THE
QUARTERLY JOURNAL
OF ECONOMICS

Vol. 130 May 2015 Issue 2

AGGREGATE DEMAND, IDLE TIME, AND UNEMPLOYMENT*

Pascal Michaillat and Emmanuel Saez

This article develops a model of unemployment fluctuations. The model keeps the architecture of the general-disequilibrium model of Barro and Grossman (1971) but takes a matching approach to the labor and product markets instead of a disequilibrium approach. On the product and labor markets, both price and tightness adjust to equalize supply and demand. Since there are two equilibrium variables but only one equilibrium condition on each market, a price mechanism is needed to select an equilibrium. We focus on two polar mechanisms: fixed prices and competitive prices. When prices are fixed, aggregate demand affects unemployment as follows. An increase in aggregate demand leads firms to find more customers. This reduces the idle time of their employees and thus increases their labor demand. This in turn reduces unemployment. We combine the predictions of the model and empirical measures of product market tightness, labor market tightness, output, and

*This article was previously circulated under the titles "A Theory of Aggregate Supply and Aggregate Demand as Functions of Market Tightness with Prices as Parameters" and "A Model of Aggregate Demand and Unemployment." We thank George Akerlof, Andrew Atkeson, Paul Beaudry, Francesco Caselli, Varanya Chaubey, James Costain, Wouter den Haan, Peter Diamond, Emmanuel Farhi, Roger Farmer, John Fernald, Xavier Gabaix, Yuriy Gorodnichenko, Pierre-Olivier Gourinchas, Philipp Kircher, David Lagakos, Etienne Lehmann, Alan Manning, Emi Nakamura, Maurice Obstfeld, Nicolas Petrosky-Nadeau, Franck Portier, Valerie Ramey, Pontus Rendhal, Kevin Sheedy, Robert Shimer, Stefanie Stantcheva, Jón Steinsson, Silvana Tenreyro, Carl Walsh, Johannes Wieland, Danny Yagan, four referees, the editor Robert Barro, and numerous seminar and conference participants for helpful discussions and comments. This work was supported by the Center for Equitable Growth at the University of California Berkeley, the British Academy, the Economic and Social Research Council [grant number ES/K008641/1], the Banque de France foundation, the Institute for New Economic Thinking, and the W.E. Upjohn Institute for Employment Research.

The Quarterly Journal of Economics (2015), 507–569. doi:10.1093/qje/qjv006.
Advance Access publication on February 8, 2015.

employment to assess the sources of labor market fluctuations in the United States. First, we find that product market tightness and labor market tightness fluctuate a lot, which implies that the fixed-price equilibrium describes the data better than the competitive-price equilibrium. Next, we find that labor market tightness and employment are positively correlated, which suggests that the labor market fluctuations are mostly due to labor demand shocks and not to labor supply or mismatch shocks. Last, we find that product market tightness and output are positively correlated, which suggests that the labor demand shocks mostly reflect aggregate demand shocks and not technology shocks. *JEL* Codes: E10, E24, E30, J2, J64.

I. Introduction

Numerous hypotheses have been formulated and empirically tested to explain the extent and persistence of unemployment in the United States between December 2008 and November 2013. Over that five-year period, the unemployment rate remained above 7 percent, peaking at 10 percent in October 2009. These hypotheses include high labor market mismatch, caused by major shocks to the financial and housing sectors; low job search effort from unemployed workers, triggered by the long extension of unemployment insurance benefits; and low aggregate demand, caused by a sudden need to repay debts or by pessimism.[1] Low technology is another natural hypothesis since technology shocks are the main source of fluctuations in the textbook model of unemployment.[2]

We have learned a lot from this work. Yet our understanding of this period of high unemployment and of the cyclical fluctuations of the labor market in general remains incomplete. There is a view that to make progress, we need a macroeconomic model that describes the many sources of labor market fluctuations, including aggregate demand, while permitting comparative-statics analysis. The aim of this article is to develop such a model and use it to assess the sources of labor market fluctuations in the United States.

Our starting point is the general-disequilibrium model of Barro and Grossman (1971). The Barro-Grossman model was the first microfounded representation of the macroeconomic theory of Keynes (1936). The model elegantly captures the link

1. On mismatch, see Sahin et al. (2014), Lazear and Spletzer (2012), and Diamond (2013). On job search effort, see Elsby, Hobijn, and Sahin (2010), Rothstein (2011), and Farber and Valletta (2013). On aggregate demand, see Farmer (2012) and Mian and Sufi (2014).

2. See for instance Pissarides (2000) and Shimer (2005).

between aggregate demand and unemployment, so it is a promising starting point.[3] However, it suffers from some limitations because it relies on disequilibrium, whereby the price is fixed and demand and supply may not be equal. First, disequilibrium raises difficult theoretical questions—for instance, how to ration those who cannot buy or sell what they would like at the prevailing price. Second, disequilibrium limits tractability because the economy can be in four different regimes, each described by a different system of equations, depending on which sides of the product and labor markets are rationed.

We keep the architecture of the Barro-Grossman model: our model is static; it has a produced good, labor, and money; the product and labor markets are formally symmetric. But to address the limitations of the Barro-Grossman model, we take a matching approach to the product and labor markets instead of a disequilibrium approach: on each market, a matching function governs the number of trades and buyers incur a matching cost.

The matching approach allows us to move into general-equilibrium theory. A matching market is analogous to a Walrasian market in which a seller takes as given not only the price but also the probability to sell, and a buyer takes as given not only the price but also a price wedge reflecting the cost of matching. The selling probability and price wedge are determined by the market tightness. Hence, the matching equilibrium is analogous to a Walrasian equilibrium in which not only prices but also tightnesses equalize supply and demand on all markets.

Although grounded in equilibrium theory, the matching approach allows us to introduce the price and real-wage rigidities required for aggregate demand to influence unemployment. Indeed, on each matching market, price and tightness adjust to equalize supply and demand. Since there are two equilibrium variables (price and tightness) but only one equilibrium condition (supply equals demand) on each matching market, many combinations of prices and tightnesses satisfy all the equilibrium conditions. This means that many price mechanisms are consistent with equilibrium; of course, once a price mechanism is specified,

3. General disequilibrium generated a vast amount of research. For surveys, see Grandmont (1977), Drazen (1980), and Bénassy (1993). For book-length treatments, see Barro and Grossman (1976) and Malinvaud (1977). For recent applications, see Mankiw and Weinzierl (2011), Caballero and Farhi (2014), and Korinek and Simsek (2014).

the equilibrium is unique. To understand the effects of aggregate demand shocks, we study an equilibrium in which the price and real wage are fixed and the product market and labor market tightnesses equalize supply and demand on all markets. In addition, we contrast the properties of this fixprice equilibrium with those of an equilibrium in which the price and real wage are competitive—they ensure that the tightnesses always maximize consumption, in the spirit of Moen (1997). We also show that the results for fixed prices hold under partially rigid prices, and the results for competitive prices hold under Nash bargained prices.

The matching approach also allows us to describe the general equilibrium of the model with one system of well-behaved equations while preserving the property of the disequilibrium approach that market conditions are favorable sometimes to buyers and sometimes to sellers. This property is essential for the propagation of aggregate demand shocks to unemployment. In the Barro-Grossman model, sellers and buyers are in a binary situation on each market—rationed or not rationed. In our model, the conditions on each matching market are captured by a tightness: a high tightness is favorable to sellers and a low tightness is favorable to buyers. Because the tightnesses are continuous and not binary variables, the equilibrium equations are well behaved and the model is tractable.

Our model generates predictions concerning the comparative static effects of aggregate demand shocks on unemployment and other variables. Despite the different formalism, our model retains the intuition of the Barro-Grossman model that negative aggregate demand shocks propagate to the labor market by making it more difficult for firms to sell goods or services. With fixed prices, a decrease in aggregate demand lowers product market tightness, which reduces sales made by firms and increases the idle time of employed workers. Since employees are idle a larger fraction of the time, they are less profitable to firms, and the labor demand falls. Finally, the decrease in labor demand reduces the labor market tightness and raises unemployment. With competitive prices, a decrease in aggregate demand is absorbed by a price change, so it has no effect on product market tightness and unemployment.

Besides aggregate demand shocks, our model generates predictions concerning the comparative static effects of technology, mismatch, and labor supply shocks, thus capturing many of the shocks cited in the context of the depressed labor market of 2008–2013. Two principles emerge from the analysis. First, tightnesses respond to shocks when prices are fixed but not when prices are competitive. Second, when prices are fixed, a demand shock on a market generates a positive correlation between tightness and quantity, whereas a supply shock generates a negative correlation.

By combining the predictions of the model with empirical evidence, we assess the sources of labor market fluctuations in the United States. Time series are available for employment, output, and labor market tightness, but not for product market tightness, so we construct a time series proxying for product market tightness. The proxy is based on the capacity utilization rate measured in the Survey of Plant Capacity (SPC) of the Census Bureau.

Our first finding is that a fixprice equilibrium describes the data better than a competitive equilibrium. This finding is based on the observation that the product market and labor market tightnesses fluctuate a lot. We therefore use the comparative statics from the fixprice equilibrium to identify the sources of labor market fluctuations. Our second finding is that labor market fluctuations are mostly due to labor demand shocks—aggregate demand or technology shocks—and not to labor supply or mismatch shocks. This finding is based on the observation that labor market tightness and employment are positively correlated. Our third finding is that labor demand shocks mostly reflect aggregate demand shocks and not technology shocks. This finding is based on the observation that product market tightness and output are positively correlated.

Our findings are consonant with those obtained by other researchers. Our first finding agrees with the result from Shimer (2005) and Hall (2005) that real-wage rigidity is important to explain unemployment fluctuations over the business cycle. Our second finding is similar to the finding of Blanchard and Diamond (1989b) that labor market fluctuations are mostly due to aggregate activity shocks and not reallocation or labor force participation shocks. Our third result echoes the finding of Galí (1999) and Basu, Fernald, and Kimball (2006) that technology shocks do not explain most business-cycle fluctuations.

To explore the sources of labor market fluctuations, many models are available. We review them here. The textbook model of unemployment is the matching model of the labor market.[4] The

4. See Pissarides (2000) for an exhaustive treatment.

matching model accurately represents the mechanics of the labor market, and it can be used to analyze many labor market shocks. But it ignores aggregate demand, thus leaving out a potentially important source of labor market fluctuations.

To introduce aggregate demand, the matching model can be augmented with a product market combining monopolistic competition and price rigidity.[5] If prices are fixed, the model is tractable. But because employment is solely determined by aggregate demand and technology, shocks to mismatch, job search effort, and labor force participation have no effect on employment, so potentially important sources of employment fluctuations are ignored. If prices sluggishly adjust to shocks, for instance with Calvo (1983) pricing, the model can account for numerous sources of employment fluctuations.[6] But this type of model is complex because it is inherently dynamic and relies on the Phillips curve, the Euler equation, and a monetary policy rule to describe aggregate demand. Its level of complexity goes far beyond that of a static model of the sort developed by Barro and Grossman (1971) or Blanchard and Kiyotaki (1987), making it difficult to analytically characterize the effects of shocks and thus inspect the mechanisms behind labor market fluctuations.

To introduce aggregate demand into the matching model of the labor market, we combine it with a matching model of the product market. The literature applying the matching approach to the product market is small and scattered, so we develop a new matching model.[7] Our model of the product market is formally symmetric to our model of the labor market. Lehmann and Van der Linden (2010) and Huo and Ríos-Rull (2013) also propose models in which aggregate demand influences unemployment through a product market with matching frictions. These models

5. See Blanchard and Kiyotaki (1987) for a classical model of product market with monopolistic competition.

6. For new Keynesian models with matching frictions on the labor market and Calvo pricing, see for instance Walsh (2003), Gertler, Sala, and Trigari (2008), and Blanchard and Galí (2010). See Galí (2010) for a survey of this literature. See Rendahl (2012) for an alternative model built around the zero lower bound on nominal interest rates.

7. The seminal contribution to this literature is Diamond (1982a), and recent models include Arseneau and Chugh (2007), Mathä and Pierrard (2011), Gourio and Rudanko (2014), and Bai, Rios-Rull, and Storesletten (2012).

are quite different from ours, especially because they focus on economies with flexible prices in which dynamics play a key role.[8]

II. A Basic Model of Aggregate Demand and Idle Time

This section presents a simplified version of the complete model, which is introduced in Section III. In this basic model we abstract from the labor market and assume that all production directly takes place within households and not within firms. This is done to simplify the presentation of the equilibrium concept and the matching frictions on the product market, which are the two most important new elements of the complete model. This section also provides empirical evidence in support of matching frictions on the product market.

II.A. Assumptions

The model is static. The assumption that the model is static will be relaxed in Section IV. The economy is composed of a measure 1 of identical households. Households produce goods or services. For concreteness, we assume that households produce services. They sell their services on a product market with matching frictions. Households also consume services, but they cannot consume their own services, so they buy services from other households on the product market. Each household also holds some money. Money is the numeraire.

1. The Product Market. The productive capacity of each household is k; that is, a household is able to produce k services. Each household visits v other households to purchase their services. The number of trades y on the product market is given by a matching function with constant returns to scale. For concreteness, we assume that the matching function takes the form

$$y = (k^{-\gamma} + v^{-\gamma})^{-\frac{1}{\gamma}},$$

where k is the aggregate productive capacity, v is the aggregate number of visits, and the parameter γ governs the elasticity of

8. Hall (2008), den Haan (2013), and Petrosky-Nadeau and Wasmer (2011) also take a matching approach to the product and labor markets, but they do not explicitly represent and study aggregate demand.

substitution of inputs in matching. We impose $\gamma > 0$ to guarantee that y is less than k and v.[9] In each trade, one service is sold at price $p > 0$.

We define the product market tightness x as the ratio of aggregate number of visits to aggregate productive capacity: $x = \frac{v}{k}$. The product market tightness is an aggregate variable taken as given by households. With constant returns to scale in matching, the tightness determines the probabilities that services are sold and that visits yield a purchase: one service is sold with probability

$$f(x) = \frac{y}{k} = (1 + x^{-\gamma})^{-\frac{1}{\gamma}},$$

and one visit yields a purchase with probability

$$q(x) = \frac{y}{v} = (1 + x^{\gamma})^{-\frac{1}{\gamma}}.$$

A useful property is that $q(x) = \frac{f(x)}{x}$. The function f is smooth and strictly increasing on $[0, +\infty)$, with $f(0) = 0$ and $\lim_{x \to +\infty} f(x) = 1$. The function q is smooth and strictly decreasing on $[0, +\infty)$, with $q(0) = 1$ and $\lim_{x \to +\infty} q(x) = 0$. The properties of the derivative f' will be useful later: $f'(x) = q(x)^{1+\gamma}$ so f' is strictly decreasing on $[0, +\infty)$ with $f'(0) = 1$ and $\lim_{x \to +\infty} f'(x) = 0$. An implication is that f is strictly concave on $[0, +\infty)$. The properties of f and q imply that when the product market tightness is higher, it is easier to sell services but harder to buy them.

We abstract from randomness at the household level: a household sells $f(x) \cdot k$ services and purchases $q(x) \cdot v$ services with certainty. Since a household does not sell its entire productive capacity, household members are idle part of the time. In fact, since a household only sells a fraction $f(x)$ of its productive capacity, household members are busy a share $f(x)$ of the time and idle a share $1 - f(x)$ of the time. Thus, the rate of idleness in the economy is $1 - f(x)$.

9. The matching function is borrowed from den Haan, Ramey, and Watson (2000). It always satisfies $y \leq \min\{k, v\}$, which is a required property for a matching function. We use this function instead of the standard Cobb-Douglas matching function, $y = k^{\gamma} \cdot v^{1-\gamma}$, because the latter must be truncated to ensure that $y \leq \min\{k, v\}$, which complicates the analysis.

We model the matching cost as follows. Each visit requires to purchase $\rho \in (0, 1)$ services. These services for matching do not contribute to the buyer's consumption, but they are purchased like the services for consumption. A buyer doing v visits and consuming c services therefore purchases a total of $c + \rho \cdot v$ services. Since the matching process limits the purchases of a buyer doing v visits to $q(x) \cdot v$ services, the number v of visits needed to consume c services satisfies $q(x) \cdot v = c + \rho \cdot v$ or, equivalently, $v = \frac{c}{q(x)-\rho}$. This means that consuming one service requires to do $\frac{1}{q(x)-\rho}$ visits and thus to buy a total of $1 + \tau(x)$ services, where

$$\tau(x) \equiv \frac{\rho}{q(x) - \rho}.$$

The function τ is positive and strictly increasing for all $x \in [0, x^m)$, where $x^m > 0$ is defined by $q(x^m) = \rho$. We also have $\tau(0) = \frac{\rho}{1-\rho}$ and $\lim_{x \to x^m} \tau(x) = +\infty$. Note that any equilibrium satisfies $x \in [0, x^m)$. Because of the matching cost, consumption is necessarily lower than output.

We define the aggregate supply as the amount of consumption traded at a given tightness:

DEFINITION 1. The *aggregate supply* c^s is the function of product market tightness defined for all $x \in [0, x^m]$ by

$$c^s(x) = \frac{f(x) \cdot k}{1 + \tau(x)}.$$

PROPOSITION 1. The aggregate supply satisfies

$$c^s(x) = (f(x) - \rho \cdot x) \cdot k \tag{1}$$

for all $x \in [0, x^m]$. We define the tightness $x^* \in (0, x^m)$ by $f'(x^*) = \rho$. The aggregate supply is strictly increasing on $[0, x^*]$ and strictly decreasing on $[x^*, x^m]$. Hence, x^* maximizes the aggregate supply. Furthermore, $c^s(0) = 0$ and $c^s(x^m) = 0$.

Proof. We have $c^s(x) = \frac{f(x) \cdot k}{1+\tau(x)}$. Using the definition of τ and $\frac{f(x)}{q(x)} = x$, we have $\frac{f(x)}{1+\tau(x)} = f(x) \cdot (1 - \frac{\rho}{q(x)}) = f(x) - \rho \cdot x$. Hence, $c^s(x) = (f(x) - \rho \cdot x) \cdot k$. As showed above, f' is strictly decreasing on $[0, +\infty)$ with $f'(0) = 1$ and $\lim_{x \to +\infty} f'(x) = 0$. Since $\frac{dc^s}{dx} = (f'(x) - \rho) \cdot k$ with $\rho \in (0, 1)$, we infer that $\frac{dc^s}{dx} > 0$ on $[0, x^*)$, $\frac{dc^s}{dx} = 0$ at $x = x^*$, and $\frac{dc^s}{dx} < 0$ on $(x^*, +\infty)$. Thus, c^s is strictly increasing on $[0, x^*]$ and strictly decreasing on $[x^*, x^m]$. Since $f(0) = 0$ and $\frac{f(x^m)}{x^m} = q(x^m) = \rho$, we have $c^s(0) = 0$ and $c^s(x^m) = 0$. $\square$

The property that the aggregate supply is first increasing then decreasing with x is unusual, but it naturally arises from the properties of the matching function. When x is low, the matching process is congested by the available productive capacity, therefore increasing x—that is, increasing the number of visits relative to available productive capacity—leads to a large increase in the probability to sell, $f(x)$, but a small increase in the price wedge faced by buyers, $\tau(x)$. Since the aggregate supply is proportional to $\frac{f(x)}{1+\tau(x)}$, it increases. Conversely when x is high, the matching process is congested by the number of visits, and increasing x leads to a small increase in $f(x)$ but a large increase in $\tau(x)$ so an overall decrease in aggregate supply.

The aggregate supply curve is depicted in Figure I; it gives the amount of consumption for each level of tightness. Figure I also illustrates the relationship between consumption, output, and productive capacity imposed by matching frictions. Output is $y = f(x) \cdot k$, an increasing and concave function of tightness. Consumption is $c = (f(x) - \rho \cdot x) \cdot k$, so it is always below output. The number of services used for matching is $\rho \cdot v = \rho \cdot x \cdot k$, an increasing function of tightness; the gap between consumption and output represents this matching cost. The number of services that could be produced if workers were not idle is $k - f(x) \cdot k = (1 - f(x)) \cdot k$, a decreasing function of tightness; the gap between output and productive capacity represents this idle capacity.

2. Households. The representative household derives utility from consuming services and holding real money balances. The household's utility is given by

$$u\left(c, \frac{m}{p}\right) = \frac{\chi}{1+\chi} \cdot c^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{1+\chi} \cdot \left(\frac{m}{p}\right)^{\frac{\epsilon-1}{\epsilon}},$$

where c is consumption of services, m are nominal money balances, $\frac{m}{p}$ are real money balances, the parameter $\chi > 0$ measures the taste for consumption relative to holding money, and the parameter $\epsilon > 1$ is the elasticity of substitution between consumption and real money balances.

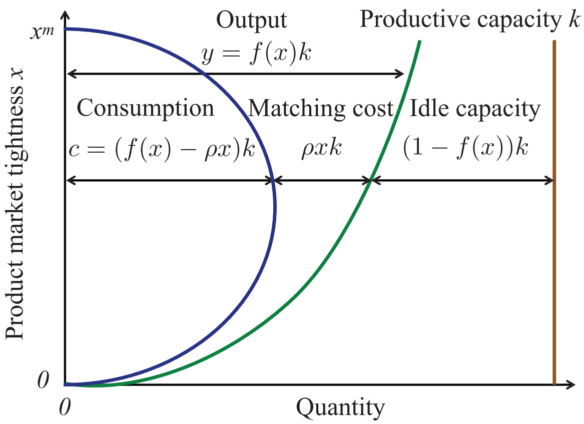


FIGURE I

The Matching Frictions on the Product Market

The desired level of consumption determines the number of visits that the household makes. Consuming c services requires to purchase $(1 + \tau(x)) \cdot c$ services in the course of $\frac{(1+\tau(x))\cdot c}{q(x)}$ visits. For simplicity, we relegate the visits to the background and focus on consumption.[10] We summarize the cost incurred by the household for the visits with a price wedge. Consuming one service requires to purchase one service for consumption plus $\tau(x)$ services to cover the cost of the visits. The total cost of consuming c therefore is $p \cdot c + p \cdot \tau(x) \cdot c = p \cdot (1 + \tau(x)) \cdot c$. From the household's perspective, it is as if it purchased c services at a unit price $p \cdot (1 + \tau(x))$. Effectively the matching frictions impose a wedge $\tau(x)$ on the price of services.

Taking as given the product market tightness and the price, the representative household chooses consumption and nominal money balances to maximize utility subject to a budget constraint. The household receives an endowment $\mu > 0$ of nominal money and income from the sale of $f(x) \cdot k$ services at price p. With these, the household purchases c services at price $(1 + \tau(x)) \cdot p$ and holds m units of nominal money balances. Hence, the household's budget constraint is

$$m + (1 + \tau(x)) \cdot p \cdot c = \mu + p \cdot f(x) \cdot k.$$

10. This representation is slightly unconventional. The matching literature usually emphasizes the role of visits or, on the labor market, of vacancies.

