Problem Set 2

Due: Thursday, September 20, 2007

1. Consider the stationary time series generated by

$$y_t = 0.9y_{t-1} - 0.2y_{t-2} + \varepsilon_t$$

where the ε_t are white noise with mean zero and variance 1.

   (a) Find the best linear predictor of y_{T+3} based on $(y_T, y_{T-1}, ...)$, the infinite past history up to time T .

   (b) Find the best linear predictor of y_0 given observations $(y_1,y_2, ...)$.

2. Consider the ARMA(2,2) process

$$y_t = y_{t-1} - .29y_{t-2} + \varepsilon_t + 3\varepsilon_{t-1} - 10\varepsilon_{t-2},$$

where the ε_t are white noise with mean zero and variance 1.

   (a) Show that the autoregresive lag polynomial is invertible but the moving average lag polynomial is not.

   (b) Find an alternative ARMA representation that has both polynomials invertible.

   (c) Define $P_s(y_t)$ to be the best linear predictor of y_t given the infinite past history $y_s, y_{s-1}, y_{s-2}, ...$ Given $y_T = y_{T-1} = 1$ and $P_{T-1}(y_T) = P_{T-2}(y_{T-1}) = 1$, find $P_T(y_{T+1})$.

3. Suppose $\{y_t\}$ is a stationary MA(1) process generated as $y_t = \varepsilon_t + \theta\varepsilon_{t-1}$, where the ε's are white noise with mean zero and variance one.

   (a) Find the best linear predictor of y_2 given y_1.

   (b) Find the best linear predictor of y_3 given y_1 and y_2.

   (c) Find a state space representation of the model.

   (d) Using the appropriate initial condition, compute the best linear predictor of part (a) from the Kalman filter.

4. Consider the MA(1) model $y_t = \varepsilon_t + \theta\varepsilon_{t-1}$ where the ε's are i.i.d. normal random variables with mean zero and variance σ^2. Suppose that, in a sample of 100 observations, the approximate maximum likelihood estimator of σ^2 turns out to be 6.25 and the approximate maximum likelihood estimator of θ turns out to be 0.3.

   (a) Is the estimate of θ significantly different from zero?

   (b) How would your answer change if the model were $y_t = \varepsilon_t + \theta\varepsilon_{t-2}$?

5. Suppose the observed time series $y_1, y_2, ..., y_T$ was generated as

$$y_t = \eta_t + \theta(\eta_{t-1}^2 - 1)$$

where $\eta_0, \eta_1, ..., \eta_T$ are unobserved i.i.d. N(0,1) innovations and θ is an unknown constant.

(a) Are the y_t stationary? White noise? Mutually indpendent? Normal? Explain.

(b) Find the best *linear* predictor of y_t given its past history $(y_{t-1}, y_{t-2}, ...)$.

(c) Suppose the model were modified slightly by assuming $\eta_0 = 1$. Derive the joint probability density function for $y_1, , , y_T$ under this specification. Explain (in detail) how you could compute the maximum likelihood estimate of θ using the Gauss-Newton algorithm.

(d) Using the likelihood fucntion of part (c), explain how you would compute the score statistic for testing the hypothesis that $\theta = 0$.