Department of Economics
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Spring 2000

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Economics 136
Problem Set #1
(Due in lecture Tuesday, February 1)

(1) Robinson Crusoe has just been shipwrecked on a desert island. Crusoe expects that he will remain on the island for 2 periods, after which time he will be rescued. The only resources available to Crusoe are the coconuts that wash up onto the beach. 60 coconuts are available this period and 36 will be available next period--that is, his endowments are $y_t = 60$ and $y_{t+1} = 36$. Crusoe's preferences for consuming coconuts this and next period is captured by a utility function of the form $U = \ln C_t + \beta \ln C_{t+1}$, where the first term is the natural logarithm of the amount of coconuts consumed in the first period, and the second term is a constant β times the natural log of consumption in the next period. The constant β is called Crusoe's *time discount factor*, since it indicates the relative importance of current and future consumption to him. For example, if $\beta = 0$, Crusoe only values current consumption, whereas if $\beta = 1$, he equally values consumption in each period. It is frequently assumed that $0 < \beta < 1$, which would mean that future consumption is relatively less valuable.

Case (I): *Autarky* (endowment economy; no trade or production)

a. Assume initially that the only technology available to Crusoe is a simple storage technology. Specifically, for every coconut he saves this period, 9/10 will be edible next period. Draw Crusoe's budget constraint on a graph, being sure to label everything.

b. Solve for Crusoe's optimal consumption level in each period if $\beta = 9/11$. (HINT: you can either set up a Lagrangian or change this constrained optimization problem into an unconstrained one by substituting C_t for C_{t+1} using the budget constraint.)

Case (II): *Exchange*

Now suppose that Crusoe's friend, Mr. Friday, arrives from a nearby island. Mr. Friday also collects coconuts from the beach of his island, and he offers to trade coconuts between periods with Crusoe. Specifically, Mr. Friday proposes that for every coconut borrowed or lent 1/10 of a coconut must be repaid--that is, he proposes a real interest rate of 10%.

c. Redraw Crusoe's budget constraint and compare it to his budget constraint in part (a). On the basis of this comparison can you say if trade will make Crusoe better off or worse off?

d. (i) Find Crusoe's demand for coconuts in each period under this arrangement (that is, find his new optimal consumption levels with this new budget constraint; continue to use $\beta = 9/11$). (ii) Verify that Crusoe is indeed better-off than in case (I). (iii) Can you say if the substitution effect or the income effect has dominated in this change?

e. If Mr. Friday has the same preferences as Crusoe, and endowments (from his island) of $y_t = 40$ and $y_{t+1} = 54$, show that $r = 10\%$ is an equilibrium interest rate. (Hint: here you need to show that demand for savings equals supply of savings, that is the amount that agents wish to borrow equals the amount that agents wish to lend. Equivalently, you can show that total demand for consumption in each period equals the total supply of coconuts (why are these equivalent?)).

Case (III): *Production*

Crusoe discovers that if he plants coconuts today trees will grow that will bear coconuts next period. Specifically, if he plants *k* coconuts this period, he will have $k + \sqrt{k}$ next period. (Note: we are assuming here that capital fully depreciates--a coconut planted cannot be eaten later.)

f. Derive and graph Crusoe's consumption possibilities frontier (You will need to be careful here in light of the note above--it just requires a little thought!)

g. Assuming Crusoe can still borrow or lend at an interest rate of 10%, (i) How many coconuts will Crusoe want to plant? (ii) How many coconuts will he want to consume in each period? (iii) How many coconuts will he want to borrow or lend?

(2) Consider two assets, which have random returns that are represented by the variables **X** and **Y**. There are five possible states of the world: (a), (b), (c), (d), and (e). In each of the five different states, the assets have a different return.

State	*Occurs with Probability*	*Observed Return on First Asset (X)*	*Observed Return on Second Asset (Y)*
(a)	*0.1*	*40*	*-50*
(b)	*0.2*	*21*	*70*
(c)	*0.4*	*12*	*30*
(d)	*0.2*	*-16*	*-80*
(e)	*0.1*	*-30*	*40*

a. Calculate expected values, variances, and covariance of the returns on assets X and Y.

b. Suppose there are assets, U and Z, where returns on those assets are represented by
$U = 0.5X + 1$ and $Z = 2Y-2$.
Calculate means and variances of returns on U and Z.

c. Suppose you want to construct a portfolio with X and Y. When constructing a portfolio, you need to decide portfolio weights (or shares) of X and Y in the portfolio. If a portfolio weight on X is ω, then the return on this portfolio is $\omega X+(1-\omega)Y$. Calculate the means and variances of the returns on portfolios where $\omega = .5$ and $\omega = .25$.

d. Find the portfolio weight which would minimize the variance of a portfolio which combined the two assets X and Y.