Solving the utility-maximization problem gives

$$(2) \qquad \frac{1}{1+\chi} \cdot \left(\frac{m}{p}\right)^{-\frac{1}{\epsilon}} = \frac{1}{1+\tau(x)} \cdot \frac{\chi}{1+\chi} \cdot c^{-\frac{1}{\epsilon}}.$$

This equation implies that at the margin, the household is indifferent between consumption and holding money.

We define the aggregate demand as the utility-maximizing level of consumption at a given product market tightness and price, accounting for the fact that the money market clears:

DEFINITION 2. The *aggregate demand* c^d is the function of product market tightness and price defined by

$$(3) \qquad c^d(x,p) = \left(\frac{\chi}{1+\tau(x)}\right)^{\epsilon} \cdot \frac{\mu}{p}$$

for all $(x,p) \in [0, x^m] \times (0, +\infty)$, where $x^m > 0$ satisfies $\rho = q(x^m)$.

PROPOSITION 2. The aggregate demand is strictly decreasing in x and p. Furthermore, $c^d(0,p) = \chi^{\epsilon} \cdot (1-\rho)^{\epsilon} \cdot \frac{\mu}{p}$ and $c^d(x^m, p) = 0$.

Proof. Obvious from equation (3), since τ is strictly increasing in x. □

The aggregate demand is the level of consumption that satisfies equation (2) when $m = \mu$. The properties of the aggregate demand reflect the household's indifference between consumption and holding $\frac{\mu}{p}$ real money balances. First, a higher p leads to lower real money balances. Households' indifference between consumption and holding money implies that they desire lower consumption when p is higher. Hence the aggregate demand decreases with p. Second, $1+\tau(x)$ is effectively the price of consumption relative to real money balances. A higher x leads to a higher relative price that reduces the attractiveness of consumption relative to holding real money balances, whose quantity is fixed at $\frac{\mu}{p}$. Hence the aggregate demand decreases with x. The aggregate demand is plotted later in Figure III; it slopes downward in the (c, x) and (c, p) planes.

II.B. Discussion of the Assumptions

We discuss two critical assumptions of the model: matching frictions on the product market and money in the utility function.

To represent the matching frictions, we assume that the number of trades is governed by a matching function and that buyers face a matching cost; we discuss matching function and matching cost in turn.

1. The Matching Function. Much in the same way the production function summarizes how inputs are transformed into output through the production process, the matching function summarizes how productive capacity and visits are transformed into trades through the matching process. The matching function provides a tractable representation of a very complex process. Its main implication is that not all productive capacity is sold and not all visits are successful. Formally, households only sell a fraction $f(x) < 1$ of their productive capacity and the visits of buyers to sellers are only successful with probability $q(x) < 1$. The matching function is a useful modeling tool only if we find convincing evidence that at all times some employed workers are idle and some visits are unsuccessful.[11]

The prediction that not all productive capacity is sold can be examined empirically. In U.S. data, we find that some productive capacity is idle at all time. Panel A of Figure II displays the rates of idleness in nonmanufacturing sectors and in the manufacturing sector. These rates indicate the share of time when employed workers are idle due to a lack of activity. These rates are constructed as one minus the operating rates measured by the Institute for Supply Management (ISM) for nonmanufacturing sectors and for the manufacturing sector. The operating rate indicates the actual production level of firms as a share of their maximum production level given current capital and labor. On average the rate of idleness is 14.8% in nonmanufacturing sectors and 17.3% in the manufacturing sector. The rate of idleness is the product market equivalent of the rate of unemployment; for comparison, the panel also displays the rate of unemployment constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). Perhaps surprisingly, the rates of idleness prevailing in the manufacturing and nonmanufacturing sectors are much higher than the rate of unemployment.

11. Pissarides (1985) pioneered the concept of matching function on the labor market. Pissarides (1986) and Blanchard and Diamond (1989a) first explored the empirical properties of the matching function on the labor market. See Petrongolo and Pissarides (2001) for a survey of this literature.

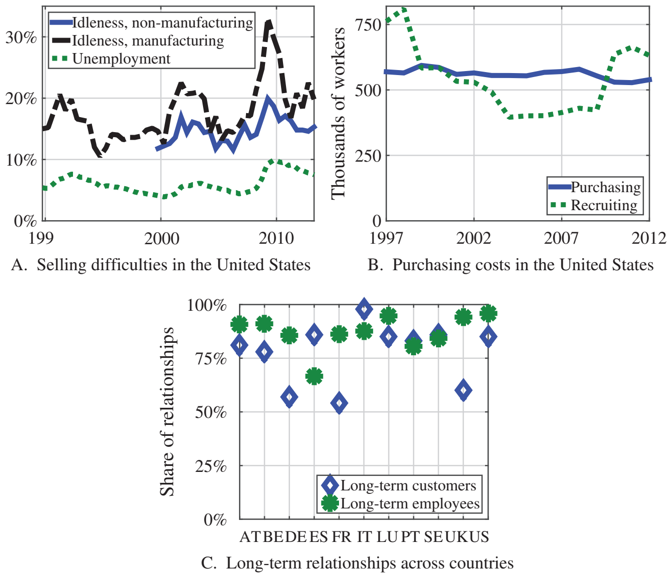


FIGURE II

Evidence of Matching Frictions on the Product and Labor Markets

Panel A: The time period is 1989:Q4–2013:Q2. The rate of idleness is one minus the operating rate measured by the ISM. For nonmanufacturing sectors, the operating rate is only available after 1999:Q4. The rate of unemployment is constructed by the BLS from the CPS. Panel B: The time period is 1997–2012. The number of workers in recruiting and purchasing occupations is from the OES database constructed by the BLS. Recruiting occupations include human resource managers, specialists, and assistants. Purchasing occupations include purchasing managers, buyers and purchasing agents, and procurement clerks. Panel C: The share of sales to long-term customers is from the following firm surveys: Kwapil, Baumgartner, and Scharler (2005) for Austria (AT); Aucremanne and Druant (2005) for Belgium (BE); Stahl (2005) for Germany (DE); Alvarez and Hernando (2005) for Spain (ES); Loupias and Ricart (2004) for France (FR); Fabiani, Gattulli, and Sabbatini (2004) for Italy (IT); Lunnemann and Matha (2006) for Luxembourg (LU); Martins (2005) for Portugal (PT); Apel, Friberg, and Hallsten (2005) for Sweden (SE); Hall, Walsh, and Yates (2000) for the United Kingdom; and Blinder et al. (1998) for the United States. All the surveys were conducted between 2000 and 2004, except in the United Kingdom and the United States where they were conducted in 1995 and 1990–1992. The share of workers in long-term employment is from the OECD data set on the incidence of permanent employment for 2005.

In addition, evidence suggests that firms in the United States face difficulties in selling their output. Using output and price microdata from the Census of Manufacturers, Foster, Haltiwanger, and Syverson (2012) find that despite similar or lower prices, new plants grow more slowly than similar plants with an established customer base because it is difficult for new plants to attract customers.

Visits are the product market equivalent of vacancies. A visit represents the process that a buyer must follow to buy an item. These visits can take different forms, depending on the buyer. For an individual consumer, a visit may be an actual visit to a restaurant, a hair salon, a bakery, or a car dealer. A visit could also be an inquiry to an intermediary, such as a travel agent, a real estate agent, or a stockbroker. For a firm, a visit could be an actual visit to a potential supplier. A visit could also be the preparation and processing of a request for proposal or request for tender or any other sourcing process. Unlike for vacancies, however, visits are not recorded in any data set. It is therefore difficult to provide quantitative evidence on the share of visits that are unsuccessful. The only quantitative evidence that we found is the average stockout rate provided by Bils (2004). Using the monthly microdata underlying the Consumer Price Index, Bils finds that temporary stockouts are quite common: the average stockout rate for consumer durables over the 1988–2004 period is 9 percent. A stockout is an item not available for sale, continuing to be carried by the outlet, and not seasonally unavailable; hence, a stockout indicates that a buyer's visit to a store would not result in a purchase because the desired product would be unavailable.

Casual observation also suggests that many visits do not generate a trade. At a restaurant, a consumer sometimes need to walk away because no tables are available or the queue is too long. The same may happen at a hair salon if no slots are available or if the salon is not open for business. At a bakery, the type of bread or cake desired by a consumer may not be available at the time of the visit, either because it was not prepared on the day or because the bakery has sold out of it. At a car dealer, the specific car desired by the consumer may not be in inventory and may therefore not be available before a long time. Buyers employed by firms travel the world to visit the production facilities of potential suppliers and assess their quality, and many of these visits do not lead to a contract. Finally, when a firm issues a request for proposal or a request for tender, it considers the applications of many potential suppliers, but only one supplier is eventually selected.

2. The Matching Cost. Empirical evidence indicates that buyers incur a broad range of matching costs on the product market. In this article we make the assumption that the matching cost is incurred in services. This representation of the matching cost is crude but convenient: it is tractable because the cost appears as a price wedge for buyers; it is portable because we could similarly represent the matching cost incurred by a firm or a government (other representations of the matching cost, such as a utility cost, would not offer this portability); and it is isomorphic to the representation of the matching cost on the labor market in Section III. It is also conventional in the matching literature to measure matching costs in terms of output and to define consumption as output net of matching costs, as we do here (for example, Gertler and Trigari 2009).

Of all the matching costs incurred by buyers on the product market, some are indeed service costs. For a consumer using a travel agency to book a vacation, the matching cost of purchasing hospitality services is the travel agent's fee; for a consumer who goes to a hair salon in a taxicab, the matching cost of purchasing hairdressing services is the cab fare; and for a firm recruiting a manager with an executive search agency, the cost of purchasing labor services is the agency's fee. The travel agent's fee, cab fare, and executive search agency's fee are service costs.

Besides service costs, buyers incur other types of matching costs on the product market. For consumers, the cost of a visit to a seller could be a traveling time or the time spent in a queue at a restaurant or hair salon. These time costs are not negligible: on average between 2003 and 2011 in the American Time Use Survey conducted by the BLS, people spend 47 minutes a day shopping for goods and services. For firms, a large share of the cost of sourcing goods and services is a labor cost. To quantify this cost, we use data from the Occupational Employment Survey (OES) database constructed by the BLS. We measure the number of workers whose occupation is buying, purchasing, and procurement.[12] Panel B of Figure II displays the results; on average between 1997 and 2012, 560,600 workers were employed in such occupations. For comparison, we use the same methodology to evaluate the matching cost incurred by firms on the labor market. We measure the number of workers devoted to recruiting in the OES database; on average between 1997 and 2012, 543,200 workers were employed in an occupation involving recruitment,

12. Note that the classification of occupations evolves over time so comparisons across years are not meaningful.

placement, screening, and interviewing. Hence, the numbers of buyers and recruiters have the same magnitude.

Since matching costs take various forms, we could model the matching cost differently. For example, in Online Appendix E we study an alternative model in which the matching cost is a time cost instead of a service cost. In that model, households share their time between supplying services and matching with other households who sell services. We find that this alternative representation of the matching cost does not modify the properties of the model.

Finally, sellers could also incur a matching cost. Indeed, firms spend substantial resources on sales and marketing. These resources are used by firms to increase their sales for a given productive capacity. In Online Appendix F we extend the model to include an endogenous marketing effort for sellers. We model the marketing effort as a continuous variable that increases sellers' selling probability at a cost. This extension does not alter the structure of the model or its properties.

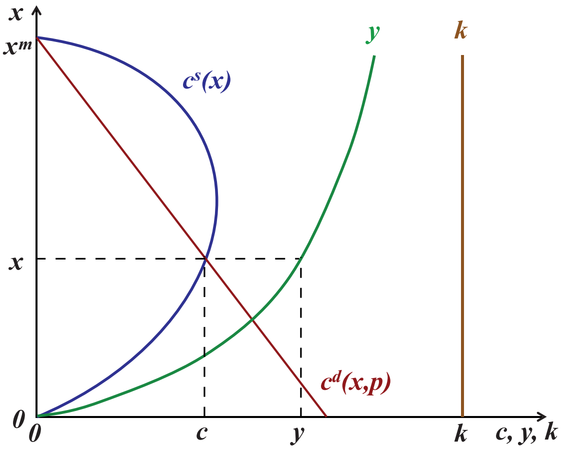
3. Money in the Utility Function. The assumption that households derive utility from holding real money balances is borrowed from Barro and Grossman (1971). This assumption was also used by Blanchard and Kiyotaki (1987), among many others. Introducing money in the utility function crudely but conveniently captures the fact that money provides transaction services to households. The presence of money in the utility function is necessary to obtain an interesting concept of aggregate demand in a static environment because without money, consumers would mechanically spend all their income on the produced good (Say's law). Here households choose between buying consumption and holding money, and the aggregate demand is the desired level of consumption.

II.C. Definition of the Equilibrium

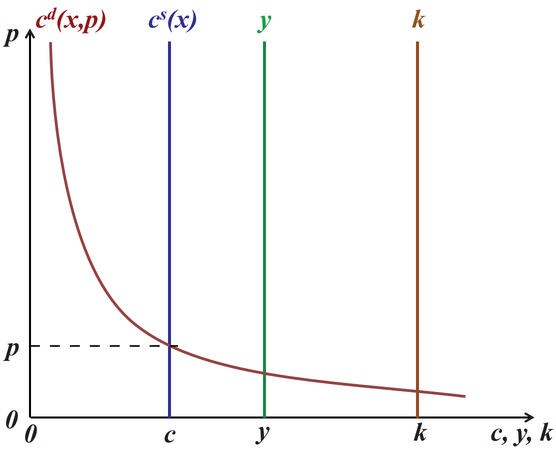
DEFINITION 3. An *equilibrium* consists of a product market tightness and a price (x, p) such that aggregate supply is equal to aggregate demand:

$$c^s(x) = c^d(x, p).$$

Since the equilibrium has two variables but only one condition, infinitely many combinations of price and tightness satisfy



A. Representation in a (c, x) plane



B. Representation in a (c, p) plane

FIGURE III

Aggregate Demand, Aggregate Supply, and Equilibrium in the Basic Model of Section II the equilibrium condition. To select an equilibrium, we specify a price mechanism. In Sections II.E–II.G, we study the equilibria selected by different mechanisms.

Figure III represents aggregate demand and supply, and the equilibrium. The equilibrium tightness is at the intersection of aggregate demand and supply with positive consumption in the (c, x) plane.[13] The equilibrium price is at the intersection of aggregate supply and demand in the (c, p) plane.

Since many equilibrium prices are possible, we categorize equilibria into the following regimes:

DEFINITION 4. An equilibrium is *efficient* if it maximizes consumption. An inefficient equilibrium can be either *slack*, if an increase in tightness at the equilibrium point raises consumption, or *tight*, if an increase in tightness at the equilibrium point lowers consumption. Equivalently, an equilibrium is efficient if $x = x^*$, slack if $x < x^*$, and tight if $x > x^*$.

Figure IV illustrates the three regimes in which equilibria may fall. In the efficient equilibrium, consumption is maximized. An efficient equilibrium also maximizes welfare taking real money balances as given. In a slack equilibrium, aggregate demand is too low and tightness is below its efficient level. In a tight equilibrium, aggregate demand is too high and tightness is above its efficient level. The slack and tight equilibria are inefficient because their consumption levels are below the efficient consumption level. As illustrated in Figure I, higher output is not equivalent to higher consumption. Compared to the efficient equilibrium, a slack equilibrium has lower output and a tight equilibrium has higher output, but both have lower consumption. Given that the aggregate demand is decreasing in price, the price is too high when the equilibrium is slack and too low when the equilibrium is tight. The property that an equilibrium can be efficient, slack, or tight is true in any matching model (Pissarides 2000, chapter 8).

13. There is another equilibrium at the intersection with zero consumption. In that equilibrium, the tightness is x^m. We do not study that equilibrium because it is uninteresting.

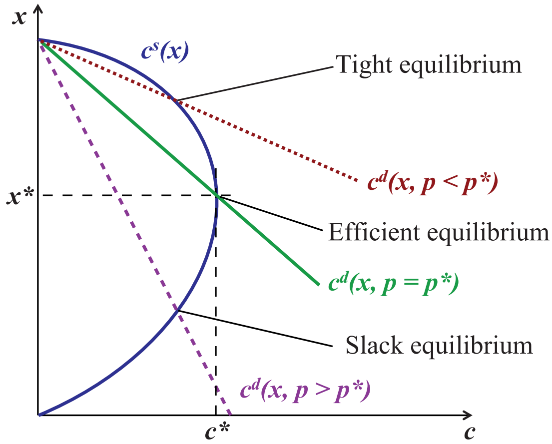


FIGURE IV

The Three Regimes in the Basic Model of Section II

The figure compares the equilibria obtained for different equilibrium prices. The price p^* is given by equation (5).

II.D. Discussion of the Equilibrium Concept

This section proposes a more detailed definition of the equilibrium concept. To make the definition more transparent, we generalize our model slightly and consider a measure 1 of households indexed by $i \in [0, 1]$. Household i has productive capacity $k(i)$ and an endowment of money $\mu(i)$. We define the equilibrium by analogy to a Walrasian equilibrium:[14]

DEFINITION 5. An *equilibrium* is a price p, a tightness x, visits $\{v(i), i \in [0, 1]\}$, and nominal money balances $\{m(i), i \in [0, 1]\}$ such that the following conditions are satisfied:

(i) Taking x and p as given, household $i \in [0, 1]$ chooses $v(i)$ and $m(i)$ to maximize its utility function subject to a budget constraint and the constraints imposed by matching frictions. The matching frictions impose that the output bought by household i is $y^b(i) = v(i) \cdot q(x)$, the output sold by household i is $y^s(i) = k(i) \cdot f(x)$, and the consumption

14. For a standard definition of a Walrasian equilibrium, see Mas-Colell, Whinston, and Green (1995).

of household i is $c(i) = \frac{y^b(i)}{1+\tau(x)}$. The budget constraint is $m(i) + p \cdot y^b(i) = \mu(i) + p \cdot y^s(i)$.

(ii) Quoted tightness equals actual tightness: $x = \frac{\int_0^1 v(i)di}{\int_0^1 k(i)di}$.

As in Walrasian theory, we make the institutional assumption that a price and a tightness are quoted on the product market, and we make the behavioral assumption that households take the quoted price and tightness as given. It is natural for households to take tightness as given because the tightness is the ratio of aggregate number of visits to aggregate productive capacity, and each household is small relative to the size of the market. The issue is more complicated for the price since a buyer and a seller could bargain the transaction price once they have matched. However, the actual transaction price has no influence on households' decisions because the decisions are made before the match is realized; what matters is the price at which households expect to trade. Hence, we assume that households take the expected transaction price as given, and to ensure the consistency of the equilibrium, we require that actual and expected transaction prices are the same.

As in a Walrasian equilibrium, condition (i) imposes that households behave optimally given the quoted price and tightness. The difference with Walrasian theory is that households cannot choose the quantities that they trade. These quantities are constrained by matching frictions: as buyers, households only choose how many sellers to visit, knowing that the purchasing probability is $q(x)$ and that the purchase of one unit of output yields $\frac{1}{1+\tau(x)}$ unit of consumption; and as sellers, households only choose how much productive capacity to bring to the market, knowing that the selling probability is $f(x)$.[15]

Condition (ii) is the equivalent of the market-clearing condition of the Walrasian equilibrium. The Walrasian market-clearing condition imposes that at the quoted price, the quan-

15. Here the productive capacity of household i is fixed to $k(i)$, but the model could be extended to have household i choose $k(i)$.

tity that buyers desire to buy equals the quantity that sellers desire to sell. This condition is required to ensure the consistency of the Walrasian equilibrium because sellers and buyers make their decisions expecting to be able buy and sell any quantity at the quoted price. Similarly, condition (ii) is required to ensure the consistency of our equilibrium. Given $\{v(i),\ i \in [0,1]\}$ and $\{k(i),\ i \in [0,1]\}$, the number of trades is

$$\left[\left(\int v(i)di\right)^{-\gamma} + \left(\int k(i)di\right)^{-\gamma}\right]^{-\frac{1}{\gamma}} = \int k(i)di \cdot f\left(\frac{\int v(i)di}{\int k(i)di}\right) = \int v(i)di \cdot q\left(\frac{\int v(i)di}{\int k(i)di}\right).$$

These equations imply that the actual selling probability faced by households is $f\left(\frac{\int v(i)di}{\int k(i)di}\right)$ and the actual purchasing probability faced by households is $q\left(\frac{\int v(i)di}{\int k(i)di}\right)$. To ensure the consistency of the equilibrium, these probabilities must match the probabilities $f(x)$ and $q(x)$ on which households base their calculations; equivalently, the quoted tightness, x, must be equal to the actual tightness, $\frac{\int v(i)di}{\int k(i)di}$.

Our equilibrium has one more variable than the Walrasian equilibrium—the tightness. But the equilibrium does not have one more equation, which explains why many price-tightness pairs are consistent with the equilibrium and why a price mechanism is needed to select an equilibrium. At a microeconomic level, it is impossible to add an equilibrium condition to determine a price because each seller-buyer pair decides the price in a situation of bilateral monopoly. This situation arises because the pairing of a buyer and a seller generates a positive surplus. Since the solution to the bilateral monopoly problem is indeterminate, it cannot be used to impose a condition on the price.[16] What this means is that there is no obvious economic criterion that can determine the price. For instance, when a buyer and a seller meet, there is no deviation from the quoted price that generates

16. The indeterminacy of the solution to the bilateral monopoly problem has been known since Edgeworth (1881). The indeterminacy is discussed by Howitt and McAfee (1987) and Hall (2005) in the context of matching models.

a Pareto improvement. Of course, a seller would be better off with a higher price, but a buyer would be worse off with that price.

In a symmetric equilibrium, Definition 5 implies that $c^s(x) = c^d(x, p)$. First, the budget constraints of all households are satisfied, and sales of services equal purchases, so the money market clears: $m = \mu$. Given the definition of the aggregate demand and the fact that $m = \mu$, condition (i) imposes that $v(x, p) = \frac{(1+\tau(x))\cdot c^d(x,p)}{q(x)}$. Next, condition (ii) imposes that $x = \frac{v(x,p)}{k} = \frac{(1+\tau(x))\cdot c^d(x,p)}{k\cdot q(x)}$. Last, since $f(x) = q(x) \cdot x$, this equation implies that

$$c^d(x, p) = \frac{x \cdot q(x)}{1 + \tau(x)} \cdot k = \frac{f(x)}{1 + \tau(x)} \cdot k = c^s(x).$$

II.E. Fixprice Equilibrium

We first study a simple equilibrium in which the price is a parameter.[17] In this equilibrium, only the product market tightness equilibrates the market.

