412	0.4389	0.8244	0.6871	0.8703	0.7029

103	0.4357	0.4135	0.7201	0.7238	0.8734	0.7474
104	0.4303	0.473	0.5235	0.7507	0.9726	0.741
105	0.4134	0.4591	0.5108	0.733	0.9712	0.7214
106	0.452	0.5108	0.4867	0.7773	0.9634	0.7531
107	0.4545	0.5214	0.4437	0.8104	0.95	0.7654
108	0.3232	0.3427	0.4046	0.7212	0.9657	0.7204
109	0.2168	0.2478	0.1945	0.6611	0.9165	0.6907
110	0.1852	0.1882	0.1817	0.6266	0.9296	0.6689
111	0.1829	0.1912	0.1947	0.6069	0.9281	0.6443
112	0.1316	0.1785	0.0957	0.6858	0.871	0.6749
113	0.0796	0.0759	0.0786	0.6324	0.9539	0.6083
114	0.5288	0.6087	0.6156	0.9139	0.9311	0.8603
115	0.264	0.2741	0.2966	0.7347	0.9901	0.7302
116	0.2833	0.2949	0.316	0.7504	0.9904	0.7439
117	0.0218	0.0219	0.0615	0.4527	0.9649	0.4569
118	0.6339	0.6569	0.5729	0.8286	0.986	0.8158
119	0.0793	0.0814	0.0569	0.6599	0.9912	0.6518
120	0.2821	0.2967	0.2632	0.7351	0.9852	0.7235
121	-0.0004	0.0007	0.1036	0.2642	0.9734	0.2445
122	0.6449	0.6732	0.5692	0.8349	0.9889	0.8213
123	0.0242	0.0263	0.0588	0.5574	0.9742	0.542
124	0.0364	0.0356	0.2172	0.3279	0.9671	0.3277
125	0.3036	0.3109	0.2654	0.771	0.9848	0.757
126	0.6334	0.6511	0.5024	0.8488	0.9916	0.8431
127	0.5824	0.6004	0.5246	0.8198	0.9857	0.808
128	0.6372	0.6647	0.5359	0.8291	0.9895	0.819
129	0.0163	0.0209	0.0454	0.5307	0.958	0.4847
130	0.6828	0.7039	0.5455	0.8449	0.9907	0.8359
131	0.0852	0.0925	0.1441	0.5238	0.9776	0.5111
132	0.2675	0.3751	0.4279	0.8325	0.9008	0.7214
133	0.3626	0.3897	0.5481	0.7041	0.9841	0.6871
134	0.23	0.247	0.4103	0.6295	0.9864	0.6137
135	0.182	0.191	0.4429	0.5765	0.9797	0.5624
136	0.1009	0.1119	0.1045	0.7097	0.9448	0.7012
137	0.1908	0.2008	0.3419	0.5938	0.9852	0.5813
138	0.6577	0.7324	0.6881	0.9026	0.94	0.8659
139	0.71	0.8267	0.7605	0.9401	0.9255	0.8699
140	0.7093	0.8429	0.8057	0.9315	0.9231	0.8546
141	0.5761	0.7534	0.7613	0.9163	0.906	0.8063
142	0.6671	0.852	0.8994	0.9305	0.8844	0.8361
143	0.6792	0.8552	0.9104	0.9301	0.8775	0.8409
144	0.7032	0.8496	0.9087	0.9325	0.8691	0.8632
145	0.6404	0.8514	0.8978	0.9426	0.8644	0.8439
146	0.6837	0.8601	0.8902	0.941	0.8799	0.8527
mean	0.3875	0.4365	0.5179	0.7644	0.9295	0.7288
sd	0.2207	0.2557	0.2587	0.1682	0.0468	0.1513

Note: The table reports the adjusted R2 in a regression of the variable on the common component and the correlation between each pair of estimated common components
See the Data Appendix for variable definitions.

Figure 1: The role of factors in monetary VARs



Note: Impulse response function to an interest rate shock in the base case (no factors in VAR), with static principal components (PCA), dynamic principal components (DPCA), and state space factors (SSS).

DATA APPENDIX

This appendix lists the variables used in the empirical analysis,
with a short description and the transformation applied.
The transformation codes are: 1 = no transformation; 2 = first difference; 3= second difference; 4 = logarithm; 5 = first difference of logarithm; 6 = second difference of logarithm.

