Problem Set 4

Due: Tuesday, October 16, 2007

1. Consider the model

$$x_t = \varepsilon_t + a\eta_{t-1} \qquad \text{and} \qquad y_t = \eta_t + b\varepsilon_{t-1} \quad .$$

where $\{\varepsilon_t\}$ and $\{\eta_t\}$ are independent white noise processes with zero mean and unit variance.

   (a) Show that, individually, $\{x_t\}$ and $\{y_t\}$ are white noise series.

   (b) Find the best linear predictor of y_t given all past, present, and future values of x_t

   (c) For what parameter values will there be one-way Granger causality from x to y?

2. Consider the bivariate time-series model

$$\begin{aligned} y_t &= \beta x_{t-1} + u_t \\ x_t &= \gamma x_{t-1} + v_t \end{aligned}$$

where $|\beta|$ is less than one and $0 < \gamma < 1$. The processes $\{u_t\}$ and $\{v_t\}$ are independent normal white noise, each with known variance one. We observe $2T$ successive observations $y_1, y_2, ..., y_{2T-1}, y_{2T}$ but only the T even dated values $x_2, x_4, ..., x_{2T}$. Solving out the unobserved x's, we obtain, for $t \geq 3$,

$$\begin{aligned} y_t &= \beta x_{t-1} + u_t \qquad (t \text{ odd}) \\ y_t &= \beta\gamma x_{t-2} + u_t + \beta v_{t-1} \qquad (t \text{ even}) \\ x_t &= \gamma^2 x_{t-2} + v_t + \gamma v_{t-1} \qquad (t \text{ even}) \end{aligned}$$

It seems reasonable to estimate the parameters (β, γ) by generalized method of moments; that is, one might minimize $g_T(\beta,\gamma)' W_T g_T(\beta,\gamma)$ for some appropriate vector g_T and matrix W_T.

   (a) Find an appropriate 3-dimensional "moment" vector g_T based on the orthogonality condition that, in each of the above three equations, the observed right-hand variable is uncorrelated with the unobserved error component.

   (b) Find an expression for V, the approximate variance matrix for your vector g_T.

   (c) Efficiency considerations suggest using some preliminary consistent estimate of (β, γ) to obtain an estimate of V and set $W_T = \widehat{V}^{-1}$. Suggest a preliminary estimate based on two OLS regressions.

   (d) Find a general expression for the asymptotic variance of the efficient GMM estimator of (β, γ) using your vector g_T. Evaluate this expression when β happens to be zero.