DEFINITION 6. A *fixprice* equilibrium parameterized by $p_0 > 0$ consists of a product market tightness and a price (x, p) such that aggregate supply equals aggregate demand and the price is given by the parameter p_0: $c^s(x) = c^d(x, p)$ and $p = p_0$.

PROPOSITION 3. For any $p_0 > 0$, there exists a unique fixprice equilibrium parameterized by p_0 with positive consumption.

Proof. In equilibrium, x satisfies $c^s(x) = c^d(x, p_0)$. We look for an equilibrium with positive consumption, so we restrict the search to $x \in (0, x^m)$. The equilibrium condition is equivalent to $(1 + \tau(x))^\epsilon \cdot \left(c^s(x) - c^d(x, p_0)\right) = 0$ because $x \in (0, x^m)$ so $(1 + \tau(x))^\epsilon \in (0, +\infty)$. This equation is equivalent to

$$(1 + \tau(x))^{\epsilon-1} \cdot f(x) = \frac{\chi^\epsilon}{k} \cdot \frac{\mu}{p_0}. \tag{4}$$

Since $\epsilon > 1$, the function $x \mapsto (1 + \tau(x))^{\epsilon-1} \cdot f(x)$ is strictly increasing from 0 to $+\infty$ on $[0, x^m)$. Thus, there is a unique $x \in (0, x^m)$ that solves equation (4). □

17. In matching models of the labor market, several researchers have assumed that the wage is a parameter or a function of the parameters. See for instance Hall (2005), Blanchard and Galí (2010), and Michaillat (2012, 2014).

We study the comparative static effects of aggregate demand and supply shocks in the fixprice equilibrium. We parameterize an increase in aggregate demand by an increase in money supply, μ, or in the taste for consumption, χ. We parameterize an increase in aggregate supply by an increase in productive capacity, k. The following proposition summarizes the comparative statics:

PROPOSITION 4. Consider a fixprice equilibrium with positive consumption.

- An increase in aggregate demand has the following effects: output and product market tightness increase; the rate of idleness decreases; consumption increases in a slack equilibrium, decreases in a tight equilibrium, and does not change in the efficient equilibrium.
- An increase in aggregate supply has the following effects: output increases but product market tightness decreases; the rate of idleness increases; consumption increases.

Proof. In a fixprice equilibrium, x is the unique solution to equation (4). Since the functions τ and f are strictly increasing and $\epsilon > 1$, equation (4) implies that $\frac{dx}{d\mu} > 0$, $\frac{dx}{d\chi} > 0$, but $\frac{dx}{dk} < 0$. The rate of idleness is $1 - f(x)$ so its comparative statics follow from those of x. Since $y = f(x) \cdot k$, $\frac{dy}{d\mu} > 0$ and $\frac{dy}{d\chi} > 0$. Since $y = (1 + \tau(x)) \cdot c^d(x, p) = (1 + \tau(x))^{1-\epsilon} \cdot \chi^{\epsilon} \cdot \frac{\mu}{p}$, we infer that $\frac{dy}{dk} > 0$. Given that $c = c^s(x)$ and the properties of c^s, we infer that $\frac{dc}{d\mu} > 0$ if $x < x^*$, $\frac{dc}{d\mu} = 0$ if $x = x^*$, $\frac{dc}{d\mu} < 0$ if $x > x^*$. The same is true for $\frac{dc}{d\chi}$. As $c = c^d(x, p)$ and c^d decreases with x, we have $\frac{dc}{dk} > 0$. □

The comparative statics are summarized in Panel A of Table I and illustrated in Figure V.

Panel A in Figure V depicts an increase in aggregate demand. The aggregate demand curve rotates outward. Indeed, households want to consume more for a given price and tightness, either because they hold more money or because they value consumption more. Since the price is fixed, they want to consume more for a given tightness, explaining the rotation of the curve. To reach a new equilibrium, the product market tightness necessarily increases. Since tightness increases, workers sell a larger fraction of their productive capacity, which is fixed, so output

TABLE I

COMPARATIVE STATICS IN THE BASIC MODEL OF SECTION II

	Effect on:			
Increase in:	Output y	Product market tightness x	Idleness $1 - f(x)$	Consumption c
Panel A: Fixprice equilibrium and equilibrium with partially rigid price				
Aggregate demand	+	+	–	+ (slack) 0 (efficient) – (tight)
Aggregate supply	+	–	+	+
Panel B: Competitive equilibrium and equilibrium with Nash bargaining				
Aggregate demand	0	0	0	0
Aggregate supply	+	0	0	+

Notes. An increase in aggregate demand is an increase in money supply, μ, or in the taste for consumption, χ. An increase in aggregate supply is an increase in productive capacity, k. This table summarizes the results of Propositions 4 and 6 and the results discussed in Section II.G.

increases and the rate of idleness decreases. The equilibrium point moves upward along the aggregate supply curve, so the response of consumption depends on the regime: in the slack regime consumption increases; but in the tight regime consumption decreases, because the increase in tightness raises the amount of output dissipated in matching more than it raises total output.

Panel B in Figure V depicts an increase in aggregate supply. The aggregate supply curve expands outward because households' productive capacity increases. To reach a new equilibrium, the product market tightness necessarily decreases. Consumption increases as the equilibrium point moves downward along the aggregate demand curve. The effect on output is not obvious on the graph: productive capacity increases but tightness falls, so households sell a smaller fraction of a larger capacity. However, the proposition establishes that output increases. Since tightness decreases, the rate of idleness increases.

The proposition implies that aggregate demand matters when the price is fixed. This result echoes the findings of a vast body of work in macroeconomics, including the contributions of Barro and Grossman (1971) and Blanchard and Kiyotaki (1987), that aggregate demand matters in the presence of price rigidity. The proposition also implies that aggregate demand shocks and aggregate supply shocks have different macroeconomic effects: product

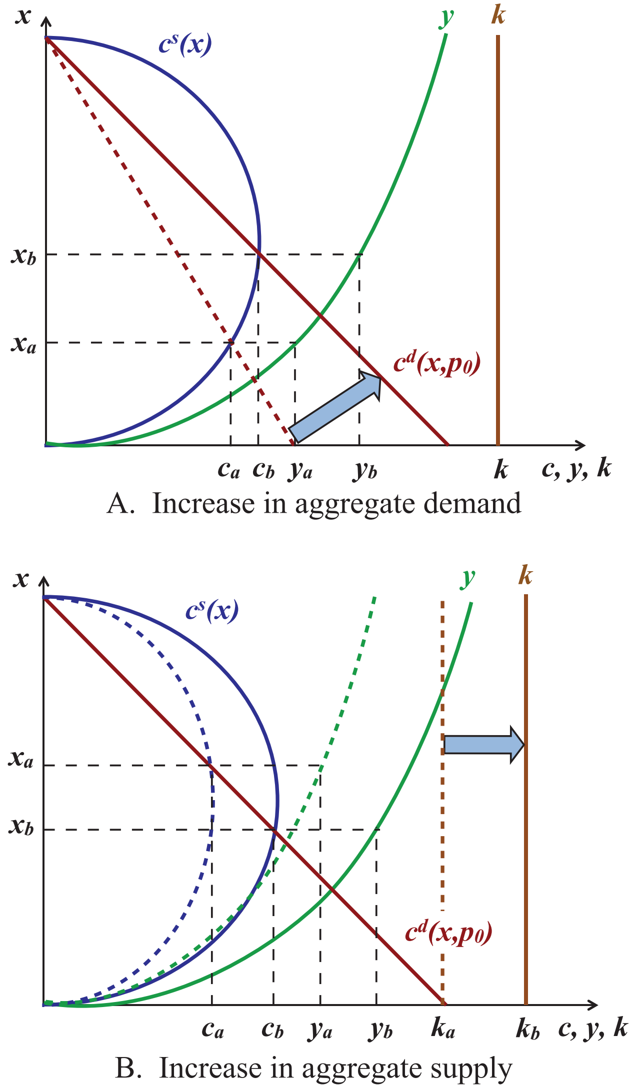


FIGURE V

Shocks in the Fixprice Equilibrium of Section II

market tightness and output are positively correlated under aggregate demand shocks but negatively correlated under aggregate supply shocks. In Section V, we exploit this property to identify aggregate demand and aggregate supply shocks in the data.

II.F. Competitive Equilibrium

We study an equilibrium in which the price mechanism is the polar opposite of the fixed price. In a fixprice equilibrium, the price is fixed and tightness alone equilibrates the market. In the equilibrium that we study now, the price is flexible enough to maintain the market in an efficient situation. The efficient tightness is invariant to the shocks considered, so in practice the tightness is fixed at its efficient level and the price alone equilibrates the market.

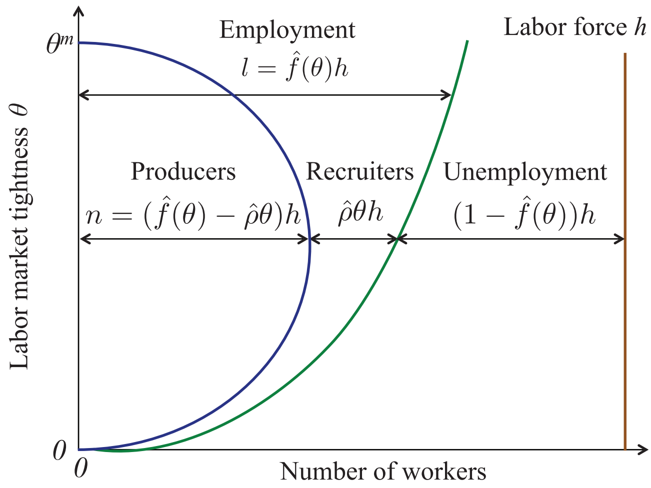
DEFINITION 7. A *competitive equilibrium* consists of a product market tightness and a price (x, p) such that aggregate supply equals aggregate demand and the product market tightness is efficient: $c^s(x) = c^d(x, p)$ and $x = x^*$.

PROPOSITION 5. There exists a unique competitive equilibrium. The competitive price is

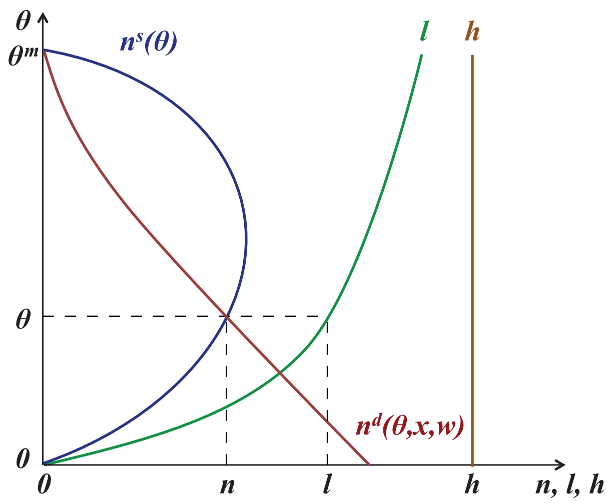
$$p^* = \frac{(1 + \tau(x^*))^{1-\epsilon}}{f(x^*)} \cdot \frac{\chi^\epsilon}{k} \cdot \mu. \tag{5}$$

Proof. Obvious using equation (4). □

In Definition 7 we simply assume that the price adjusts to maintain the product market tightness at its efficient level, but market forces could achieve this through the competitive search mechanism of Moen (1997). (We label the equilibrium as competitive in reference to the competitive search mechanism.) The mechanism lies beyond the scope of the model because it relies on directed search, whereby buyers search for the submarket offering the best price-tightness compromise, whereas our model assumes random search. Nevertheless, the mechanism is simple to understand. Starting from an equilibrium (p_a, x_a), a subset of sellers can deviate and offer a different price, p_b. Buyers will flee or flock to the new submarket until they are indifferent between the old and new submarkets. Indifference happens when $p_b \cdot (1 + \tau(x_b)) = p_a \cdot (1 + \tau(x_a))$. By deviating, sellers obtain a revenue $p_b \cdot f(x_b)$; thus, sellers' optimal choice is to select p_b to maximize $p_b \cdot f(x_b)$ subject to $p_b \cdot (1 + \tau(x_b)) = p_a \cdot (1 + \tau(x_a))$. This is equivalent to selecting x_b to maximize $\frac{f(x_b)}{1+\tau(x_b)} = f(x_b) - \rho \cdot x_b$, that is, to selecting the efficient tightness. Under the competitive search mechanism, tightness is always efficient in equilibrium, and prices



A. The matching frictions on the labor market



B. Labor demand and supply in a $(n;\theta)$ plane

FIGURE VI

The Labor Market in the Model of Section III cannot be rigid because market forces provide an incentive for sellers to adjust their price if tightness changes.

The competitive price ensures that the aggregate demand curve is always in the position depicted in Figure IV, where it intersects the aggregate supply curve at its maximum. This price necessarily exists because by increasing the price from 0 to $+\infty$, the aggregate demand curve rotates around the point $(0, x^m)$ from a horizontal to a vertical position.

The following proposition summarizes the comparative statics in the competitive equilibrium:

PROPOSITION 6. Consider a competitive equilibrium.

- An increase in aggregate demand has the following effects: output, product market tightness, the rate of idleness, and consumption remain the same; the price increases.
- An increase in aggregate supply has the following effects: output and consumption increase; product market tightness and the rate of idleness remain the same; the price decreases.

Proof. The efficient tightness x^* satisfies $f'(x^*) = \rho$ so x^* is independent of χ, μ, and k. The comparative statics for the competitive equilibrium follow because in this equilibrium, $x = x^*$, $y = f(x^*) \cdot k$, $c = (f(x^*) - \rho \cdot x^*) \cdot k$, and p is given by equation (5). □

The comparative statics are summarized in Panel B of Table I. The comparative statics follow from the properties that the tightness is efficient in a competitive equilibrium and that the efficient tightness responds neither to aggregate demand shocks nor to aggregate supply shocks.

The proposition implies that aggregate demand shocks have no effect on real outcomes in a competitive equilibrium. This result is reminiscent of those obtained by Blanchard and Galí (2010) and Shimer (2010, Chapter 2) in the context of matching models of the labor market. They find that labor demand shocks in the form of technology shocks have no effect on the efficient labor market tightness and unemployment rate.

II.G. Other Equilibria

We have considered a fixed price and a competitive price, but many other price mechanisms are possible. We study two of them here: a partially rigid price and Nash bargaining. The partially rigid price is a generalization of the fixed price that partially responds to shocks. Nash bargaining is the typical price mechanism in the matching literature.[18] We show that the comparative statics with a partially rigid price are the same as those with a fixed price, and the comparative statics with a Nash bargained price are the same as those with a competitive price.

1. Equilibrium with Partially Rigid Price. We consider the following partially rigid price:

$$p = p_0 \cdot \left(\frac{\chi^\epsilon}{k} \cdot \mu\right)^\xi, \tag{6}$$

where the parameter $p_0 > 0$ governs the price level and the parameter $\xi \in [0, 1)$ governs the rigidity of the price. If $\xi = 0$, the price is fixed. If $\xi = 1$, the price is proportional to and therefore as flexible as the competitive price, given by equation (5).[19] In the general case with $0 < \xi < 1$, the price is more rigid than the competitive price but less rigid than the fixed price.

In equilibrium, tightness equalizes aggregate demand and supply with the price given by equation (6). As in the fixprice case, there exists a unique equilibrium with positive consumption. Combining $c^s(x) = c^d(x, p)$ with equation (6) implies that in equilibrium the product market tightness satisfies

$$(1 + \tau(x))^{\epsilon - 1} \cdot f(x) = \left(\frac{\chi^\epsilon}{k} \cdot \mu\right)^{1-\xi} \cdot \frac{1}{p_0}.$$

Since $\xi < 1$ the comparative statics for the product market tightness are the same here and in the fixprice equilibrium, where tightness satisfies equation (4). Hence, all the comparative statics of the fixprice equilibrium remain valid in this equilibrium even though the price is not fixed but partially rigid.

The comparative statics of the fixprice equilibrium are therefore robust: they hold whenever the price responds less than proportionally to $\frac{\chi^\epsilon \cdot \mu}{k}$, and they only break down in the knife-edge case in which the price is proportional to $\frac{\chi^\epsilon \cdot \mu}{k}$. This finding

18. Nash bargaining was first used in the seminal work of Diamond (1982b), Mortensen (1982), and Pissarides (1985).

19. The competitive price is obtained by setting $\xi = 1$ and $p_0 = \frac{(1+\tau(x^*))^{1-\epsilon}}{f(x^*)}$ in equation (6).

echoes results obtained by Blanchard and Galí (2010) and Michaillat (2012): they show in matching models of the labor market that the comparative static effects of technology are the same when the real wage is fixed and when the real wage responds less than proportionally to technology.

2. *Equilibrium with Nash Bargaining.* In an equilibrium with Nash bargaining, the price is the generalized Nash solution to the bargaining problem between a buyer and a seller with bargaining power $\beta \in (0, 1)$. After a match is made, the marginal surplus to the household of buying one service at price $\tilde{p}$ is

$$\mathscr{B}(\tilde{p}) = \frac{\partial u}{\partial c} - \frac{\tilde{p}}{p} \cdot \frac{\partial u}{\partial(\frac{m}{p})} = \frac{\epsilon}{\epsilon - 1} \cdot \frac{1}{1+\chi} \cdot \left[\chi \cdot c^{-\frac{1}{\epsilon}} - \frac{\tilde{p}}{p} \cdot \left(\frac{m}{p}\right)^{-\frac{1}{\epsilon}} \right],$$

and the marginal surplus to the household of selling one service at price $\tilde{p}$ is

$$\mathscr{S}(\tilde{p}) = \frac{\tilde{p}}{p} \cdot \frac{\partial u}{\partial(\frac{m}{p})} = \frac{\epsilon}{\epsilon - 1} \cdot \frac{1}{1+\chi} \cdot \frac{\tilde{p}}{p} \cdot \left(\frac{m}{p}\right)^{-\frac{1}{\epsilon}},$$

where p is the price level on the product market. The Nash solution maximizes $\mathscr{B}(\tilde{p})^{1-\beta} \cdot \mathscr{S}(\tilde{p})^{\beta}$, so $\mathscr{S}(\tilde{p}) = \beta \cdot [\mathscr{S}(\tilde{p}) + \mathscr{B}(\tilde{p})] = \beta \cdot \frac{\epsilon}{\epsilon-1} \cdot \frac{\chi}{1+\chi} \cdot c^{\frac{-1}{\epsilon}}$, and the bargained price is $\tilde{p} = p \cdot \beta \cdot \chi \cdot c^{\frac{-1}{\epsilon}} \cdot (\frac{m}{p})^{\frac{1}{\epsilon}}$. In equilibrium $\tilde{p} = p$, so combining the condition on the bargained price with the aggregate demand condition, given by equation (2), yields

$$\beta \cdot (1 + \tau(x)) = 1. \tag{7}$$

This equation determines the product market tightness in an equilibrium with Nash bargaining.

Equation (7) implies that the product market tightness responds neither to aggregate demand shocks nor to aggregate supply shocks, exactly as in the competitive equilibrium. Since the comparative statics for the product market tightness are the same in the equilibrium with Nash bargaining and in the competitive equilibrium, all the comparative statics are in fact the same.

The comparative statics are the same in the competitive equilibrium and in the equilibrium with Nash bargaining despite the fact that the former is always efficient whereas the latter is generally inefficient. Indeed, the efficient tightness satisfies $f'(x^*) = \rho$; using $\frac{f(x)}{x} = q(x)$, we rewrite this condition as $x^* \cdot \frac{f'(x^*)}{f(x^*)} = \frac{\rho}{q(x^*)}$ and then as

$$\eta(x^*) = \frac{1}{1 + \tau(x^*)},$$

where $1 - \eta(x)$ is the elasticity of $f(x)$. Comparing this equation with equation (7) indicates that the equilibrium with Nash bargaining is only efficient if $\beta = \eta(x)$—this is the Hosios (1990) condition for efficiency. Hence, the equilibrium with Nash bargaining is efficient only for a specific value of the bargaining power, not in general.

The result that aggregate demand shocks have no effect on tightness, output, and consumption in the equilibrium with Nash bargaining is reminiscent of a result obtained by Blanchard and Galí (2010), Shimer (2010, chapter 2), and Michaillat (2012): they show in different matching models of the labor market that labor demand shocks in the form of technology shocks have no effect on labor market tightness and unemployment when real wages are determined by Nash bargaining.

The result of Blanchard and Galí, Shimer, and Michaillat does not hold in any matching model of the labor market, however. If the value of unemployment (unemployment benefits plus the value from leisure) is positive and fixed (independent of technology), then labor market tightness and unemployment respond to technology shocks under Nash bargaining. Indeed, in that case, the bargained wage increases less than proportionally with technology, so the labor demand increases with technology. Yet if the fixed value of unemployment is calibrated to the generosity of the unemployment insurance system in the United States, the responses of labor market tightness and unemployment are negligible, much smaller than in the data (Shimer 2005). It is only if the fixed value of unemployment is very close to the value of employment—that is, if the higher value from leisure obtained by unemployed workers almost offsets their lower income—that the responses of labor market tightness and unemployment can be large (Hagedorn and Manovskii 2008).

To generate realistic labor market fluctuations, Hagedorn and Manovskii rely on two strong assumptions: individuals are almost indifferent between working and being unemployed, and the value of unemployment is fixed. As will become apparent in Section III when we introduce our complete model, an advantage of our approach is that an equilibrium with fixed or partially rigid prices can generate large responses of labor market tightness and unemployment to shocks without any of these two assumptions.

II.H. The Case with No Matching Cost

We describe the case with no matching cost ($\rho = 0$). This case is useful to clarify the relations between a matching model and Walrasian and disequilibrium models. Without matching cost, there is no price wedge so the aggregate demand is independent of tightness. Furthermore, no output is dissipated in matching so consumption equals output and the aggregate supply increases with tightness everywhere; the efficient tightness, which maximizes the aggregate supply, is infinite. Formally, $\tau(x) = 0$ so the aggregate demand and supply are given by $c^d(p) = \frac{\chi^\epsilon \cdot \mu}{p}$ and $c^s(x) = f(x) \cdot k$. In Panel A of Figure III, the aggregate supply curve would take the shape of the output curve, and the aggregate demand curve would become vertical at $c = \frac{\chi^\epsilon \cdot \mu}{p}$.