Variable	*Transf*
1 INDUSTRIAL PRODUCTION: TOTAL INDEX(1992=100,SA)	5
2 INDUSTRIAL PRODUCTION: PRODUCTS,TOTAL(1992=100,SA)	5
3 INDUSTRIAL PRODUCTION: FINAL PRODUCTS(1992=100,SA)	5
4 INDUSTRIAL PRODUCTION: CONSUMER GOODS(1992=100,SA)	5
5 INDUSTRIAL PRODUCTION: DURABLE CONSUMER GOODS(1992=100,SA)	5
6 INDUSTRIAL PRODUCTION: NONDURABLE CONDSUMER GOODS(1992=100,SA)	5
7 INDUSTRIAL PRODUCTION: BUSINESS EQUIPMENT(1992=100,SA)	5
8 INDUSTRIAL PRODUCTION: INTERMEDIATE PRODUCTS(1992=100,SA)	5
9 INDUSTRIAL PRODUCTION: MATERIALS(1992=100,SA)	5
10 INDUSTRIAL PRODUCTION: NONDURABLE GOODS MATERIALS(1992=100,SA)	5
11 INDUSTRIAL PRODUCTION: MANUFACTURING(1992=100,SA)	5
12 INDUSTRIAL PRODUCTION: DURABLE MANUFACTURING(1992=100,SA)	5
13 INDUSTRIAL PRODUCTION: NONDURABLE MANUFACTURING(1992=100,SA)	5
14 INDUSTRIAL PRODUCTION: MINING(1992=100,SA)	5
15 INDUSTRIAL PRODUCTION: UTILITIES(1992-=100,SA)	5
16 CAPACITY UTIL RATE: MANUFACTURING,TOTAL(%OF CAPACITY,SA)(FRB)	1
17 PURCHASING MANAGERS' INDEX (SA)	1
18 NAPM PRODUCTION INDEX (PERCENT)	1
19 PERSONAL INCOME (CHAINED) (BIL 92$, SAAR)	5
20 INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS "(1967=100;SA)"	5
21 EMPLOYMENT: "RATIO;" HELP-WANTED ADS:NO.UNEMPLOYED CLF	4
22 CIVILIAN LABOR FORCE:EMPLOYED,TOTAL (THOUS.,SA)	5
23 CIVILIAN LABOR FORCE:EMPLOYED,NONAGRIC.INDUSTRIES(THOUS.,SA)	5
24 UNEMPLOYMENT RATE:ALL WORKERS,16 YEARS & OVER(%,SA)	1
25 UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS(SA)	1
26 UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5WKS(THOUS.,SA)	1
27 UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS(THOUS.,SA)	1
28 UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS +(THOUS.,SA)	1
29 UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS(THOUS.,SA)	1
30 EMPLOYEES ON NONAG.PAYROLLS:TOTAL(THOUS.,SA)	5
31 EMPLOYEES ON NONAG.PAYROLLS:TOTAL,PRIVATE (THOUS,SA)	5
32 EMPLOYEES ON NONAG.PAYROLLS:GOODS-PRODUCING(THOUS.,SA)	5
33 EMPLOYEES ON NONAG.PAYROLLS:CONTRACT CONSTRUCTION(THOUS.,SA)	5
34 EMPLOYEES ON NONAG.PAYROLLS:MANUFACTURING(THOUS.,SA)	5
35 EMPLOYEES ON NONAG.PAYROLLS:DURABLE GOODS(THOUS.,SA)	5
36 EMPLOYEES ON NONAG.PAYROLLS:NONDURABLE GOODS(THOUS.,SA)	5
37 EMPLOYEES ON NONAG.PAYROLLS:SERVICE-PRODUCING(THOUS.,SA)	5
38 EMPLOYEES ON NONAG.PAYROLLS:WHOLESALE & RETAIL TRADE (THOUS.,SA)	5
39 EMPLOYEES ON NONAG.PAYROLLS:FINANCE,INSUR.&REAL ESTATE (THOUS.,SA	5
40 EMPLOYEES ON NONAG.PAYROLLS:SERVICES(THOUS.,SA)	5
41 EMPLOYEES ON NONAG.PAYROLLS:GOVERNMENT(THOUS.,SA)	5
42 AVG. WEEKLY HRS. OF PRODUCTION WKRS.: MANUFACTURING (SA)	1
43 AVG. WEEKLY HRS. OF PROD. WKRS.:MFG., OVERTIME HRS. (SA)	1
44 NAPM employment index (percent)	1
45 MANUFACTURING & TRADE: TOTAL(MIL OF CHAINED 1992 DOLLARS)(SA)	5