A first result is that the competitive equilibrium of the model with no matching cost achieves the price and consumption of a Walrasian equilibrium. In the competitive equilibrium of the model with no matching cost, the tightness is efficient so $x = x^* = +\infty$ and $c = \lim_{x \to +\infty} c^s(x) = k$ as $\lim_{x \to +\infty} f(x) = 1$; furthermore, since $c^d(p) = c = k$, the price satisfies $p = \frac{\chi^\epsilon \cdot \mu}{k}$. In a Walrasian equilibrium, households are indifferent between consumption and money and the money market clears, so $c = \frac{\chi^\epsilon \cdot \mu}{p}$; furthermore, the product market clears, so $c = k$ and $p = \frac{\chi^\epsilon \cdot \mu}{k}$. Hence, c and p are the same in the two equilibria.

Consider a price $p_0 > \frac{\chi^\epsilon \cdot \mu}{k}$. A second result is that the fixprice equilibrium at p_0 in the model with no matching cost yields the same consumption as the excess supply situation at p_0 in the disequilibrium model. In the fixprice equilibrium of the model with no matching cost, consumption is given by $c^d(p_0) = \frac{\chi^\epsilon \cdot \mu}{p_0} < k$. In the excess supply situation of the disequilibrium model, the price is too high for the market to clear, so consumption is determined by the level of demand at p_0: $c = \frac{\chi^\epsilon \cdot \mu}{p_0} < k$. Hence, c is the same in the two cases (by assumption, $p = p_0$ is also the same). Michaillat (2012) obtains a similar result in a matching model of the labor market.

The models with matching cost ($\rho > 0$) and without matching cost ($\rho = 0$) share many properties. In fact, the properties of all the observable variables (price, output, tightness) are the same in the two models. However, imposing $\rho = 0$ has several disadvantages. It is unrealistic because empirical evidence suggests that buyers face matching costs. It impoverishes the model by eliminating the tight regime and thus the model's ability to describe an economy that "overheats." Finally, it makes the model less tractable by imposing a constraint on the equilibrium price (the equilibrium only exists if $p \geq \frac{\chi^{\epsilon} \cdot \mu}{k}$) and by making the efficient tightness infinite.

III. A Model of Aggregate Demand, Idle Time, and Unemployment

This section develops the main model of the article. The model keeps the architecture of the Barro and Grossman (1971) model but takes a matching approach to the labor and product markets instead of a disequilibrium approach.

III.A. Assumptions

The economy has a measure 1 of identical households and a measure 1 of identical firms, owned by the households. The product and labor markets are matching markets that are formally symmetric. Product market and households are the same as in Section II. Labor market and firms are described below.

1. The Labor Market. In each household, $h \in (0, 1)$ members are in the labor force and $1 - h$ members are out of the labor force. All the workers in the labor force are initially unemployed and search for a job. Each firm posts $\hat{v}$ vacancies to hire workers. The number l of workers who find a job is given by the following matching function: $l = (h^{-\hat{\gamma}} + \hat{v}^{-\hat{\gamma}})^{\frac{-1}{\hat{\gamma}}}$, where h is the aggregate number of workers who are initially unemployed, $\hat{v}$ is the aggregate number of vacancies, and the parameter $\hat{\gamma} > 0$ governs the elasticity of substitution of inputs in matching.

We define the labor market tightness θ as the ratio of aggregate number of vacancies to aggregate number of workers who are initially unemployed: $\theta = \frac{\hat{v}}{h}$. The labor market tightness is an aggregate variable taken as given by the firms and households. The labor market tightness determines the probabilities that a worker finds a job and that a vacancy is filled: a worker finds a job with probability $\hat{f}(\theta) = \frac{l}{h} = (1 + \theta^{-\hat{\gamma}})^{\frac{-1}{\hat{\gamma}}}$, and a vacancy is filled with probability $\hat{q}(\theta) = \frac{l}{\hat{v}} = (1 + \theta^{\hat{\gamma}})^{\frac{-1}{\hat{\gamma}}}$. The properties of the functions $\hat{f}$

and $\hat{q}$ imply that when the labor market tightness is higher, it is easier to find a job but harder to fill a vacancy. We abstract from randomness at the firm and household levels: a firm hires exactly $\hat{v} \cdot \hat{q}(\theta)$ workers, and exactly $\hat{f}(\theta) \cdot h$ household members find a job.

Each firm has two types of employees: n producers and $l - n$ recruiters. The job of recruiters is to post vacancies.[20] Posting a vacancy requires $\hat{\rho} \in (0, 1)$ recruiter, so the number of recruiters required to post $\hat{v}$ vacancies is $l - n = \hat{\rho} \cdot \hat{v}$. Since hiring l employees requires posting $\frac{l}{\hat{q}(\theta)}$ vacancies, the number n of producers in a firm with l employees is limited to $n = l - \hat{\rho} \cdot \frac{l}{\hat{q}(\theta)}$. This relationship can be written as $l = (1 + \hat{\tau}(\theta)) \cdot n$, where $\hat{\tau}(\theta) \equiv \frac{\hat{\rho}}{\hat{q}(\theta) - \hat{\rho}}$ is the number of recruiters per producer.

We define the labor supply as the number of producers employed at a given labor market tightness:

DEFINITION 8. The *labor supply* n^s is the function of labor market tightness defined by $n^s(\theta) = \frac{\hat{f}(\theta) \cdot h}{1 + \hat{\tau}(\theta)}$ for all $\theta \in [0, \theta^m]$, where $\theta^m > 0$ satisfies $\hat{\rho} = \hat{q}(\theta^m)$.

PROPOSITION 7. The labor supply satisfies

$$n^s(\theta) = \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h$$

for all $\theta \in [0, \theta^m]$. We define the tightness $\theta^* \in (0, \theta^m)$ by $\hat{f}'(\theta^*) = \hat{\rho}$. The labor supply is strictly increasing on $[0, \theta^*]$ and strictly decreasing on $[\theta^*, \theta^m]$. Hence, the tightness θ^* maximizes the labor supply. Furthermore, $n^s(0) = 0$, and $n^s(\theta^m) = 0$.

Proof. Similar to the proof of Proposition 1. □

The labor supply is depicted in Figure VI. In Panel A, the labor supply curve gives the number of producers. The panel also displays the numbers of recruiters and unemployed workers as a function of labor market tightness. Employment is $l = \hat{f}(\theta) \cdot h$,

20. In the literature, firms usually pay the cost of posting vacancies in output. Here, firms pay the cost of posting vacancies in labor as they need to employ recruiters to fill vacancies. We make this assumption because it greatly simplifies the analysis and seems more realistic. Farmer (2008) and Shimer (2010) make the same assumption.

an increasing and concave function of tightness. The number of producers is $n = \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right) \cdot h$ so it is always below the number of employed workers. The number of recruiters is $l - n = \hat{\rho} \cdot \hat{v} = \hat{\rho} \cdot \theta \cdot h$, an increasing function of tightness; this number is represented by the gap between the labor supply and employment curves. The number of unemployed workers is $h - l = (1 - \hat{f}(\theta)) \cdot h$, a decreasing function of tightness; this number is represented by the gap between the employment and labor force curves. The unemployment rate is $1 - \frac{l}{h} = 1 - \hat{f}(\theta)$. Comparing this panel with Figure I shows that the matching frictions on the product and labor markets are isomorphic.

2. Firms. The representative firm hires l workers. Some of the workers are engaged in production while others are engaged in recruiting. More precisely, $n < l$ producers generate a productive capacity k according to the production function $k = a \cdot n^{\alpha}$. The parameter $a > 0$ measures the technology of the firm and the parameter $\alpha \in (0, 1)$ captures decreasing marginal returns to labor. Because of the product market frictions, the firm only sells a fraction $f(x)$ of its productive capacity.

The firm pays its l workers a real wage w; the nominal wage is $p \cdot w$. The real wage bill of the firm is $w \cdot l = (1 + \hat{\tau}(\theta)) \cdot w \cdot n$. From this perspective, matching frictions in the labor market impose a wedge $\hat{\tau}(\theta)$ on the wage of producers.

Taking as given the labor market tightness, product market tightness, price, and real wage, the representative firm chooses employment to maximize its profits

$$p \cdot f(x) \cdot a \cdot n^{\alpha} - (1 + \hat{\tau}(\theta)) \cdot p \cdot w \cdot n.$$

The profit-maximizing number of producers satisfies

$$f(x) \cdot \alpha \cdot a \cdot n^{\alpha - 1} = (1 + \hat{\tau}(\theta)) \cdot w. \tag{8}$$

This equation implies that the real marginal revenue of one producer equals its real marginal cost. The real marginal revenue is the marginal product of labor, $\alpha \cdot a \cdot n^{\alpha-1}$, times the selling probability, $f(x)$. The real marginal cost is the real wage, w, plus the recruiting cost, $\hat{\tau}(\theta) \cdot w$.

We define the labor demand as the profit-maximizing number of producers at a given product market tightness, labor market tightness, and real wage:

DEFINITION 9. The *labor demand* n^d is the function of labor market tightness, product market tightness, and real wage defined by

$$n^d(\theta, x, w) = \left[\frac{f(x) \cdot \alpha \cdot a}{(1 + \hat{\tau}(\theta)) \cdot w}\right]^{\frac{1}{1-\alpha}} \tag{9}$$

for all $(\theta, x, w) \in [0, \theta^m] \times (0, +\infty) \times (0, +\infty)$, where $\theta^m > 0$ satisfies $\hat{\rho} = \hat{q}(\theta^m)$.

PROPOSITION 8. The labor demand is strictly increasing in x and strictly decreasing in θ and w. Furthermore, $n^d(0, x, w) = \left[\frac{f(x) \cdot \alpha \cdot a \cdot (1-\hat{\rho})}{w}\right]^{\frac{1}{1-\alpha}}$ and $n^d(\theta^m, x, w) = 0$.

Proof. Obvious from equation (9). □

The labor demand is the number of producers that satisfies equation (8). The labor demand is strictly increasing in x because when x increases, the probability $1 - f(x)$ that a producer is idle decreases, so producers become more profitable to firms. It is strictly decreasing in w because when w increases, the wage of producers increases, so producers become less profitable to firms. It is strictly decreasing in θ because when θ increases, the number $\hat{\tau}(\theta)$ of recruiters that firms must hire for each producer increases, so producers become less profitable to firms. The labor demand is depicted in Panel B of Figure VI. The labor demand curve slopes downward in the (n, θ) plane.

Unemployment is traditionally decomposed into three components: a Keynesian component caused by deficient aggregate demand, a classical component caused by excessively high real wages, and a frictional component caused by recruiting costs. In our model this decomposition is not meaningful because equilibrium unemployment is simultaneously determined by aggregate demand, real wage, and recruiting cost. Yet our model of the labor demand incorporates Keynesian, classical, and frictional factors. The Keynesian factor operates through $f(x)$ in equation (9), because $f(x)$ describes how easy or difficult it is for firms to find customers. The classical factor operates through w in equation (9). The frictional factor operates through $\hat{\tau}(\theta)$ in equation (9), because $\hat{\tau}(\theta)$ describes how costly it is for firms to recruit workers. The cost of recruiting workers can be high either because the cost of posting a vacancy is high or because vacancies are filled with low probability—this happens when the labor market tightness is high.

III.B. *Definition of the Equilibrium*

Employed and unemployed household members pool their income before jointly deciding consumption; therefore, despite the unemployment risk, the aggregate demand is still given by equation (3). Firms' productive capacity is not exogenous but is endogenously determined by firms' employment level; the aggregate supply is given by

$$c^s(x,\theta) = (f(x) - \rho \cdot x) \cdot a \cdot \left(\hat{f}(\theta) - \hat{\rho} \cdot \theta\right)^{\alpha} \cdot h^{\alpha}.$$

This expression is obtained from equation (1) by setting the productive capacity to $a \cdot n^{\alpha}$ and expressing n as a function of θ using the labor supply. The equilibrium is defined as follows:

DEFINITION 10. An *equilibrium* consists of a product market tightness, a price, a labor market tightness, and a real wage (x, p, θ, w) such that aggregate supply is equal to aggregate demand and labor supply is equal to labor demand:

$$\begin{cases} c^s(x,\theta) & = & c^d(x,p) \\ n^s(\theta) & = & n^d(\theta, x, w). \end{cases}$$

Since the equilibrium is composed of four variables that satisfy two conditions, infinitely many combinations of (x, p, θ, w) are consistent with the equilibrium conditions. To select an equilibrium, we specify a price and a wage mechanism. In Sections III.C–III.E, we study the equilibria selected by different mechanisms.

Many equilibrium prices and wages are possible, so the equilibrium may be in different regimes:

DEFINITION 11. The equilibrium is *efficient* if $\theta = \theta^*$ and $x = x^*$, *labor-slack* and *product-slack* if $\theta < \theta^*$ and $x < x^*$, *labor-slack and product-tight* if $\theta < \theta^*$ and $x > x^*$, *labor-tight* and *product-slack* if $\theta > \theta^*$ and $x < x^*$, and *labor-tight* and *product-tight* if $\theta > \theta^*$ and $x > x^*$.

These four inefficient regimes are reminiscent of the four regimes in the Barro-Grossman model. In both models, whether the price and the real wage are inefficiently high or low determines which regime prevails. In Online Appendix B we characterize the four regions of a (w, p) plane that correspond to the four

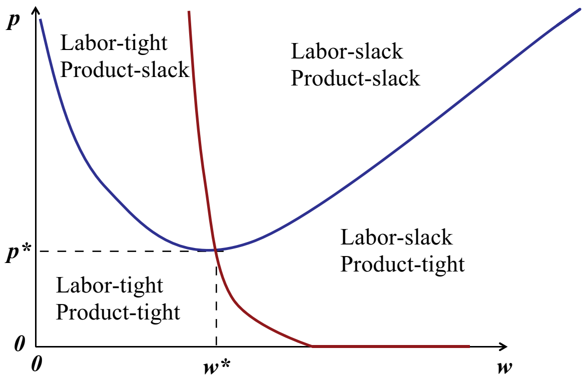


FIGURE VII

The Four Inefficient Regimes in the Model of Section III

inefficient regimes. These regions are depicted in Figure VII. The region of the labor-slack and product-slack equilibria has high prices and high real wages, the region of labor-tight and product-tight equilibria has low prices and low real wages, and so on. The efficient equilibrium is at the intersection of the two curves delimiting the inefficient regimes.

Despite the similarities between our model and the Barro-Grossman model, our model is more tractable because it describes the economy in the four regimes more compactly. In our model the equilibrium is described by the same system of smooth equations in all the regimes. In contrast, in the Barro-Grossman model the disequilibrium is described by four different systems of equations, one for each regime. These four systems are required to describe all the possible disequilibrium situations as either supply or demand can be rationed in each market. Studying the model is therefore difficult because each regime requires a different analysis.

III.C. Fixprice Equilibrium

DEFINITION 12. A *fixprice equilibrium* parameterized by $p_0 > 0$ and $w_0 > 0$ consists of a product market tightness, a price, a labor market tightness, and a real wage (x, p, θ, w) such that supply equals demand on the product and labor markets and price and real wage are given by the parameter

p_0 and w_0: $c^s(x,\theta) = c^d(x,p)$, $n^s(\theta) = n^d(\theta, x, w)$, $p = p_0$, and $w = w_0$.

PROPOSITION 9. For any $p_0 > 0$ and $w_0 > 0$, there exists a unique fixprice equilibrium parameterized by p_0 and w_0 with positive consumption.

Proof. See Online Appendix A. □

We use comparative statics to describe the response of the fixprice equilibrium to aggregate demand, technology, labor supply, and mismatch shocks. We parameterize an increase in aggregate demand by an increase in money supply, μ, or in the taste for consumption, χ. We parameterize an increase in technology by an increase in the production function parameter, a. We parameterize an increase in labor supply by an increase in the size of the labor force, h. An increase in h captures increases in labor force participation caused by demographic factors, changes to the taste for leisure and work, or changes to policies such as disability insurance. An increase in h also captures increases in job search effort caused by changes to policies such as unemployment insurance.[21] We parameterize an increase in mismatch by a decrease of the matching efficacy on the labor market along with a corresponding decrease in recruiting cost: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and ρ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$ with $\lambda < 1$.[22] Note that the function $\hat{\tau}$ and tightness θ^* remain the same after a mismatch shock. The interpretation of an increase in mismatch is that a fraction of potential workers are not suitable to employers, which reduces matching efficacy, and these unsuitable workers can be spotted at

21. Assume that workers receiving unemployment insurance search for a job with effort 0 or 1. A change to the generosity of unemployment insurance affects the share of workers searching with effort 1. But only workers searching with effort 1 are part of h because only these workers contribute to the matching process. Hence, changing the generosity of unemployment insurance affects h. Note that our classification of the workers receiving unemployment insurance is consistent with the definitions used in official statistics. In the statistics constructed by the BLS from the CPS, job seekers are counted as unemployed if they search with effort 1 and as out of the labor force if they search with effort 0, irrespective of their receipt of unemployment insurance.

22. See Shimer (2007) and Sahin et al. (2014) for microfounded models of labor market mismatch.

no cost, which reduces the cost of managing a vacancy.[23] The following proposition summarizes the comparative statics:[24]

PROPOSITION 10. Consider a fixprice equilibrium with positive consumption.

- An increase in aggregate demand has the following effects: output, product market tightness, employment, and labor market tightness increase; the rate of idleness and the rate of unemployment decrease.
- An increase in technology has the following effects: output increases but product market tightness decreases; employment and labor market tightness increase; the rate of idleness increases but the rate of unemployment decreases.
- An increase in labor supply has the following effects: output and employment increase, but product market tightness and labor market tightness decrease; the rate of idleness and the rate of unemployment increase.
- A decrease in mismatch has the following effects: output and employment increase, but product market tightness and labor market tightness decrease; the rate of idleness increases but the rate of unemployment decreases.

Proof. See Online Appendix A. □

The comparative statics are summarized in Panel A of Table II. Here we explain these comparative statics with the help of the equilibrium diagrams in Figures III and VI. We concentrate on the effects of shocks on tightnesses; the effects of shocks on quantities follow.

First consider an increase in aggregate demand. As explained in Section II.E, the aggregate demand curve rotates upward in Figure III, Panel A, and the product market tightness rises. Therefore, the rate of idleness among the producers employed by firms falls, and hiring a producer becomes more profitable. Consequently, the labor demand curve rotates outward in

23. Another possible parameterization of mismatch shocks is a decrease in matching efficacy with no change in recruiting cost. Such a parameterization leads to less clearcut results because the mismatch shock affects both labor demand and labor supply.

24. With a linear production function ($\alpha = 1$), all the comparative statics would remain the same.

TABLE II

COMPARATIVE STATICS IN THE MODEL OF SECTION III

	Effect on:			
	Output	Product market tightness	Employment	Labor market tightness
Increase in:	y	x	l	θ
Panel A: Fixprice equilibrium and equilibrium with partially rigid price and real wage				
Aggregate demand	+	+	+	+
Technology	+	–	+	+
Labor supply	+	–	+	–
Mismatch	–	+	–	+
Panel B: Competitive equilibrium and equilibrium with Nash bargaining				
Aggregate demand	0	0	0	0
Technology	+	0	0	0
Labor supply	+	0	+	0
Mismatch	–	0	–	0

Notes. An increase in aggregate demand is an increase in money supply, μ, or in the taste for consumption, χ. An increase in technology is an increase in the production-function parameter, a. An increase in labor supply is an increase in the size of the labor force, h. An increase in mismatch is a decrease of the matching efficacy on the labor market along with a corresponding decrease in recruiting cost. After an increase in mismatch, $\hat{f}(\theta)$, $\hat{q}(\theta)$, and $\hat{\rho}$ become $\lambda \cdot \hat{f}(\theta)$, $\lambda \cdot \hat{q}(\theta)$, and $\lambda \cdot \hat{\rho}$, with $\lambda < 1$. This table summarizes the results of Propositions 10 and 12 and the results discussed in Section III.E.

Figure VI, Panel B, which raises the labor market tightness. As a result, the number of producers change, which shifts the aggregate supply curve in Figure III, Panel A. Hence, the initial increase in product market tightness may be dampened (if the labor market is slack and the number of producers increases) or accentuated (if the labor market is tight and the number of producers decreases).

Second, consider an increase in technology. Firms' productive capacity rises, so, as explained in Section II.E, the aggregate supply curve shifts outward in Figure III, Panel A, and the product market tightness falls. At the same time, producers' productivity increases while their real wage remains fixed; hence hiring a producer becomes more profitable. Consequently, the labor demand curve rotates outward in Figure VI, Panel B, which raises the labor market tightness. The initial responses of the tightnesses spill over across markets. First, the decrease in product market tightness increases the rate of idleness among firms' producers, which pushes the labor demand curve back inward and attenuates the initial increase in labor market tightness.

Second, the increase in labor market tightness changes the number of producers, which shifts the aggregate supply curve in Figure III, Panel A. Thus, the initial decrease in product market tightness may be dampened (if the labor market is tight) or accentuated (if the labor market is slack).

Our model retains the intuition of the Barro-Grossman model that negative aggregate demand shocks propagate to the labor market by making it more difficult for firms to sell services. But the mechanism of propagation from the product market to the labor market is quite different in the two models; the response of employment to an increase in technology make this difference visible. In our model, a positive technology shock always increases employment. In contrast, in the Keynesian unemployment regime of the Barro-Grossman model, a positive technology shock decreases employment (Bénassy 1993). In that regime, firms are demand constrained: fixed price and aggregate demand determine the output $\bar{y}$ that firms can sell. As firms have a production function $y = a \cdot l^{\alpha}$, employment is determined by the demand constraint: $l = (\frac{\bar{y}}{a})^{\frac{1}{\alpha}}$. An increase in technology a therefore reduces employment. The same property is true in some new Keynesian models (Galí 1999).[25] Technology shocks have opposite effects on employment in the Barro-Grossman model and our model because aggregate demand constrains firms differently in the two models: in the Barro-Grossman model, firms take as given the number of services that they can sell; in our model, firms take as given the probability to sell a service offered for sale.

Third, consider an increase in labor supply. The labor force and labor supply curves shift outward in Figure VI, Panel B. Thus, the labor market tightness falls, but the number of producers increases. With more producers, firms' productive capacity increases; therefore, the aggregate supply curve shifts outward in Figure III, Panel A, which reduces the product market tightness. Since the product market tightness falls, the labor demand curve rotates inward in Figure VI, Panel B, which further reduces the labor market tightness and attenuates the initial increase in the number of producers.

25. New Keynesian models feature monopolistic firms that can only change their prices at intermittent intervals. When its price is fixed, a firm faces a demand constraint as the firms in the Keynesian unemployment regime of the Barro-Grossman model. This explains why some new Keynesian models have inherited the property of the Barro-Grossman model.

Finally, consider a decrease in mismatch. In Figure VI, Panel B, the labor supply curve shifts outward but the labor demand curve remains the same. As after an increase in labor supply, the labor market and product market tightnesses decrease. But unlike after an increase in labor supply, the unemployment rate decreases. This is because the reduction in mismatch increases employment without affecting the size of the labor force; an increase in labor supply also increases employment, but not as much as the underlying increase in the size of the labor force. In fact, the mismatch shock is the only shock generating a positive correlation between labor market tightness and unemployment rate.

III.D. Competitive Equilibrium

DEFINITION 13. A *competitive equilibrium* consists of a product market tightness, a price, a labor market tightness, and a real wage (x, p, θ, w) such that supply equals demand on the product and labor markets and the labor and product market tightnesses are efficient: $c^s(x, \theta) = c^d(x, p)$, $n^s(\theta) = n^d(\theta, x, w)$, $x = x^*$, and $\theta = \theta^*$.

PROPOSITION 11. There exists a unique competitive equilibrium. The competitive price and real wage are given by

$$p^* = \frac{(1+\tau(x^*))^{1-\epsilon}}{f(x^*)} \cdot \left(\frac{1+\hat{\tau}(\theta^*)}{\hat{f}(\theta^*)}\right)^{\alpha} \cdot \frac{\chi^{\epsilon}}{a \cdot h^{\alpha}} \cdot \mu$$

$$w^* = f(x^*) \cdot \frac{\hat{f}(\theta^*)^{\alpha-1}}{(1+\hat{\tau}(\theta^*))^{\alpha}} \cdot \alpha \cdot a \cdot h^{\alpha-1}.$$

Proof. See Online Appendix A. □

PROPOSITION 12. Consider a competitive equilibrium.

- An increase in aggregate demand has no effect on output, product market tightness, the rate of idleness, employment, labor market tightness, and the rate of unemployment.
- An increase in technology has the following effects: output increases; product market tightness, the rate of idleness, employment, labor market tightness, and the rate of unemployment remain the same.

- An increase in labor supply has the following effects: output and employment increase; product market tightness, the rate of idleness, labor market tightness, and the rate of unemployment remain the same.
- A decrease in mismatch has the following effects: output and employment increase; product market tightness, the rate of idleness, and labor market tightness remain the same; the rate of unemployment decreases.

Proof. Similar to the proof of Proposition 6. □

The comparative statics are summarized in Table II, Panel B. The competitive equilibrium has three notable properties. First, aggregate demand shocks have no real effects. Second, the product market and labor market tightnesses do not respond to any of the shocks considered, not even mismatch shocks. Third, employment only responds to labor supply and mismatch shocks, and the unemployment rate only responds to mismatch shocks.

III.E. Other Equilibria

1. Equilibrium with Partially Rigid Price and Real Wage. We consider the following partially rigid price and real wage:

$$p = p_0 \cdot \left(\frac{\chi^{\epsilon}}{a \cdot h^{\alpha}} \cdot \mu\right)^{\xi}$$

$$w = w_0 \cdot \left(\alpha \cdot a \cdot h^{\alpha-1}\right)^{\xi}.$$

The parameter $\xi \in [0, 1)$ governs the rigidity of the price and the rigidity of the real wage.[26] We show in Online Appendix A that even though price and real wage are only partially rigid, the equilibrium conditions have the same properties as when price and real wage are fixed. Hence, the comparative statics of the fixprice equilibrium remain valid in this equilibrium with partial rigidity.

2. Equilibrium with Nash Bargaining. The real wage is the generalized Nash solution of the bargaining problem between a

26. We confine our analysis to the case in which price and real wage have the same rigidity. This case shows that the comparative statics of the fixprice equilibrium may also be valid when price and real wage are only partially rigid.

firm and a marginal worker with bargaining power $\hat{\beta} \in (0, 1)$.[27] After a match is made, the surplus to the firm of employing a marginal worker is $\mathscr{F}(w) = f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1} - w$, and the surplus to the worker of being employed is $\mathscr{W}(w) = w$. The Nash solution maximizes $\mathscr{F}(w)^{1-\hat{\beta}} \cdot \mathscr{W}(w)^{\hat{\beta}}$, so $\mathscr{W}(w) = \hat{\beta} \cdot [\mathscr{W}(w) + \mathscr{F}(w)] = \hat{\beta} \cdot f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1}$ and the real wage satisfies $w = \hat{\beta} \cdot f(x) \cdot a \cdot \alpha \cdot n^{\alpha-1}$. With this wage the labor demand condition, given by equation (8), becomes

$$\hat{\beta} \cdot (1 + \hat{\tau}(\theta)) = 1.$$

This equation determines the labor market tightness. Equation (7) determines the product market tightness. In equilibrium the tightnesses are solely determined by the functions τ and $\hat{\tau}$; therefore, they do not respond to aggregate demand, technology, labor supply, or mismatch shocks. We conclude that all the comparative statics are the same as in the competitive equilibrium.

IV. A Dynamic Model with Long-Term Relationships

In this section we embed the static model of Section III into a dynamic environment to represent long-term customer and employment relationships. Such relationships are prevalent, as showed in Figure II, Panel C. The panel displays the fraction of sales going to long-term customers in eleven countries, including the United States; on average, 77 percent of sales go to long-term customers. The panel also displays the share of workers engaged in long-term employment relationships in the same eleven countries; on average, 87 percent of workers have long-term employment contracts.

We use the dynamic model in the empirical analysis of Section V because, compared to the static model, the dynamic model offers a better mapping between theoretical and empirical variables. The mapping is better because the matching process in the dynamic model features long-term relationships and thus corresponds more closely to what we see in real world. Although the dynamic model is more complex, its comparative statics at the

27. Although a firm and its workers are engaged in multilateral intrafirm bargaining, we abstract from possible strategic behavior. Such behavior is analyzed in Stole and Zwiebel (1996). Instead, we assume that a firm bargains with each of its workers individually, taking each worker as marginal.

limit without time discounting are the same as those of the static model.

IV.A. Matching Process on the Product and Labor Markets

We work in continuous time. Firms engage in long-term relationships with customers on the product market, and they engage in long-term relationships with employees on the labor market.

At time t, there are h workers in the labor force, $l(t)$ employed workers, and $h - l(t)$ unemployed workers. Firms post $\hat{v}(t)$ vacancies. New employment relationships are formed at a rate $\left[(h - l(t))^{-\hat{\gamma}} + \hat{v}(t)^{-\hat{\gamma}}\right]^{\frac{-1}{\hat{\gamma}}}$. We define the labor market tightness as $\theta(t) = \frac{\hat{v}(t)}{h - l(t)}$. Unemployed workers find a job at rate $\hat{f}(\theta(t))$, and vacancies are filled at rate $\hat{q}(\theta(t))$. Employment relationships are destroyed at rate $\hat{s} > 0$. The law of motion of employment is therefore given by

$$\dot{l}(t) = \hat{f}(\theta(t)) \cdot (h - l(t)) - \hat{s} \cdot l(t). \tag{10}$$

In this law of motion, $\hat{f}(\theta(t)) \cdot (h - l(t))$ is the number of employment relationships created at t and $\hat{s} \cdot l(t)$ is the number of employment relationships destroyed at t.

The product market operates exactly like the labor market. All purchases take place through long-term customer relationships. At time t, firms have a productive capacity $k(t) = a \cdot n(t)^{\alpha}$ and sell output $y(t) < k(t)$. Idle capacity is $k(t) - y(t)$. Households create new customer relationships by visiting $v(t)$ firms that have $k(t) - y(t)$ productive capacity available. New customer relationships are formed at a rate $[(k(t) - y(t))^{-\gamma} + v(t)^{-\gamma}]^{\frac{-1}{\gamma}}$. We define the product market tightness as $x(t) = \frac{v(t)}{k(t) - y(t)}$. The $k(t) - y(t)$ units of available productive capacity yield new customer relationships at rate $f(x(t))$ and the $v(t)$ visits are successful at rate $q(x(t))$. Customer relationships are destroyed at rate s. The law of motion of output is therefore given by

$$\dot{y}(t) = f(x(t)) \cdot (k(t) - y(t)) - s \cdot y(t). \tag{11}$$

In this law of motion, $f(x(t)) \cdot (k(t) - y(t))$ is the number of customer relationships created at t and $s \cdot y(t)$ is the number of customer relationships destroyed at t.

IV.B. Households

The utility of the representative household is given by $\int_{t=0}^{+\infty} e^{-\delta \cdot t} \cdot u(c(t), \frac{m(t)}{p(t)})dt$, where $\delta > 0$ is the time discount factor, $c(t)$ is consumption at time t, and $\frac{m(t)}{p(t)}$ are real money balances at time t. To consume $c(t)$, the household must make $y(t) \geq c(t)$ purchases. The $y(t)$ purchases are used for consumption, $c(t)$, and to cover the matching costs. At time t, the household adjusts its number of customer relationships by $\dot{y}(t)$, and it also replaces the $s \cdot y(t)$ relationships that have just been destroyed. Making these $\dot{y}(t) + s \cdot y(t)$ new relationships requires $\frac{\dot{y}(t)+s \cdot y(t)}{q(x(t))}$ visits, each costing ρ purchases. Hence, purchases and consumption are related by

$$y(t) = c(t) + \frac{\rho}{q(x(t))} \cdot (\dot{y}(t) + s \cdot y(t)). \tag{12}$$

Nominal money balances are an asset with law of motion

$$\dot{m}(t) = p(t) \cdot w(t) \cdot l(t) - p(t) \cdot y(t) + T(t), \tag{13}$$

where $T(t)$ includes firms' nominal profits, which are rebated to the household, and transfers from the government. Given $[p(t), w(t), x(t), l(t), T(t)]_{t=0}^{+\infty}$ the household chooses $[y(t), c(t), m(t)]_{t=0}^{+\infty}$ to maximize utility subject to equations (12) and (13).

IV.C. Firms

The representative firm employs $n(t)$ producers and $l(t) - n(t)$ recruiters. At time t, the firm adjusts its number of employees by $\dot{l}(t)$, and it also replaces the $\hat{s} \cdot l(t)$ employees that have just left the firm. Hiring these $\dot{l}(t) + \hat{s} \cdot l(t)$ new workers requires to post $\frac{\dot{l}(t)+\hat{s} \cdot l(t)}{\hat{q}(\theta(t))}$ vacancies. Each vacancy takes the time of $\hat{\rho}$ recruiters. Hence, the firm needs the following number of recruiters:

$$l(t) - n(t) = \frac{\hat{\rho}}{\hat{q}(\theta(t))} \cdot \left(\dot{l}(t) + \hat{s} \cdot l(t)\right). \tag{14}$$

The firm sells output $y(t)$ to customers. The amount of sales depend on the product market tightness and the productive capacity of the firm:

$$\dot{y}(t) = f(x(t)) \cdot (a \cdot n(t)^{\alpha} - y(t)) - s \cdot y(t). \tag{15}$$

Given $[w(t), x(t), \theta(t)]_{t=0}^{+\infty}$, the firm chooses $[l(t), n(t), y(t)]_{t=0}^{+\infty}$ to maximize the discounted stream of real profits, $\int_{t=0}^{+\infty} e^{-\delta \cdot t} \cdot (y(t) - w(t) \cdot l(t))dt$, subject to equations (14) and (15).

IV.D. Discussion of the Assumptions

We assume that in a long-term relationship, the buyer does not incur the matching cost and the seller sells one unit of good (labor or output) per unit time with certainty. These assumptions are standard in dynamic matching models of the labor market. They describe well long-term employment relationships given the nature of labor contracts.

We also think that the assumptions describe long-term customer relationships well. First, a sizable share of transactions on the product market are conducted under contract, and our assumptions describe well the terms of an explicit contract.[28] Second, observations from a survey of bakers that we conducted in France in summer 2007 suggest that even when no explicit contract is signed, long-term customer relationships are governed by implicit contracts that alleviate matching frictions in line with our assumptions.[29] A first observation is that customer relationships alleviate the uncertainty associated with random demand. A baker told us that demand is difficult to predict and that having a large clientele of loyal customers who make it a habit to purchase bread in the shop was therefore important. In fact, "good" customers are expected to come every day to the bakery. A second observation is that customer relationships alleviate the uncertainty associated with random supply. Being a customer means having the assurance that your usual bread will be available, even on days when supply runs low. Of course, this is possible because bakers know exactly what customers order every day through their long association. In fact, one baker said that it would be "unacceptable" to run out of bread for a customer, and that customers would probably "leave the bakery" if that happened.

28. Using BLS data on contractual arrangements between firms trading intermediate goods, Goldberg and Hellerstein (2011) find that one-third of all transactions are conducted under contract.

29. This survey is described in Eyster, Madarasz, and Michaillat (2015).

IV.E. Steady-State Equilibrium

We focus on a steady-state equilibrium with no time discounting and a money supply growing at a constant rate $\frac{\dot{\mu}(t)}{\mu(t)} = \pi > 0$.[30] To maintain real money balances constant, the rate of price inflation must be $\frac{\dot{p}(t)}{p(t)} = \pi$; hence, the price level satisfies the differential equation $\dot{p}(t) = \pi \cdot p(t)$, where $\pi > 0$ is growth rate of the money supply and $p(0)$ is determined by the price mechanism. The real wage is constant: $w(t) = w$, where w is determined by the price mechanism. Given a price mechanism, the variables $\{l, n, y, c, \theta, x\}$ satisfy $y = \frac{f(x)}{s+f(x)} \cdot a \cdot n^{\alpha}$, $l = \frac{\hat{f}(\theta)}{\hat{s}+\hat{f}(\theta)} \cdot h$, $y = (1 + \tau(x)) \cdot c$ where $\tau(x) \equiv \frac{s \cdot \rho}{q(x) - s \cdot \rho}$, $l = (1 + \hat{\tau}(\theta)) \cdot n$ where $\hat{\tau}(\theta) \equiv \frac{\hat{s} \cdot \hat{\rho}}{\hat{q}(\theta) - \hat{s} \cdot \hat{\rho}}$,

$$c = \left(\frac{\chi \cdot \pi}{1 + \tau(x)}\right)^{\epsilon} \cdot \frac{\mu(0)}{p(0)}$$

$$n = \left[\frac{f(x)}{s + f(x)} \cdot \frac{\alpha \cdot a}{(1 + \hat{\tau}(\theta)) \cdot w}\right]^{\frac{1}{1-\alpha}}.$$

The first two equations are obtained by setting $\dot{l}(t) = 0$ and $\dot{y}(t) = 0$ in equations (10) and (11). The next two are obtained by setting $\dot{y}(t) = 0$ and $\dot{l}(t) = 0$ in equations (12) and (14). The last two describe the household's optimal consumption choice combined with the market-clearing condition for money and the firm's optimal employment choice. These last two equations are derived in Online Appendix C.

These equations describe the output, employment, aggregate supply, labor supply, aggregate demand, and labor demand curves. These curves correspond exactly to the curves of the static model of Section III once χ, $f(x)$, and $\hat{f}(\theta)$ are replaced by $\chi \cdot \pi$, $\frac{f(x)}{s+f(x)}$ and $\frac{\hat{f}(\theta)}{\hat{s}+\hat{f}(\theta)}$, and once the parameters ρ and $\hat{\rho}$ are replaced by the parameters $s \cdot \rho$ and $\hat{s} \cdot \hat{\rho}$ in τ and $\hat{\tau}$. All the relevant properties of the functions f and $\hat{f}$ are preserved by the

30. Introducing positive inflation ensures that households consume some produced good even when they become infinitely patient at the limit without time discounting. Without inflation, infinitely patient households would use all their income to increase their money balances, and aggregate demand would be zero.

transformation to $\frac{f}{s+f}$ and $\frac{\hat{f}}{\hat{s}+\hat{f}}$. Hence, the comparative statics of the dynamic model are the same as those of the static model of Section III. This is true both in a fixprice equilibrium, in which $p(0)$ and w are fixed, and in a competitive equilibrium, in which $p(0)$ and w ensure that the tightnesses are efficient.

V. Exploration of the Sources of Labor Market Fluctuations in the United States

In this section we combine the comparative static predictions of the dynamic model of Section IV with empirical evidence to assess the sources of the labor market fluctuations observed in the United States.[31] We find that aggregate demand shocks are the main source of these fluctuations.

V.A. A Proxy for Product Market Tightness

The empirical analysis relies on the cyclical behavior of the product market tightness x_t. We are not aware of any measure of product market tightness, so we construct a proxy for the cyclical component of the product market tightness in the United States.[32] Our proxy is the cyclical component of the labor utilization rate $\frac{f(x_t)}{s+f(x_t)}$. The labor utilization rate is 1 minus the rate of idleness of employed workers.

We construct our proxy from the capacity utilization rate cu_t measured by the Census Bureau in the SPC from 1973:Q4 to 2013:Q2. We choose the measure of capacity utilization from the SPC because, compared to other measures of utilization, it is available for the longest period and uses the broadest sample of firms. The measure applies to the manufacturing sector. The SPC measures fourth-quarter capacity utilization rates until 2007 and quarterly capacity utilization rates after that. To obtain a

31. The assumption underlying our analysis is that the comparative statics provide a good approximation to the actual dynamic effects of shocks. This assumption is justified if the labor and product markets quickly reach their steady states. Shimer (2005) and Pissarides (2009) argue that this assumption is justified for the labor market because the rates of inflow to and outflow from unemployment are large. Michaillat (2012) uses numerical simulations to validate this assumption for the labor market. There is little evidence on the size of customer flows, making it difficult to validate the assumption for the product market.

32. For a measure of the tightness on the capital market, see Ottonello (2014).

quarterly series for the entire period, we use a linear interpolation of the annual series into a quarterly series for 1973:Q4–2007:Q4 and combine it with the quarterly series for 2008:Q1–2013:Q2.

We need to correct cu_t to obtain $\frac{f(x_t)}{s+f(x_t)}$ because $\frac{f(x_t)}{s+f(x_t)}$ is the share of the productive capacity at current employment that is actually sold, whereas cu_t is the share of the productive capacity at full employment that is actually sold (Morin and Stevens 2004). Let $g(a, n, k) = a \cdot n^{\alpha} \cdot k^{1-\alpha}$ be a firm's productive capacity with technology a, employment n, and capital k. Let k_t be the current stock of capital, which is also the stock of capital that respondents take into account when they report cu_t. Let N_t be the full-employment level that respondents take into account when they report cu_t. Let n_t be the current level of employment. We will assume that N_t moves slowly over time so that its cyclical component is zero. The firm's capacity is $g(a_t, n_t, k_t)$ under current employment and $g(a_t, N_t, k_t)$ under full employment. We can write the firm's output in two different ways:

$$y_t = cu_t \cdot g(a_t, N_t, k_t) = \frac{f(x_t)}{s + f(x_t)} \cdot g(a_t, n_t, k_t).$$

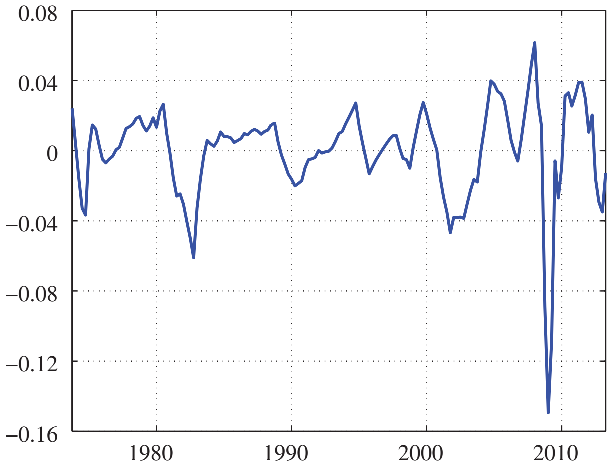
Taking log and recombining, we find that

$$\ln\left(\frac{f(x_t)}{s + f(x_t)}\right) = \ln(cu_t) - \alpha \cdot \ln(n_t) + \alpha \cdot \ln(N_t). \tag{16}$$

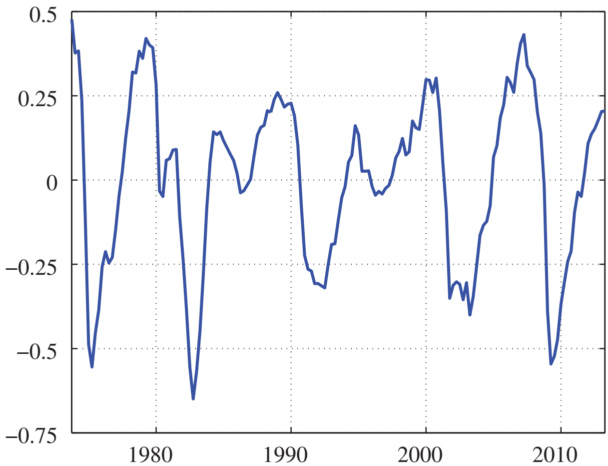
We use equation (16) to construct the cyclical component of $\frac{f(x_t)}{s+f(x_t)}$, which is our proxy for the cyclical component of the product market tightness. We denote this proxy by x_t^c. First, we measure n_t as the quarterly average of the seasonally adjusted monthly employment level in the manufacturing sector constructed by the BLS from the Current Employment Statistics survey. Second, we remove from $\ln(cu_t)$ and $\ln(n_t)$ the low-frequency trends produced by a Hodrick-Prescott (HP) filter with smoothing parameter 1600; this procedure yields the cyclical components of cu_t and n_t, which we denote by cu_t^c and n_t^c. Third, we assume that the cyclical component of N_t is zero because N_t is a slow-moving variable. Following conventions, we set $\alpha = \frac{2}{3}$. Last, using equation (16), we obtain

$$x_t^c = cu_t^c - \alpha \cdot n_t^c. \tag{17}$$

Panel A of Figure VIII plots the proxy for 1973:Q4–2013:Q2.



A. Cyclical component of product market tightness



B. Cyclical component of labor market tightness

FIGURE VIII

Product Market Tightness and Labor Market Tightness in the United States, 1973:Q4–2013:Q2

Panel A displays the proxy for the cyclical component of the product market tightness, x_t^c. The proxy x_t^c is computed using equation (17). Panel B displays the cyclical component of the labor market tightness, θ_t^c. The labor market tightness is constructed as $\theta_t = \frac{v_t}{u_t}$, where v_t is the quarterly average of the monthly vacancy index constructed by Barnichon (2010), and u_t is the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. We construct θ_t^c by removing from $\ln(\theta_t)$ the trend produced by a HP filter with smoothing parameter 1600.