46	MANUFACTURING & "TRADE: MANUFACTURING;TOTAL	5
47	MANUFACTURING & "TRADE: MFG;" DURABLE GOODS	5
48	MANUFACT.& "TRADE:MFG;NONDURABLE" GOODS	5
49	MERCHANT WHOLESALERS: TOTAL (MIL OF CHAINED 1992 DOLLARS)(SA)	5
50	MERCHANT WHOLESALERS:DURABLE GOODS TOTAL	5
51	MERCHANT WHOLESALERS:NONDURABLE GOODS	5
52	RETAILTRADE: TOTAL (MIL OF CHAINED 1992 DOLLARS)(SA)	5
53	RETAILTRADE: NONDURABLE GOODS (MIL OF 1992 DOLLARS)(SA)	5
54	PERSONAL CONSUMPTION EXPEND (CHAINED)-TOTAL(BIL 92$,SAAR)	5
55	PERSONAL CONSUMPTION EXPEND (CHAINED)-TOTAL DURABLES(BIL 92$,SAAR)	5
56	PERSONAL CONSUMPTION EXPEND (CHAINED)-NONDURABLES(BIL 92$,SAAR)	5
57	PERSONAL CONSUMPTION EXPEND (CHAINED)-SERVICES(BIL 92$,SAAR)	5
58	PERSONAL CONS EXPEND (CHAINED)-NEW CARS (BIL 92$,SAAR)	5
59	HOUSING "STARTS: NONFARM(1947-58);TOTAL" FARM&NONFARM(1959-)(THOUS.,SA	4
60	HOUSING STARTS: NORTHEAST (THOUS.U.)S.A.	4
61	HOUSING STARTS: MIDWEST (THOUS.U.)S.A.	4
62	HOUSING STARTS: SOUTH (THOUS.U.)S.A.	4
63	HOUSING STARTS: WEST (THOUS.U.)S.A.	4
64	HOUSING AUTHORIZED:TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)	4
65	MOBILE HOMES: MANUFACTURERS' SHIPMENTS(THOUS.OF UNITS,SAAR)	4
66	MANUFACTURING & TRADE INVENTORIES:TOTAL(MIL OF CHAINED 1992)(SA)	5
67	INVENTORIES,BUSINESS,MFG(MIL OF CHAINED 1992 DOLLARS, SA)	5
68	INVENTORIES,BUSINESS DURABLES(MIL OF CHAINED 1992 DOLLARS, SA)	5
69	INVENTORIES,BUSINESS,NONDURABLES(MIL OF CHAINED 1992 DOLLARS, SA)	5
70	MANUFACTURING & TRADE INV:MERCHANT WHOLESALERS	5
71	MANUFACTURING & TRADE INV:RETAIL TRADE (MIL OF CHAINED 1992 DOLLARS)(SA)	5
72	RATIO FOR MFG & TRADE:INVENTORY/SALES (CHAINED 1992 DOLLARS, SA)	2
73	RATIO FOR MFG & "TRADE:MFG;INVENTORY/SALES"(87$)(S.A.)	2
74	RATIO FOR MFG & "TRADE:WHOLESALER;INVENTORY/SALES(87$)(S.A.)"	2
75	RATIO FOR MFG & TRADE:RETAIL"TRADE;INVENTORY/SALES(87$)(S.A.)"	2
76	NAPM INVENTORIES INDEX (PERCENT)	1
77	NAPM NEW ORDERS INDEX (PERCENT)	1
78	NAPM VENDOR DELIVERIES INDEX (PERCENT)	1
79	NEW ORDERS (NET)-CONSUMER GOODS & MATERIALS, 1992 DOLLARS(BCI)	5
80	NEW ORDERS, DURABLE GOODS INDUSTRIES, 1992 DOLLARS(BCI)	5
81	NEW ORDERS, NONDEFENSE CAPITAL GOODS,IN 1992 DOLLARS(BCI)	5
82	MFG NEW ORDERS:ALL MANUFACTURING INDUSTRIES,TOTAL(MIL$,SA)	5
83	MFG NEW ORDERS:MFG INDUSTRIES WITH UNFILLED ORDERS(MIL$,SA)	5
84	MFG NEW ORDERS:DURABLE GOODS INDUSTRIES, TOTAL(MIL$,SA)	5
85	MFG NEW ORDERS:DURABLE GOODS INDUST WITH UNFILLED ORDERS(MIL$,SA)	5
86	MFG NEW ORDERS:NONDURABLE GOODS INDUSTRIES, TOTAL (MIL$,SA)	5
87	MFG NEW ORDERS:NONDURABLE GDS IND.WITH UNFILLED ORDERS(MIL$,SA)	5
88	MFG UNFILLED ORDERS: ALL MANUFACTURING INDUSTRIES,TOTAL(MIL$,SA)	5
89	MFG UNFILLED ORDERS: DURABLE GOODS INDUSTRIES,TOTAL(MIL$,SA)	5
90	MFG UNFILLED ORDERS: NONDURABLE GOODS INDUSTRIES, TOTAL(MIL$,SA)	5
91	CONTRACTS & ORDERS FOR PLANT & EQUIPMENT (BIL$,SA)	5
92	CONTRACTS & ORDERS FOR PLANT & EQUIPMENT IN 1992 DOLLARS(BCI)	5
93	NYSE COMMON STOCK PRICE INDEX: COMPOSITE (12/31/65=50)	5
94	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)	5
95	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS(1941-43=10)	5
96	S&P'S COMMON STOCK PRICE INDEX: CAPITAL GOODS (1941-43=10)	5
97	S&P'S COMMON STOCK PRICE INDEX: UTILITIES (1941-43=10)	5
98	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD(% PER ANNUM)	1
99	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO(%,NSA)	1