Our proxy for the product market tightness is not ideal. First, it is constructed from a measure of capacity utilization instead of a direct measure of labor utilization. Second, the measure of capacity utilization applies to the manufacturing sector, and it may therefore be influenced by some logistical issues, such as peak load and inventory management. We address these problems in Online Appendix D. There we show that all our empirical results are robust to using an alternative proxy for product market tightness. This alternative proxy is constructed from the operating rate in nonmanufacturing sectors measured by the ISM and published in their Semiannual Reports. The operating rate is the actual production level of firms as a share of their maximum production level given current capital and labor, so it exactly corresponds to our concept of labor utilization. Unfortunately, the time series for the operating rate only starts in 1999:Q4, so it is too brief to permit a thorough empirical analysis.

The empirical analysis also requires measures of output, employment, and labor market tightness for the United States from 1973:Q4 to 2013:Q2. We measure output and employment using seasonally adjusted quarterly indexes for real output and employment for the nonfarm business sector constructed by the MSPC program of the BLS. We construct the labor market tightness as the ratio of vacancies to unemployment. We measure vacancies with the quarterly average of the monthly vacancy index constructed by Barnichon (2010). This index combines the online and print help-wanted indexes of the Conference Board. We measure unemployment with the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. We construct the cyclical components of these series by taking their log and removing the low-frequency trend produced by a HP filter with smoothing parameter 1600.

V.B. Evidence of Price and Real-Wage Rigidity

The equilibria that we have studied can be sorted in two groups, based on their comparative statics. The first group includes the fixprice equilibrium and the equilibrium with partially rigid price and real wage. Their comparative statics are reported in Table II, Panel A. Since shocks are not entirely absorbed by price and real wage and transmit to tightnesses, we say that these equilibria exhibit price and real-wage rigidity. The second group includes the competitive equilibrium and the equilibrium with Nash bargaining. Their comparative statics are reported in Panel B of Table II. Since shocks are entirely absorbed by price and real wage and do not transmit to tightnesses, we say that these equilibria exhibit price and real-wage flexibility.

The two groups of equilibria have starkly different comparative statics, so the first step of the empirical analysis is to determine which group describes the data better. To do so, we observe the cyclical behavior of the product market and labor market tightnesses, and we exploit the property that the tightnesses respond to shocks only in equilibria with price and real-wage rigidity.

Figure VIII displays the cyclical components of the product market and labor market tightnesses. Panel A shows that the cyclical component of the product market tightness is subject to fluctuations.[33] Panel B confirms the well-known fact that the labor market tightness is subject to large fluctuations over the business cycle.[34] For instance, the cyclical component of the labor market tightness fell to –0.5 in 2009, which indicates that the labor market tightness was broadly 50 percent below trend in 2009. While the drop in labor market tightness in 2008–2009 was commensurate to the drops in previous recessions, the drop in product market tightness in 2008–2009 was unprecedented—it was three times as large as the drops in 1981–1982 and 2001.

The cyclical fluctuations of the product market and labor market tightnesses suggest that the equilibria with price and real-wage rigidity are more appropriate to describe business cycles than the equilibria with price and real-wage flexibility. Relatedly, Shimer (2005) and Hall (2005) observe that the labor market tightness is subject to large fluctuations in the United States, and they conclude that real wages must be somewhat rigid. In the rest of the analysis, we therefore use the predictions of the equilibria with price and real-wage rigidity. These predictions are reported in Table II, Panel A.

33. The cyclical fluctuations of the product market tightness have never been analyzed before. Yet the observation that the product market tightness fluctuates a lot is not very surprising: everybody knows that queues at restaurants systematically vary depending on the time of the day or the day of the week, which indicates that prices do not adjust sufficiently to absorb variations in demand.

34. See, for instance, the empirical work of Blanchard and Diamond (1989b) and Shimer (2005).

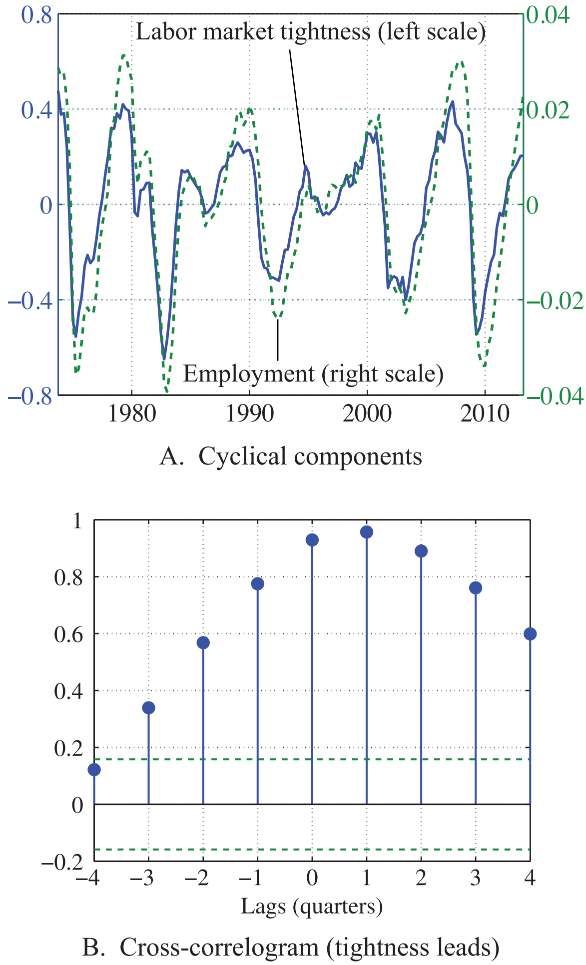


FIGURE IX

Correlation between Labor Market Tightness and Employment in the United States, 1973:Q4–2013:Q2

Panel A displays the cyclical component of the labor market tightness, θ_t^c, and the cyclical component of employment, l_t^c. The labor market tightness is constructed as $\theta_t = \frac{v_t}{u_t}$, where v_t is the quarterly average of the monthly vacancy index constructed by Barnichon (2010), and u_t is the quarterly average of the seasonally adjusted monthly unemployment level constructed by the BLS from the CPS. Employment, l_t, is the seasonally adjusted quarterly index for employment in the nonfarm business sector constructed by the BLS MSPC program. We construct θ_t^c and l_t^c by removing from $\ln(\theta_t)$ and $\ln(l_t)$ the trends produced by a HP filter with smoothing parameter 1600. Panel B displays the cross-correlogram between θ_t^c and l_t^c. The cross-correlation at lag i is the correlation between θ_{t-i}^c and l_t^c. The horizontal dashed lines are the 2-standard-deviation confidence bounds.

V.C. *Evidence of Labor Demand Shocks*

We evaluate whether labor market fluctuations are caused by labor demand, labor supply, or mismatch shocks. Labor demand shocks encompass aggregate demand and technology shocks.

Our model with price and real-wage rigidity predicts that the effects of labor demand shocks are different from those of labor supply and mismatch shocks. Labor demand shocks produce a positive correlation between labor market tightness and employment. In contrast, labor supply and mismatch shocks produce a negative correlation between labor market tightness and employment.

To assess the prevalence of labor demand, labor supply, and mismatch shocks, we therefore measure the correlation between the cyclical components of labor market tightness and employment. This correlation is displayed in Figure IX. In Panel A the cyclical components of labor market tightness and employment appear strongly positively correlated. Panel B formalizes this observation by reporting the cross-correlogram of labor market tightness and employment: labor market tightness leads employment by one lag; at one lag, the correlation is large, 0.95; the contemporaneous correlation is broadly the same, 0.93; and all the correlations are statistically different from 0.

In the context of our model, these positive correlations imply that it is labor demand shocks and not labor supply shocks or mismatch shocks that generate labor market fluctuations. Relatedly, Blanchard and Diamond (1989b) observe that the vacancy and unemployment rates are negatively correlated in U.S. data, and they conclude that labor market fluctuations must be caused by aggregate activity shocks and not by labor force participation shocks or reallocation shocks.

V.D. *Evidence of Aggregate Demand Shocks*

Having found that labor demand shocks are the prevalent source of labor market fluctuations, we determine whether these labor demand shocks are aggregate demand shocks or technology shocks.

Our model with price and real-wage rigidity predicts that the effects of aggregate demand shocks are different from those of technology shocks. Aggregate demand shocks produce a positive correlation between product market tightness and output. In

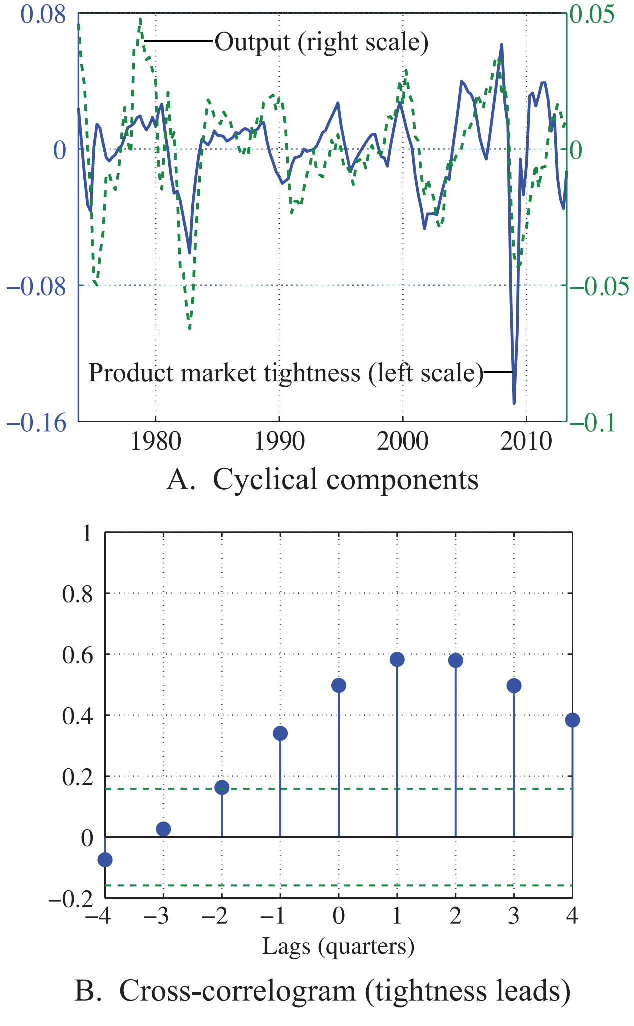


FIGURE X

Correlation between Product Market Tightness and Output in the United States, 1973:Q4–2013:Q2

Panel A displays the proxy for the cyclical component of the product market tightness, x_t^c, and the cyclical component of output, y_t^c. The proxy x_t^c is computed using equation (17). Output, y_t, is the seasonally adjusted quarterly index for real output in the nonfarm business sector constructed by the BLS MSPC program. We construct y_t^c by removing from $\ln(y_t)$ the trend produced by a HP filter with smoothing parameter 1600. Panel B displays the cross-correlogram between x_t^c and y_t^c. The cross-correlation at lag i is the correlation between x_{t-i}^c and y_t^c. The horizontal dashed lines are the 2-standard-deviation confidence bounds.

contrast, technology shocks produce a negative correlation between product market tightness and output.

To determine the nature of labor demand shocks, we therefore measure the correlation between the cyclical components of product market tightness and output. This correlation is displayed in Figure X. In Panel A the cyclical components of product market tightness and output appear positively correlated. Particularly, large drops in product market tightness followed the output drops of 1981–1982, 2001, and 2008–2009, suggesting that these recessions were caused by a negative aggregate demand shock. There are some exceptions, however. From 2004 to 2006, output was increasing while product market tightness was falling. This observation suggests a positive technology shock in the 2004–2006 period. Panel B reports the cross-correlogram of product market tightness and output: product market tightness leads output by one lag; at one lag, the correlation is quite large, 0.59; the contemporaneous correlation is 0.49; and all the correlations are statistically different from 0.

In the context of our model, these positive correlations imply that it is aggregate demand shocks and not technology shocks that are the main source of labor market fluctuations. Our conclusion coincides with the conclusions of Galí (1999) and Basu, Fernald, and Kimball (2006) that technology shocks are not the main source of business cycle fluctuations, despite the fact that the three analyses follow different approaches based on entirely different models.

VI. Conclusion

We use a simple model and direct empirical evidence to explore the sources of the unemployment fluctuations observed in the United States. The model makes predictions about the comovements of product market tightness, labor market tightness, output, and employment for a broad set of shocks that could potentially explain the fluctuations. We compare these predictions with the comovements observed in U.S. data. The comparison suggests that aggregate demand shocks are the main source of unemployment fluctuations, whereas technology, labor supply, and mismatch shocks are not an important source of fluctuations. Our analysis also confirms the prevalence of price and real-wage rigidities in the data; the rigidities allow aggregate demand shocks to propagate to the labor market.

In our model, aggregate demand arises from a choice between consumption and holding money. Usually, we think that aggregate demand arises from a choice between consumption, holding money, and saving with interest-bearing assets. In Michaillat and Saez (2014), we extend the model in that direction and show that the properties of the aggregate demand and equilibrium are robust. We also use the extended model to study the roles and limitations of conventional and unconventional fiscal and monetary policies in stabilizing unemployment fluctuations.

London School of Economics
University of California, Berkeley

Supplementary Material

An Online Appendix for this article can be found at QJE online (qje.oxfordjournal.org).

References

Alvarez, Luis J., and Ignacio Hernando, "Price Setting Behaviour of Spanish Firms: Evidence from Survey Data," European Central Bank Working Paper 538, 2005.

Apel, Mikael, Richard Friberg, and Kerstin Hallsten, "Microfoundations of Macroeconomic Price Adjustment: Survey Evidence from Swedish Firms," *Journal of Money, Credit and Banking*, 37 (2005), 313–338.

Arseneau, David M., and Sanjay K. Chugh, "Bargaining, Fairness, and Price Rigidity in a DSGE Environment," Federal Reserve Board International Finance Discussion Paper 900, 2007.

Aucremanne, Luc, and Martine Druant, "Price-Setting Behaviour in Belgium: What Can Be Learned from an Ad Hoc Survey?," European Central Bank Working Paper 448, 2005.

Bai, Yan, Jose-Victor Rios-Rull, and Kjetil Storesletten, "Demand Shocks as Productivity Shocks," (2012), available at http://www.econ.umn.edu/~vr0j/papers/brs_Sept_27_2012.pdf.

Barnichon, Regis, "Building a Composite Help-Wanted Index," *Economics Letters*, 109 (2010), 175–178.

Barro, Robert J., and Herschel I. Grossman, "A General Disequilibrium Model of Income and Employment," *American Economic Review*, 61 (1971), 82–93.

Barro, Robert J. *Money, Employment and Inflation* (Cambridge: Cambridge University Press, 1976).

Basu, Susanto, John G. Fernald, and Miles S. Kimball, "Are Technology Improvements Contractionary?," *American Economic Review*, 96 (2006), 1418–1448.

Bénassy, Jean-Pascal, "Nonclearing Markets: Microeconomic Concepts and Macroeconomic Applications," *Journal of Economic Literature*, 31 (1993), 732–761.

Bils, Mark, "Studying Price Markups from Stockout Behavior," 2004, available at http://www.econ.rochester.edu/people/BilsPapers/st1.pdf.

Blanchard, Olivier J., and Peter A. Diamond, "The Aggregate Matching Function," in *Growth, Productivity and Unemployment*, Peter A. Diamond, ed. (Cambridge, MA: MIT Press, 1989a).

———, "The Beveridge Curve," *Brookings Papers on Economic Activity*, 20 (1989b), 1–76.
Blanchard, Olivier J., and Jordi Galí, "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, 2 (2010), 1–30.
Blanchard, Olivier J., and Nobuhiro Kiyotaki, "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, 77 (1987), 647–666.
Blinder, Alan S., Elie R. D. Canetti, David E. Lebow, and Jeremy B. Rudd, *Asking about Prices* (New York, NY: Russell Sage Foundation, 1998).
Caballero, Ricardo, and Emmanuel Farhi, "The Safety Trap," NBER Working Paper 19927, 2014.
Calvo, Guillermo A., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12 (1983), 383–398.
den Haan, Wouter J., "Inventories and the Role of Goods-Market Frictions for Business Cycles," (2013), available at http://www.wouterdenhaan.com/papers/inventories.pdf.
den Haan, Wouter J., Garey Ramey, and Joel Watson, "Job Destruction and Propagation of Shocks," *American Economic Review*, 90 (2000), 482–498.
Diamond, Peter A., "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90 (1982a), 881–894.
———, "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 49 (1982b), 217–227.
———, "Cyclical Unemployment, Structural Unemployment," *IMF Economic Review*, 61 (2013), 410–455.
Drazen, Allan, "Recent Developments in Macroeconomic Disequilibrium Theory," *Econometrica*, 48 (1980), 283–306.
Edgeworth, Francis Y. *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (London: Kegan Paul, 1881).
Elsby, Michael W. L., Bart Hobijn, and Aysegul Sahin, "The Labor Market in the Great Recession," *Brookings Papers on Economic Activity*, 41 (2010), 1–69.
Eyster, Erik, Kristof Madarasz, and Pascal Michaillat, "Preferences for Fair Prices, Cursed Inferences, and the Nonneutrality of Money," (2015), available at http://works.bepress.com/pascal/13.
Fabiani, Silvia, Angela Gattulli, and Roberto Sabbatini, "The Pricing Behavior of Italian Firms: New Survey Evidence on Price Stickiness," European Central Bank Working Paper 333, 2004.
Farber, Henry S., and Robert Valletta, "Do Extended Unemployment Benefits Lengthen Unemployment Spells? Evidence from Recent Cycles in the U.S. Labor Market," NBER Working Paper 19048, 2013.
Farmer, Roger E. A., "Aggregate Demand and Supply," *International Journal of Economic Theory*, 4 (2008), 77–93.
———, "The Stock Market Crash of 2008 Caused the Great Recession: Theory and Evidence," *Journal of Economic Dynamics and Control*, 36 (2012), 693–707.
Foster, Lucia, John C. Haltiwanger, and Chad Syverson, "The Slow Growth of New Plants: Learning about Demand?," NBER Working Paper 17853, 2012.
Galí, Jordi, "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?," *American Economic Review*, 89 (1999), 249–271.
——— "Monetary Policy and Unemployment," in *Handbook of Monetary Economics*, vol. 3A, Benjamin M. Friedman and Michael Woodford, eds. (Amsterdam: Elsevier, 2010).
Gertler, Mark, Luca Sala, and Antonella Trigari, "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 40 (2008), 1713–1764.
Gertler, Mark, and Antonella Trigari, "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117 (2009), 38–86.

Goldberg, Pinelopi K., and Rebecca Hellerstein, “How Rigid Are Producer Prices?,” 2011, available at http://www.econ.yale.edu/~pg87/Producer Prices.pdf.

Gourio, Francois, and Leena Rudanko, “Customer Capital,” *Review of Economic Studies*, 81 (2014), 1102–1136.

Grandmont, Jean-Michel, “The Logic of the Fix-Price Method,” *Scandinavian Journal of Economics*, 79 (1977), 169–186.

Hagedorn, Marcus, and Iourii Manovskii, “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98 (2008), 1692–1706.

Hall, Robert E., “Employment Fluctuations with Equilibrium Wage Stickiness,” *American Economic Review*, 95 (2005), 50–65.

———, “General Equilibrium with Customer Relationships: A Dynamic Analysis of Rent-Seeking,” (2008), available at http://www.stanford.edu/~rehall/GECR_no_derivs.pdf.

Hall, Simon, Mark Walsh, and Anthony Yates, “Are UK Companies' Prices Sticky?,” *Oxford Economic Papers*, 52 (2000), 425–446.

Hosios, Arthur J., “On the Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies*, 57 (1990), 279–298.

Howitt, Peter, and R. Preston McAfee, “Costly Search and Recruiting,” *International Economic Review*, 28 (1987), 89–107.

Huo, Zhen, and José-Víctor Ríos-Rull, “Paradox of Thrift Recessions,” NBER Working Paper No. 19443, 2013.

Keynes, John Maynard, *The General Theory of Employment, Interest and Money* (New York: Macmillan, 1936).

Korinek, Anton, and Alp Simsek, “Liquidity Trap and Excessive Leverage,” NBER Working Paper No. 19970, 2014.

Kwapil, Claudia, Joseph Baumgartner, and Johann Scharler, “The Price-Setting Behaviour of Austrian Firms: Some Survey Evidence,” European Central Bank Working Paper 464, 2005.

Lazear, Edward P., and James R. Spletzer, “The United States Labor Market: Status Quo or A New Normal?,” *Proceedings of the Economic Policy Symposium of Jackson Hole* Federal Reserve Bank of Kansas City, 2012.

Lehmann, Etienne, and Bruno Van der Linden, “Search Frictions on Product and Labor Markets: Money in the Matching Function,” *Macroeconomic Dynamics*, 14 (2010), 56–92.

Loupias, Claire, and Roland Ricart, “Price Setting in France: New Evidence from Survey Data,” European Central Bank Working Paper 448, 2004.

Lunnemann, Patrick, and Thomas Y. Matha, “New Survey Evidence on the Pricing Behavior of Luxembourg Firms,” European Central Bank Working Paper 617, 2006.

Malinvaud, Edmond, *The Theory of Unemployment Reconsidered* (Oxford: Basil Blackwell, 1977).

Mankiw, N. Gregory, and Matthew Weinzierl, “An Exploration of Optimal Stabilization Policy,” *Brookings Papers on Economic Activity*, 42 (2011), 209–272.

Martins, Fernando, “The Price Setting Behaviour of Portuguese Firms: Evidence From Survey Data,” European Central Bank Working Paper 562, 2005.

Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green *Microeconomic Theory* (Oxford: Oxford University Press, 1995).

Mathä, Thomas Y., and Olivier Pierrard, “Search in the Product Market and the Real Business Cycle,” *Journal of Economic Dynamics and Control*, 35 (2011), 1172–1191.

Mian, Atif, and Amir Sufi, “What Explains the 2007–2009 Drop in Employment?,” *Econometrica*, 82 (2014), 2197–2223.

Michaillat, Pascal, “Do Matching Frictions Explain Unemployment? Not in Bad Times,” *American Economic Review*, 102 (2012), 1721–1750.

———, “A Theory of Countercyclical Government Multiplier,” *American Economic Journal: Macroeconomics*, 6 (2014), 190–217.

Michaillat, Pascal, and Emmanuel Saez, “An Economical Business-Cycle Model,” NBER Working Paper No. 19777, 2014.

Moen, Espen R., "Competitive Search Equilibrium," *Journal of Political Economy*, 105 (1997), 385–411.
Morin, Norman, and John Stevens, "Estimating Capacity Utilization from Survey Data," Federal Reserve Board Finance and Economics Discussion Paper 2004-49, 2004.
Mortensen, Dale T., "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 72 (1982), 968–979.
Ottonello, Pablo, "Capital Unemployment, Financial Shocks, and Investment Slumps," (2014), available at http://www.columbia.edu/~po2171/Ottonello_JMP_13Oct2014.pdf.
Petrongolo, Barbara, and Christopher A. Pissarides, "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39 (2001), 390–431.
Petrosky-Nadeau, Nicolas, and Etienne Wasmer, "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions," IZA Discussion Paper 5763, 2011.
Pissarides, Christopher A., "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, 75 (1985), 676–690.
———, "Unemployment and Vacancies in Britain," *Economic Policy*, 1 (1986), 500–559.
———, *Equilibrium Unemployment Theory*. 2nd ed. (Cambridge, MA: MIT Press, 2000).
———, "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," *Econometrica*, 77 (2009), 1339–1369.
Rendahl, Pontus, "Fiscal Policy in an Unemployment Crisis," (2012), available at http://sites.google.com/site/pontusrendahl/stimulus.pdf.
Rothstein, Jesse, "Unemployment Insurance and Job Search in the Great Recession," *Brookings Papers on Economic Activity*, 43 (2011), 143–213.
Sahin, Aysegul, Joseph Song, Giorgio Topa, and Giovanni L. Violante, "Mismatch Unemployment," *American Economic Review*, 104 (2014), 3529–3564.
Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95 (2005), 25–49.
———, "Mismatch," *American Economic Review*, 97 (2007), 1074–1101.
———, *Labor Markets and Business Cycles* (Princeton, NJ: Princeton University Press, 2010).
Stahl, Harald, "Price Setting in German Manufacturing: New Evidence from New Survey Data," European Central Bank Working Paper 561, 2005.
Stole, Lars A., and Jeffrey Zwiebel, "Intra-Firm Bargaining under Non-Binding Contracts," *Review of Economic Studies*, 63 (1996), 375–410.
Walsh, Carl E., "Labor Market Search and Monetary Shocks," in *Elements of Dynamic Macroeconomic Analysis*, Sumru Altug, Jagjit S. Chadha, and Charles Nolan, eds. (Cambridge: Cambridge University Press, 2003).

AGGREGATE DEMAND, IDLE TIME, AND UNEMPLOYMENT: ONLINE APPENDICES

Pascal Michaillat and Emmanuel Saez

January 2015

ONLINE APPENDIX A: LONG PROOFS

Proof of Proposition 9. In a fixprice equilibrium parameterized by $p_0 > 0$ and $w_0 > 0$, (x, θ) satisfies $n^s(\theta) = n^d(\theta, x, w_0)$ and $c^s(x, \theta) = c^d(x, p_0)$. We look for equilibria with positive consumption. These equilibria necessarily have $\theta \in (0, \theta^m)$ and $x \in (0, x^m)$.

Equation $n^d(\theta, x, w_0) = n^s(\theta)$ is equivalent to $\left[n^d(\theta, x, w_0)\right]^{1-\alpha} = [n^s(\theta)]^{1-\alpha}$. Following the logic of the proof of Proposition 3, we can show that the latter equation is equivalent to

$$F(\theta, x, a, h, w) \equiv f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha} \cdot h^{1-\alpha} \cdot \frac{w}{\alpha \cdot a} = 0. \tag{A1}$$

Since $\alpha < 1$, the function $\theta \mapsto \hat{f}(\theta)^{1-\alpha} \cdot (1 + \hat{\tau}(\theta))^{\alpha}$ is strictly increasing from 0 to $+\infty$ on $[0, \theta^m)$. Hence, (A1) implicitly defines θ as a function Θ^F of $x \in [0, +\infty)$. Since f is strictly increasing from 0 to 1 on $[0, +\infty)$, Θ^F is strictly increasing on $[0, +\infty)$, $\Theta^F(0) = 0$, and $\lim_{x \to +\infty} \Theta^F(x) = \theta^F$ where $\theta^F \in (0, \theta^m)$ is defined by $\hat{f}(\theta^F)^{1-\alpha} \cdot \left(1 + \hat{\tau}(\theta^F)\right)^{\alpha} \cdot h^{1-\alpha} \cdot w/(\alpha \cdot a) = 1$.

The proof of Proposition 3 shows that the equation $c^s(x, \theta) = c^d(x, p_0)$, combined with $k = a \cdot n^{\alpha}$ is equivalent to

$$f(x) \cdot a \cdot \left(\frac{\hat{f}(\theta)}{1 + \hat{\tau}(\theta)} \cdot h\right)^{\alpha} \cdot (1 + \tau(x))^{\varepsilon - 1} = \chi^{\varepsilon} \cdot \frac{\mu}{p}.$$

Using (A1), we transform this equation to

$$G(\theta, x, h, \chi, \mu, w, p) \equiv \hat{f}(\theta) \cdot (1 + \tau(x))^{\varepsilon - 1} - \frac{\alpha}{w \cdot h} \cdot \chi^{\varepsilon} \cdot \frac{\mu}{p} = 0. \tag{A2}$$

If $\alpha \cdot \chi^{\varepsilon} \cdot (\mu/p)/(w \cdot h) \geq 1$, we define $x^G(p,w)$ by $\left(1+\tau(x^G)\right)^{\varepsilon-1} = \alpha \cdot \chi^{\varepsilon} \cdot (\mu/p)/(w \cdot h)$. If $\alpha \cdot \chi^{\varepsilon} \cdot (\mu/p)/(w \cdot h) < 1$, we set $x^G(p,w) = 0$. Since $\varepsilon > 1$, the function $x \mapsto (1+\tau(x))^{\varepsilon-1}$ is strictly increasing from 1 to $+\infty$ on $[0,x^m)$; therefore, x^G is well defined and $x^G(p,w) \in (0,x^m)$. Since $\hat{f}$ is strictly increasing from 0 to 1 on $(0,+\infty)$, (A2) implicitly defines θ as a function Θ^G of $x \in (x^G(p,w),x^m)$. Moreover, Θ^G is strictly decreasing on $(x^G(p,w),x^m)$, $\lim_{x \to x^G(p,w)} \Theta^G(x) = +\infty$, and $\lim_{x \to x^m} \Theta^G(x) = 0$.

The system of (A1) and (A2) is equivalent to the system of $\Theta^F(x) = \Theta^G(x)$ and $\theta = \Theta^F(x)$. The properties of Θ^F and Θ^G imply that this system admits a unique solution (x,θ) with $x \in (x^G(p,w),x^m)$ and $\theta \in (0,\theta^F)$.

Proof of Proposition 10.

Aggregate Demand Shocks. We parameterize an increase in aggregate demand by an increase in χ or μ. The function F in (A1) satisfies $\partial F/\partial \theta < 0$, $\partial F/\partial x > 0$, $\partial F/\partial a > 0$, and $\partial F/\partial h < 0$. Using the implicit function theorem, we write the solution θ to $F(\theta,x,a,h,w) = 0$ as a function $\Theta^F(x,a,h)$ with $\partial \Theta^F/\partial x > 0$, $\partial \Theta^F/\partial a > 0$, and $\partial \Theta^F/\partial h < 0$.

The function G in (A2) satisfies $\partial G/\partial \theta > 0$, $\partial G/\partial x > 0$, $\partial G/\partial h > 0$, $\partial G/\partial \chi < 0$, and $\partial G/\partial \mu < 0$. Using the implicit function theorem, we write the solution θ to $G(\theta,x,h,\chi,\mu,w,p) = 0$ as a function $\Theta^G(x,h,\chi,\mu)$ with $\partial \Theta^G/\partial x < 0$, $\partial \Theta^G/\partial h < 0$, $\partial \Theta^G/\partial \chi > 0$, and $\partial \Theta^G/\partial \mu > 0$.

In equilibrium, x satisfies $G(\Theta^F(x,a,h),x,h,\chi,\mu) = 0$. Given that $\partial \Theta^F/\partial x > 0$, $\partial G/\partial \theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial \chi < 0$, the implicit function theorem implies that $\partial x/\partial \chi > 0$. We can show similarly that $\partial x/\partial \mu > 0$. Since $\theta = \Theta^F(x,a,h)$ with $\partial \Theta^F/\partial x > 0$, we also have $\partial \theta/\partial \chi > 0$ and $\partial \theta/\partial \mu > 0$. Equation (8) yields $y = \hat{f}(\theta) \cdot h \cdot w/\alpha$; therefore, the comparative statics for θ imply that $\partial y/\partial \chi > 0$ and $\partial y/\partial \mu > 0$. Since $l = \hat{f}(\theta) \cdot h$, the comparative statics for θ also imply that $\partial l/\partial \chi > 0$ and $\partial l/\partial \mu > 0$.

Technology Shocks. We parameterize an increase in technology by an increase in a. In equilibrium, x satisfies $F(\Theta^G(x,h,\chi,\mu),x,a,h) = 0$. Given that $\partial \Theta^G/\partial x < 0$, $\partial F/\partial \theta < 0$, $\partial F/\partial x > 0$, and $\partial F/\partial a > 0$, the implicit function theorem implies that $\partial x/\partial a < 0$. Since $\theta = \Theta^G(x,h,\chi,\mu)$ with $\partial \Theta^G/\partial x < 0$, we obtain $\partial \theta/\partial a > 0$. The logic presented for aggregate demand shocks implies that since $\partial \theta/\partial a > 0$, then $\partial y/\partial a > 0$ and $\partial l/\partial a > 0$.

Labor Supply Shocks. We parameterize an increase in labor supply by an increase in h. The functions F and G both depend on h, so it is impossible to obtain comparative statics for x and θ from them. To obtain the comparative statics, we manipulate and combine (A1) and (A2), and we obtain

$$H(\theta,x) \equiv (1+\hat{\tau}(\theta))^{\alpha} - f(x)\cdot(1+\tau(x))^{(1-\alpha)\cdot(\varepsilon-1)}\cdot a\cdot\left(\frac{\alpha}{w}\right)^{\alpha}\cdot\left(\chi^{\varepsilon}\cdot\frac{\mu}{p}\right)^{\alpha-1} = 0. \tag{A3}$$

The function H satisfies $\partial H/\partial\theta > 0$ and $\partial H/\partial x < 0$. The function H does not depend on h, which resolves the earlier problem. Using the implicit function theorem, we write the solution θ to $H(\theta,x) = 0$ as a function $\Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$.

In equilibrium, x satisfies $G(\Theta^H(x),x,h) = 0$. Given that $\partial\Theta^H/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial h > 0$, the implicit function theorem implies that $\partial x/\partial h < 0$. Since $\theta = \Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$, we obtain $\partial\theta/\partial h < 0$. We find that $\partial y/\partial h > 0$ because $y = (1+\tau(x))\cdot c^d(x,p) = (1+\tau(x))^{1-\varepsilon}\cdot\chi^{\varepsilon}\cdot\mu/p$ and $1-\varepsilon < 0$ and $\partial x/\partial h < 0$. We also find that $\partial l/\partial h > 0$ because $l = \alpha\cdot y/w$ (from equation (8)) and $\partial y/\partial h > 0$.

Mismatch Shocks. We parameterize an increase in mismatch by a decrease in matching efficacy on the labor market along with a corresponding decrease in recruiting cost: $\hat{f}(\theta)$, $\hat{q}(\theta)$, and ρ become $\lambda\cdot\hat{f}(\theta)$, $\lambda\cdot\hat{q}(\theta)$, and $\lambda\cdot\hat{\rho}$ with $\lambda < 1$. Consequently, the function $\hat{\tau}$ remains the same. With the parameter λ for mismatch, the functions H and Θ^H are the same, but the function G depends on λ with $\partial G/\partial\lambda > 0$. In equilibrium, x satisfies $G(\Theta^H(x),x,\lambda) = 0$. Given that $\partial\Theta^H/\partial x > 0$, $\partial G/\partial\theta > 0$, $\partial G/\partial x > 0$, and $\partial G/\partial\lambda > 0$, the implicit function theorem implies that $\partial x/\partial\lambda < 0$. Since $\theta = \Theta^H(x)$ with $\partial\Theta^H/\partial x > 0$, we have $\partial\theta/\partial\lambda < 0$. The logic presented for labor supply shocks implies that since $\partial x/\partial\lambda < 0$, then $\partial y/\partial\lambda > 0$ and $\partial l/\partial\lambda > 0$.

Comparative Statics with Partially Rigid Price and Real Wage. Using the expressions of the partially rigid price and real wage, we rewrite (A1), (A2), and (A3) as

$$f(x) - \hat{f}(\theta)^{1-\alpha} \cdot (1+\hat{\tau}(\theta))^{\alpha} \cdot \frac{w_0 \cdot h^{(1-\alpha)\cdot(1-\xi)}}{(\alpha \cdot a)^{1-\xi}} = 0$$

$$\hat{f}(\theta) \cdot (1+\tau(x))^{\varepsilon-1} - \frac{\alpha^{1-\xi}}{w_0 \cdot h^{1-\xi}} \cdot \frac{(\chi^{\varepsilon} \cdot \mu)^{1-\xi}}{p_0} = 0$$

$$(1+\hat{\tau}(\theta))^{\alpha} - f(x) \cdot (1+\tau(x))^{(1-\alpha)\cdot(\varepsilon-1)} \cdot a^{1-\xi} \cdot \frac{\alpha^{\alpha(1-\xi)}}{w_0^{\alpha}} \cdot \left[\frac{(\chi^{\varepsilon} \cdot \mu)^{1-\xi}}{p_0}\right]^{\alpha-1} = 0.$$

The implicit functions defined by these equations have exactly the same properties as the functions F, G, and H. Hence, the comparative statics for x and θ are the same in the fixprice equilibrium and in the equilibrium with partially rigid prices. Finally, the arguments used above for the fixprice equilibrium imply that the comparative statics for y and l are the same in the fixprice equilibrium and in the equilibrium with partially rigid prices.

Proof of Proposition 11. In a competitive equilibrium, the pair (p^*, w^*) satisfies $n^s(\theta^*) = n^d(\theta^*, x^*, w^*)$ and $c^s(x^*, \theta^*) = c^d(x^*, p^*)$. Following the steps of the proof of Proposition 9, we show that (p^*, w^*) satisfies

$$w^* = f(x^*) \cdot \hat{f}(\theta^*)^{\alpha-1} \cdot (1+\hat{\tau}(\theta^*))^{-\alpha} \cdot h^{\alpha-1} \cdot \alpha \cdot a$$

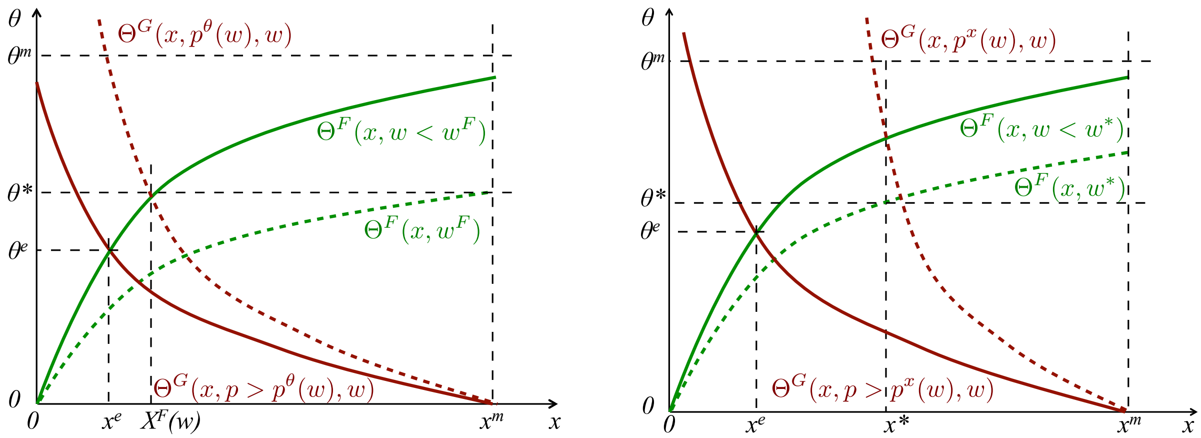
$$p^* = \frac{(1+\tau(x^*))^{1-\varepsilon}}{\hat{f}(\theta^*)} \cdot \frac{\alpha}{w^* \cdot h} \cdot \chi^{\varepsilon} \cdot \mu.$$

This system admits a unique solution. Thus, there exists a unique competitive equilibrium. Clearly, the wage w^* satisfies the expression in the proposition. Combining these two equations, we find that the price p^* also satisfies the expression in the proposition.

Online Appendix B: The Four Inefficient Regimes

This appendix establishes the boundaries in a (w, p) plane of the four inefficient regimes of the model of Section III. These boundaries are depicted in Figure VII.

Proposition A1. *There exists a function $w \mapsto p^x(w)$ such that for any $w > 0$, the product market is slack if $p > p^x(w)$ and tight if $p < p^x(w)$. There exists a function $w \mapsto p^{\theta}(w)$ such that for*



A. Condition such that $\theta < \theta^*$

B. Condition such that $x < x^*$

Figure A1: Illustration of the Proof of Proposition A1

any $w > 0$, *the labor market is slack if* $p > p^\theta(w)$ *and tight if* $p < p^\theta(w)$. *The function* p^x *is strictly decreasing for* $w \in (0, w^*]$, *strictly increasing for* $w \in [w^*, +\infty)$, $\lim_{w\to 0} p^x(w) = +\infty$, *and* $\lim_{w\to+\infty} p^x(w) = +\infty$. *The function* p^θ *is strictly decreasing for* $w \in (0, w^F]$, $\lim_{w\to 0} p^\theta(w) = +\infty$, *and* $p^\theta(w) = 0$ *for* $w \in [w^F, +\infty)$, *where* $w^F \in (w^*, +\infty)$. *Furthermore,* $p^x(w^*) = p^\theta(w^*) = p^*$.

Proof. We build on the proof of Proposition 9. We define the function $\Theta^F : [0, +\infty) \times (0, +\infty) \to (0, \theta^m)$ such that $F(\Theta^F(x, w), x, a, h, w) = 0$. The function Θ^F is strictly increasing in x and strictly decreasing in w. We define the function Θ^G: $\left\{(x, w, p) | p > 0,\ w > 0,\ x^G(p, w) < x < x^m\right\} \to (0, +\infty)$ such that $G(\Theta^G(x, w, p), x, h, \chi, \mu, w, p) = 0$. The function Θ^G is strictly decreasing in x, p, and w. The proof is illustrated in Figure A1.

First Part: Condition such that $\theta < \theta^*$. Let w^F be defined by $\Theta^F(x^m, w^F) = \theta^*$. For all $w > w^F$ and for all $x \in [0, x^m]$, $\Theta^F(x, w) < \theta^*$. For all $w \le w^F$, there exists a unique $x \in [0, x^m]$ such that $\Theta^F(x, w) = \theta^*$. We define the function $X^F : (0, w^F] \to [0, x^m]$ by $\Theta^F(X^F(w), w) = \theta^*$. The function X^F is strictly increasing, $\lim_{w\to 0} X^F(w) = 0$, $X^F(w^*) = x^*$, and $X^F(w^F) = x^m$.

Next, for all $w \le w^F$, there exists a unique $p \in (0, +\infty)$ such that $\Theta^G(X^F(w), p, w) = \theta^*$. We define the function $p^\theta : (0, w^F] \to (0, +\infty)$ by $\Theta^G(X^F(w), p^\theta(w), w) = \theta^*$. The function p^θ is strictly decreasing, $\lim_{w\to 0} p^\theta(w) = +\infty$, $p^\theta(w^*) = p^*$, and $p^\theta(w^F) = 0$. We extend the definition of p^θ by setting $p^\theta(w) = 0$ for all $w \in (w^F, +\infty)$.

Last, we denote equilibrium labor market tightness by θ^e and equilibrium product market tightness by x^e. For any $w > w^F$, $\theta^e = \Theta^F(x, w) < \theta^*$ by definition of w^F. Consider $w \le w^F$ and

$p > p^{\theta}(w)$. Then $\Theta^G(X^F(w), p, w) < \Theta^G(X^F(w), p^{\theta}(w), w) = \theta^* = \Theta^F(X^F(w), w)$ because Θ^G is strictly decreasing in p. Given that Θ^F is strictly increasing in x and Θ^G is strictly decreasing in x and $\Theta^G(x^e, p, w) = \Theta^F(x^e, w)$, we conclude that $x^e < X^F(w)$. Thus, $\theta^e = \Theta^F(x^e, w) < \Theta^F(X^F(w), w) = \theta^*$ because Θ^F is strictly increasing in x. In sum, for any $w > 0$ and $p > p^{\theta}(w)$, we have $\theta^e < \theta^*$. Following the same logic, we find that for any $w > 0$ and $p < p^{\theta}(w)$, we have $\theta^e > \theta^*$.

Second Part: Condition such that $x < x^*$. We define the function $p^x : (0, +\infty) \to (0, +\infty)$ by

$$p^x(w) = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{h \cdot \hat{f}(\Theta^F(x^*, w))} \cdot \chi^{\varepsilon} \cdot \frac{\alpha}{w} \cdot \mu.$$

The function p^x has the property that $\Theta^G(x^*, p^x(w), w) = \Theta^F(x^*, w)$. Hence, $p^x(w^*) = p^*$.

Next, we define the auxiliary function $Z : (0, +\infty) \to (0, +\infty)$ by

$$Z(w) = f(x^*) \cdot a \cdot \alpha \cdot n^s(\Theta^F(x^*, w)).$$

Given that $\Theta^F(x^*, w^*) = \theta^*$ and Θ^F is strictly decreasing in w, $\Theta^F(x^*, w) \in [\theta^*, \theta^m)$ if $w \in (0, w^*]$ and $\Theta^F(x^*, w) \in (0, \theta^*]$ if $w \in [w^*, +\infty)$. Since n^s is strictly increasing on $[0, \theta^*]$ and strictly decreasing on $[\theta^*, \theta^m]$ and Θ^F is strictly decreasing in w, we infer that Z is strictly increasing for $w \in (0, w^*]$ and strictly decreasing for $w \in [w^*, +\infty)$. Since $n^s(0) = n^s(\theta^m) = 0$, $\lim_{w \to 0} \Theta^F(x^*, w) = \theta^m$, and $\lim_{w \to +\infty} \Theta^F(x^*, w) = 0$, we infer that $\lim_{w \to 0} Z(w) = 0$ and $\lim_{w \to +\infty} Z(w) = 0$.

The definition of Θ^F implies that $Z(w) = h \cdot w \cdot \hat{f}(\Theta^F(x^*, w))$. Thus,

$$p^x(w) = \frac{(1 + \tau(x^*))^{1-\varepsilon}}{Z(w)} \cdot \chi^{\varepsilon} \cdot \alpha \cdot \mu.$$

The properties of Z imply that the function p^x is strictly decreasing for $w \in (0, w^*]$ and strictly increasing for $w \in [w^*, +\infty)$, $\lim_{w \to 0} p^x(w) = +\infty$, and $\lim_{w \to +\infty} p^x(w) = +\infty$.