100 UNITED "STATES;EFFECTIVE" EXCHANGE RATE(MERM)(INDEX NO.)	5
101 FOREIGN EXCHANGE RATE: GERMANY(DEUTSCHE MARK PER U.S.$)	5
102 FOREIGN EXCHANGE RATE: SWITZERLAND(SWISS FRANC PER U.S.$)	5
103 FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)	5
104 FOREIGN EXCHANGE RATE: CANADA(CANADIAN $ PER U.S.$)	5
105 INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)	2
106 INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)	2
107 BOND YIELD: MOODY'S AAA CORPORATE(%PER ANNUM)	2
108 BOND YIELD: MOODY'S BAA CORPORATE(%PER ANNUM)	2
109 SECONDARY MARKET YIELDS ON FHA MORTGAGES(%PER ANNUM)	2
110 Spread FYCP -FYFF	1
111 Spread FYGM3-FYFF	1
112 Spread FYGM6-FYFF	1
113 Spread FYGT1-FYFF	1
114 Spread FYGT5-FYFF	1
115 Spread FYGT10-FYFF	1
116 Spread FYAAAC-FYFF	1
117 Spread FYBAAC - FYFF	1
118 Spread FYFHA-FYFF	1
119 MONEY STOCK:M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)	6
120 MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP	6
121 MONEY STOCK:M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL$,SA)	6
122 MONEY SUPPLY-M2 IN 1992 DOLLARS (BCI)	5
123 MONETARY BASE,ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)	6
124 DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)	6
125 DEPOSITORY INST RESERVES:NONBORROW+EXT CR,ADJ RES REQ CGS(MIL$,SA)	6
126 NAPM COMMODITY PRICES INDEX (PERCENT)	1
127 PRODUCER PRICE INDEX: FINISHED GOODS(82=100,SA)	6
128 PRODUCER PRICE INDEX: FINISHED CONSUMER GOODS(82=100,SA)	6
129 INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)	6
130 CPI-U: ALL ITEMS(82-84=100,SA)	6
131 CPI-U: APPAREL & UPKEEP(82-84=100,SA)	6
132 CPI-U: TRANSPORTATION(82-84=100,SA)	6
133 CPI-U: MEDICAL CARE(82-84=100,SA)	6
134 CPI-U: COMMODITIES(82-84=100,SA)	6
135 CPI-U: DURABLES(82-84=100,SA)	6
136 CPI-U: SERVICES(82-84=100,SA)	6
137 CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)	6
138 CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)	6
139 CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)	6
140 PCE,IMPL PR DEFL:PCE (1987=100)	6
141 PCE,IMPL PR "DEFL:PCE;" DURABLES (1987=100)	6
142 PCE,IMPL PR "DEFL:PCE;" NONDURABLES (1987=100)	6
143 PCE,IMPL PR "DEFL:PCE;" SERVICES (1987=100)	6
144 AVG HR EARNINGS OF CONSTR WKRS: CONSTRUCTION ($,SA)	6
145 AVG HR EARNINGS OF PROD WKRS: MANUFACTURING ($,SA)	6
146 U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)	1