Last, we denote equilibrium labor market tightness by θ^e and equilibrium product market tightness by x^e. Consider $w \in (0, +\infty)$ and $p > p^x(w)$. Then $\Theta^G(x^*, p, w) < \Theta^G(x^*, p^x(w), w) = \Theta^F(x^*, w)$ because Θ^G is strictly decreasing in p. Given that Θ^F is strictly increasing in x and Θ^G is strictly decreasing in x and $\Theta^G(x^e, p, w) = \Theta^F(x^e, w)$, we conclude that $x^e < x^*$. In sum, for any $w > 0$ and $p > p^x(w)$, we have $x^e < x^*$. Similarly, for any $w > 0$ and $p < p^x(w)$, we have

$x^e > x^*$. $\square$

In Figure VII, the function p^θ is represented by the downward-sloping line and the function p^x is represented by the u-shaped line. The two curves intersect at (w^*, p^*).

ONLINE APPENDIX C: OPTIMAL CONTROL PROBLEMS

This appendix solves the optimal control problems of the household and firm in the dynamic model of Section IV.

The Optimal Control Problem of the Household. Let $b(t) \equiv m(t)/p(t)$ denote real money balances. The law of motion of $b(t)$ is obtained from (13):

$$\dot{b}(t) = w(t) \cdot l(t) - y(t) - \pi \cdot b(t) + \frac{T(t)}{p(t)}$$

To solve the problem, we set up the current-value Hamiltonian

$$\begin{aligned}\mathscr{H}(t, c(t), y(t), b(t)) = & \frac{\chi}{1+\chi} \cdot c(t)^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \cdot b(t)^{\frac{\varepsilon-1}{\varepsilon}} + Y(t) \cdot \left[\frac{q(x(t))}{\rho} \cdot (y(t) - c(t)) - s \cdot y(t)\right] \\ & + B(t) \cdot \left[w(t) \cdot l(t) - y(t) - \pi \cdot b(t) + \frac{T(t)}{p(t)}\right]\end{aligned}$$

with control variable $c(t)$, state variables $y(t)$ and $b(t)$, and costate variables $Y(t)$ and $B(t)$.

The necessary conditions for an interior solution to this maximization problem are $\partial \mathscr{H} / \partial c(t) = 0$, $\partial \mathscr{H} / \partial y(t) = \delta \cdot Y(t) - \dot{y}(t)$, and $\partial \mathscr{H} / \partial b(t) = \delta \cdot B(t) - \dot{b}(t)$, together with the transversality conditions $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot B(t) \cdot b(t) = 0$. Given that $\mathscr{H}$ is concave in (c, y, b), these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned}\frac{\chi}{1+\chi} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot c(t)^{\frac{-1}{\varepsilon}} &= Y(t) \cdot \frac{q(x(t))}{\rho} \\ Y(t) \cdot \left(\frac{q(x(t))}{\rho} - s\right) - B(t) &= \delta \cdot Y(t) - \dot{y}(t) \\ \frac{1}{1+\chi} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot b(t)^{\frac{-1}{\varepsilon}} - B(t) \cdot \pi &= \delta \cdot B(t) - \dot{b}(t)\end{aligned}$$

In steady state, $\dot{y}(t) = \dot{b}(t) = 0$. Hence, after eliminating the costate variables B and Y, we find that the optimal consumption decision of the household is

$$c = \chi^{\varepsilon} \cdot (\delta + \pi)^{\varepsilon} \cdot \left[1 - (\delta + s) \cdot \frac{\rho}{q(x)}\right]^{\varepsilon} \cdot b.$$

We obtain the equation in the text by setting $\delta = 0$ and $b = \mu(0)/p(0)$ in the above equation.

The Optimal Control Problem of the Firm. To solve this problem, we set up the current-value Hamiltonian

$$\begin{aligned} \mathscr{H}(t, n(t), y(t), l(t)) = & y(t) - w(t) \cdot l(t) + Y(t) \cdot [f(x(t)) \cdot (a \cdot n(t)^{\alpha} - y(t)) - s \cdot y(t)] \\ & + L(t) \cdot \left[\frac{\hat{q}(\theta(t))}{\hat{\rho}} \cdot (l(t) - n(t)) - \hat{s} \cdot l(t)\right] \end{aligned}$$

with control variable $n(t)$, state variables $y(t)$ and $l(t)$, and current-value costate variables $Y(t)$ and $L(t)$. The necessary conditions for an interior solution to this maximization problem are $\partial\mathscr{H}/\partial n(t) = 0$, $\partial\mathscr{H}/\partial y(t) = \delta \cdot Y(t) - \dot{y}(t)$, and $\partial\mathscr{H}/\partial l(t) = \delta \cdot L(t) - \dot{l}(t)$, together with the transversality conditions $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot Y(t) \cdot y(t) = 0$ and $\lim_{t \to +\infty} e^{-\delta \cdot t} \cdot L(t) \cdot l(t) = 0$. Given that $\mathscr{H}$ is concave in (n, y, l), these conditions are also sufficient.

These three conditions can be rewritten as

$$\begin{aligned} Y(t) \cdot f(x(t)) \cdot \alpha \cdot a \cdot n(t)^{\alpha - 1} &= L(t) \cdot \frac{\hat{q}(\theta(t))}{\hat{\rho}} \\ 1 &= Y(t) \cdot (f(x(t)) + s) + \delta \cdot Y(t) - \dot{y}(t) \\ L(t) \cdot \left(\frac{\hat{q}(\theta(t))}{\hat{\rho}} - \hat{s}\right) &= w(t) + \delta \cdot L(t) - \dot{l}(t) \end{aligned}$$

In steady state, $\dot{l}(t) = \dot{y}(t) = 0$. Hence, after eliminating the costate variables L and Y, we find that the optimal employment decision of the firm satisfies

$$n = \left\{\frac{\alpha \cdot a}{w} \cdot \frac{f(x)}{\delta + s + f(x)} \cdot \left[1 - (\delta + \hat{s}) \cdot \frac{\hat{\rho}}{\hat{q}(\theta)}\right]\right\}^{\frac{1}{1-\alpha}}.$$

We obtain equation in the text by setting $\delta = 0$ in the above equation.

Online Appendix D: Another Proxy for Product Market Tightness

This appendix proposes another proxy for the cyclical component of the product market tightness, and it shows that all the empirical results are robust to using this alternative proxy. The proxy is constructed from the operating rate in non-manufacturing sectors measured by the Institute for Supply Management (ISM) and published in their Semiannual Reports. This operating rate is available for the 1999:Q4–2013:Q2 period. In the text, the proxy is constructed from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the Survey of Plant Capacity (SPC).

Using the operating rate from the ISM is conceptually better than using the capacity utilization rate from the SPC for two reasons. First, the operating rate is a direct measure of labor utilization; therefore, it is directly linked to product market tightness. Second, the operating rate applies to non-manufacturing sectors, where logistical issues such as peak load and inventory management do not influence labor utilization. We do not use this alternative proxy in the text, however, because it is only available for a brief period (1999:Q4–2013:Q2) that does not cover sufficiently many business cycles to permit a thorough empirical analysis.

We construct our alternative proxy for the cyclical component of the product market tightness as follows. The operating rate or_t measured by the ISM is the actual production level of firms as a share of their maximum production level given their current capital stock and workforce. Since the operating rate takes labor as a fixed factor, it exactly corresponds to our concept of labor utilization: $or_t = f(x_t)/(s+f(x(t)))$.[35] The ISM measures or_t in the second and fourth quarter; we use a linear interpolation of the biannual series to transform it into a quarterly series for the 1999:Q4–2013:Q2 period. Then, we remove from $\ln(or_t)$ the trend produced by a HP filter with smoothing parameter 1600. The resulting detrended series is our proxy for the cyclical component of the product market tightness. This proxy is plotted in Figure A2, together with the proxy used in the text. We refer to the proxy in the text as the *SPC proxy* and to this alternative proxy as the *ISM proxy*.

Over the 1999:Q4–2013:Q2 period, the correlation between the two proxies is 0.67. As showed in Figure A2, the two proxies behaved similarly over the period: both fell after 2001, picked up in

[35]This is a major difference with the capacity utilization rate from the SPC, which takes labor as a variable factor and thus requires a correction to be converted into a labor utilization rate. Morin and Stevens (2005) discuss the difference between the capacity utilization rate collected in the SPC and the operating rate collected in the ISM survey.

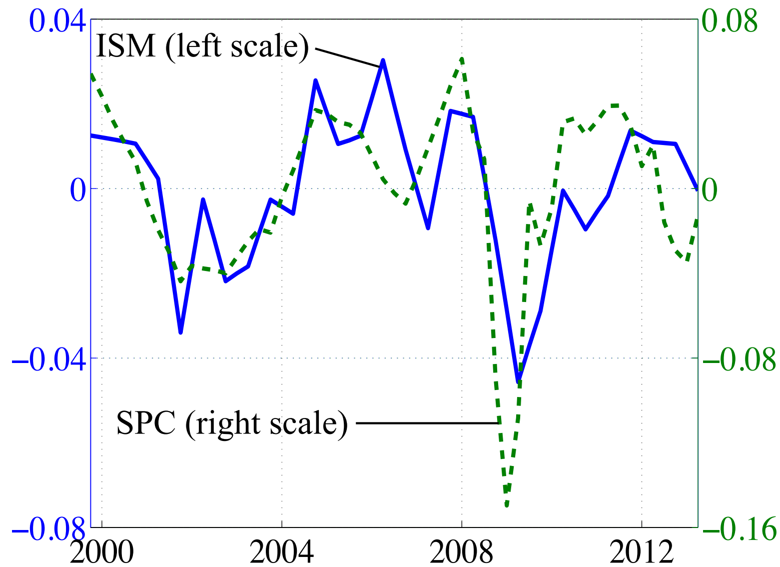


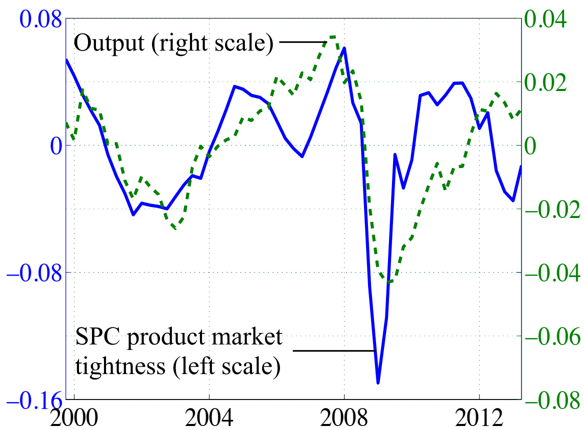
Figure A2: The Two Proxies for the Cyclical Component of Product Market Tightness

The time period is 1999:Q4–2013:Q2. The ISM proxy is constructed in Online Appendix D from the operating rate in non-manufacturing sectors constructed by the ISM. The SPC proxy is constructed in Section V from the capacity utilization rate in the manufacturing sector measured by the Census Bureau from the SPC.

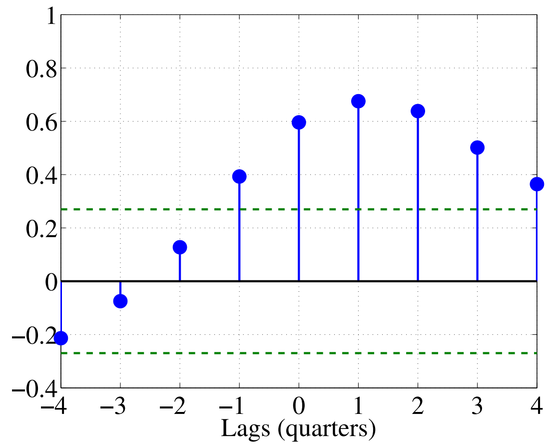
the 2004–2008 period, and collapsed in 2009, before recovering. The main difference is that the SPC proxy is subject to larger fluctuations than the ISM proxy.

In the text, we use the SPC proxy to identify aggregate demand and technology shocks. The finding reported in Figure X is that the SPC proxy and output are positively correlated, which implies that aggregate demand shocks are the main source of labor market fluctuations. Panels A and B of Figure A3 confirm that this result remains valid if we focus on the correlation between SPC proxy and output over the subperiod 1999:Q4–2013:Q2. The correlations between SPC proxy and output are slightly higher: at one lag, the correlation is 0.68; the contemporaneous correlation is 0.60. These correlations are statistically significant.

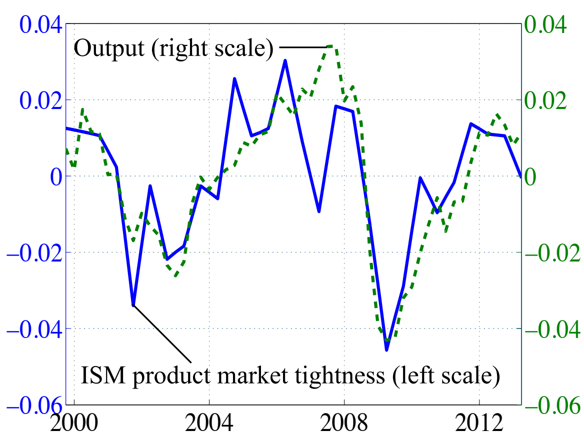
Panels C and D of Figure A3 show that we obtain the same result if we use the ISM proxy instead of the SPC proxy. The correlations of ISM proxy and output are even slightly higher than those of SPC proxy and output: the contemporaneous correlation is 0.82; at one lag, the correlation is 0.77. These correlations are statistically significant.



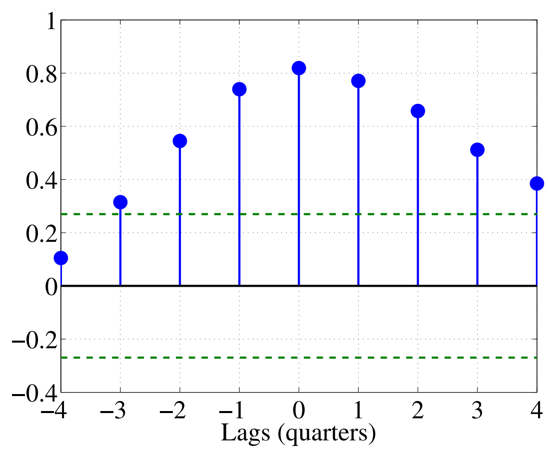
A. Cyclical components



B. Cross-correlogram (SPC tightness leads)



C. Cyclical components



D. Cross-correlogram (ISM tightness leads)

Figure A3: Correlation Between Product Market Tightness and Output

The time period is 1999:Q4–2013:Q2. Panel A displays the SPC proxy for the cyclical component of the product market tightness, $x_t^c(SPC)$, and the cyclical component of output, y_t^c. The construction of $x_t^c(SPC)$ is explained in Section V. Output, y_t, is the seasonally adjusted quarterly index for real output in the nonfarm business sector constructed by the BLS MSPC program. We construct y_t^c by removing from $\ln(y_t)$ the trend produced by a HP filter with smoothing parameter 1600. Panel B displays the cross-correlogram between $x_t^c(SPC)$ and y_t^c. The cross-correlation at lag i is the correlation between $x_{t-i}^c(SPC)$ and y_t^c. Panel C displays the ISM proxy for the cyclical component of the product market tightness, $x_t^c(ISM)$, and y_t^c. The ISM proxy is constructed in Online Appendix D. Panel D displays the cross-correlogram between $x_t^c(ISM)$ and y_t^c. The cross-correlation at lag i is the correlation between $x_{t-i}^c(ISM)$ and y_t^c. The horizontal dashed lines are the two-standard-deviation confidence bounds.

Online Appendix E: Another Type of Matching Cost

This appendix proposes an alternative to the basic model of Section II in which the cost of matching is a time cost instead of an output cost. In this alternative model, households share their time between supplying services and matching with other households who sell services. In the original model, households spend all their time supplying services and they purchase services to match with other households. Yet, all the results of Section II remain valid in this alternative model.

Households employ their time to purchase services: the more time they spend on purchasing services, the less time they have to supply services. A visit takes away an amount $\rho > 0$ of the household's productive capacity; therefore, the actual productive capacity of a household making v visits is $k - \rho \cdot v$. The number of matches on the product market is

$$y = \left[(k - \rho \cdot v)^{-\gamma} + v^{-\gamma}\right]^{-\frac{1}{\gamma}},$$

and the product market tightness is defined by

$$x = \frac{v}{k - \rho \cdot v}.$$

The probability to sell one of the $k - \rho \cdot v$ services for sale is $f(x)$. The probability that a visit is successful is $q(x)$.

To purchase c services, household need to make $c/q(x)$ visits that take away an amount $\rho \cdot c/q(x)$ of their productive capacity. Households are left with a capacity $k - \rho \cdot c/q(x)$, and they sell a fraction $f(x)$ of it. Furthermore, output is equal to consumption—and welfare—because no part of output is used for matching. Therefore, $c = y = f(x) \cdot (k - \rho \cdot c/q(x))$. The aggregate supply is the amount of consumption that solves this equation. Since $f(x)/q(x) = x$, the aggregate supply admits the following expression:

$$c^s(x) = \frac{f(x)}{1 + \rho \cdot x} \cdot k.$$

The aggregate supply is strictly increasing for $x \in [0, x^*]$ and strictly decreasing for $x \in [x^*, +\infty)$

where x^* is the unique solution to

$$q(x)^\gamma = \frac{\rho \cdot x}{1 + \rho \cdot x}.$$

This equation is obtained by rearranging $dc^s/dx = 0$ and using the fact that $f'(x) = q(x)^{1+\gamma}$. It admits a unique solution because q is strictly decreasing from 1 to 0 on $[0, +\infty)$ while $x \mapsto (\rho \cdot x)/(1 + \rho \cdot x)$ is strictly increasing on 0 to 1 on $[0, +\infty)$. The tightness x^* is the efficient tightness: it maximizes welfare for a given level of real money balances. As in the original model, x^* depends only on the matching function and on the matching cost.

Since all output is used for consumption, there is no price wedge due to matching, and the price of consumption is p. However, the selling capacity of the household, and thus its income, is reduced because of the time spent buying consumption. The household's budget constraint is therefore modified to

$$m + p \cdot c = \mu + p \cdot f(x) \cdot \left(k - \rho \cdot \frac{c}{q(x)}\right).$$

To see the parallel between this budget constraint and the budget constraint in Section II, it is convenient to rewrite this constraint as

$$m + p \cdot (1 + \rho \cdot x) \cdot c = \mu + p \cdot f(x) \cdot k.$$

From the household's perspective, the time required to buy consumption imposes a wedge $\rho \cdot x$ on the price of consumption. Accordingly, the household's optimal level of consumption is the same as in the model of Section II after replacing the old price wedge, $\tau(x)$, by the new price wedge, $\rho \cdot x$. The aggregate demand therefore is

$$c^d(x, p) = \left(\frac{\chi}{1 + \rho \cdot x}\right)^\varepsilon \cdot \frac{\mu}{p}.$$

The aggregate demand is strictly decreasing in x and p for all $x > 0$ and $p > 0$.

Given that the aggregate supply and demand are isomorphic to those in the original model once $\tau(x)$ is replaced by $\rho \cdot x$, we can analyze this alternative model by following the same steps. We can show that all the properties of the original model carry over, with one exception. In a fixprice equilibrium, the comparative statics for output and consumption are the same, and they are the same as the comparative statics for consumption in the original model. But they are different from the comparative statics for output in the original model. In the slack regime this difference is mute because consumption and output move together in the original model. But in the tight regime this difference is visible: after an increase in aggregate demand, output increases in the original model but decreases in the alternative model.

We think that the original model of Section II is more realistic than this alternative model because the result that output is higher when the economy becomes tighter seems more realistic, at least for Western economies. The main difference between the original and the alternative model is that the resources devoted to matching are marketed in the original model but not in the alternative model. The alternative model perhaps describes better the centralized economies of the Soviet Union where fewer services were marketed. It is possible that in those economies, people spent so much time queuing to buy goods and services that they had to reduce their supply of labor, and output was lower than if aggregate demand had been lower.

Online Appendix F: Endogenous Marketing Effort

This appendix proposes an extension of the basic model of Section II in which households devote marketing effort to increase their sales. In this alternative model, households share their productive capacity between supplying services and marketing these services. In the original model, households spend all their productive capacity on supplying services. All the results of Section II remain valid in this extension.

Households spend an amount $a \leq k$ of their productive capacity on marketing. The amount of productive capacity left for supplying services is $k-a$. Marketing increases the visibility of services for sale and thus their probability of being sold. The function $e:[0,k] \to [0,1]$ describes the effectiveness of marketing. We assume that e is strictly increasing and concave. To ensure an interior solution with positive marketing effort, we assume $e(0)=0$. To simplify, we assume that the function e has a constant elasticity ε. As in Pissarides (2000, Chapter 5), the number of matches on the product market is given by

$$y = \left\{[e(a)\cdot(k-a)]^{-\gamma} + v^{-\gamma}\right\}^{-\frac{1}{\gamma}},$$

and the product market tightness is defined by

$$x = \frac{v}{e(a) \cdot (k - a)}.$$

The probability to sell one service is $e(a) \cdot f(x)$. Hence, a higher amount of marketing generates more sales. The probability that a visit is successful is $q(x)$.

Households choose their marketing effort to maximize their income. Given x, they choose a to maximize $e(a) \cdot f(x) \cdot (k - a)$. The optimal a satisfies $\varepsilon = a/(k - a)$, which can be rewritten as

$$a = \frac{\varepsilon}{1 + \varepsilon} \cdot k.$$

It is optimal for households to devote a fraction $\varepsilon/(1 + \varepsilon)$ of their productive capacity to marketing.

The aggregate supply describes the amount of consumption sold given the matching process and the optimal marketing decision of households. The aggregate supply admits the following expression:

$$c^s(x) = (f(x) - \rho \cdot x) \cdot e\left(\frac{\varepsilon}{1 + \varepsilon} \cdot k\right) \cdot \frac{1}{1 + \varepsilon} \cdot k.$$

Although it admits a different expression, the aggregate supply has the same properties as in the model without marketing. Furthermore, the aggregate demand remains the same because the trade-off between consumption and holding money is not affected by the marketing effort.

We can analyze this extension with endogenous marketing effort as we analyzed the original model. Since aggregate demand and supply retain the properties of the original model, we can show that in fact all the properties of the original model carry over.

References

Morin, Norman, and John J. Stevens. 2005. "Diverging Measures of Capacity Utilization: An Explanation". *Business Economics*, 46–54.

Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory.* 2nd ed., Cambridge, MA: MIT Press.