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Economics 136
Suggested Solutions to Problem Set #3

1. (a) (i) Beginning with the formula: $P_t = \frac{E(P_{t+1} + d_{t+1})}{1+\rho}$, we can find a formula for P_{t+1} by simply advancing the subscripts one period to get: $P_{t+1} = \frac{E(P_{t+2} + d_{t+2})}{1+\rho}$

(ii) To find P_{t+2}, advance the subscripts in the pricing formula one more time:

$$P_{t+2} = \frac{E(P_{t+3} + d_{t+3})}{1+\rho}$$

We know for certain that $d_{t+3}=0$ and $P_{t+3}=0$, so $E(d_{t+3} + P_{t+3})=0$. It follows that $P_{t+2} = 0$.

(iii) Begin with $P_t = \frac{E(P_{t+1} + d_{t+1})}{1+\rho} = \frac{E(P_{t+1})}{1+\rho} + \frac{E(d_{t+1})}{1+\rho}$. Thus, we need to know $E(P_{t+1})$.

Using the formula we found in (i), we know that P_{t+1} depends on $E(d_{t+2})$ and $E(P_{t+2})$. Since $P_{t+2} = 0$ is known in advance, $E(P_{t+2})=0$. Thus, $E(d_{t+2} + P_{t+2})= E(d_{t+2})+E(P_{t+2})= E(d_{t+2})$. Therefore,

$$P_{t+1} = \frac{E(P_{t+2} + d_{t+2})}{1+\rho} = \frac{E(d_{t+2})}{1+\rho}, \text{ and}$$

$$E(P_{t+1}) = E\left\{\frac{E(d_{t+2})}{1+\rho}\right\} = \frac{1}{1+\rho} E\{E(d_{t+2})\} \qquad \text{[since } (1+\rho) \text{ is a constant].}$$

By the *law of iterated expectations,* $E\{E(d_{t+2})\}=E(d_{t+2})$. In English, this could be translated, "what you expect to expect is just what you expect." So

$$E(P_{t+1}) = \frac{E(d_{t+2})}{1+\rho}, \text{ and thus}$$

$$P_t = \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho}$$

(iv) Following the hint, you should be able to show a stock that becomes worthless in period $t+4$ has

$P_t = \frac{E(d_{t+3})}{(1+\rho)^3} + \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho}$. Similarly, a stock that becomes worthless in period $t+N+1$ has

$$P_t = \frac{E(d_{t+N})}{(1+\rho)^N} + \cdots + \frac{E(d_{t+3})}{(1+\rho)^3} + \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho} = \sum_{j=1}^{N} \frac{E(d_{t+j})}{(1+\rho)^j}$$

As you might expect, this formula generalizes to stocks with infinite dividends, that is,

$$P_t = \sum_{j=1}^{\infty} \frac{E(d_{t+j})}{(1+\rho)^j}$$

Note: to be more precise, we do not assume that the stock becomes worthless at some time in the future. Instead, we derive the formula above by first substituting our formula for P_{t+1} into the formula for P_t, i.e.

$$P_t = \frac{E(P_{t+1} + d_{t+1})}{1+\rho} = \frac{E\left[\frac{E(P_{t+2} + d_{t+2})}{1+\rho} + d_{t+1}\right]}{1+\rho} = \frac{E(P_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho}$$

Now, by substituting in for P_{t+2}, we can obtain:

$$P_t = \frac{E(P_{t+3})}{(1+\rho)^3} + \frac{E(d_{t+3})}{(1+\rho)^3} + \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho}$$

And by repeated substitution, we obtain:

$$P_t = \frac{E(P_{t+N})}{(1+\rho)^N} + \frac{E(d_{t+N})}{(1+\rho)^N} + \cdots + \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho} = \frac{E(P_{t+N})}{(1+\rho)^N} + \sum_{j=1}^{N} \frac{E(d_{t+j})}{(1+\rho)^j}.$$

Letting N go to infinity, we have:

$$P_t = \lim_{j\to\infty} \frac{E(P_{t+j})}{(1+\rho)^j} + \sum_{j=1}^{\infty} \frac{E(d_{t+j})}{(1+\rho)^j}.$$

Given the assumption shown on the problem set, $\lim_{j\to\infty} \frac{E(P_{t+j})}{(1+\rho)^j} = 0$, we obtain:

$$P_t = \sum_{j=1}^{\infty} \frac{E(d_{t+j})}{(1+\rho)^j}.$$

(b) We know from above that a stock that pays dividends only in periods *t+1* and *t+2* (and becomes worthless thereafter) has value:

$$P_t = \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho}.$$

What if the firm does not pay a dividend in period *t+1*, but instead invests the funds in asset *"Z"* with return of ρ each period? Then $d_{t+1} = 0$, and in period *t+2*, the firm pays as dividends both its regular dividend, d_{t+2}, and the proceeds from its sale of asset *"Z"*. The firm invests the amount d_{t+1} in period *t+1*, so in period *t+2* it receives $d_{t+1}(1+\rho)$. So the total payoff in period *t+2* is: $Payoff_{t+2} = d_{t+2} + d_{t+1}(1+\rho)$. Thus, the current price of the stock,

P_t = (*present value of expected period t+1 payoff*) + (*present value of expected period t+2 payoff*)

$$= \frac{E(Payoff_{t+1})}{(1+\rho)} + \frac{E(Payoff_{t+2})}{(1+\rho)^2} = 0 + \frac{E\{d_{t+2} + d_{t+1}(1+\rho)\}}{(1+\rho)^2}$$

$$= \frac{E(d_{t+2})}{(1+\rho)^2} + \frac{E(d_{t+1})}{1+\rho}.$$

2.

(a) $p_t = \frac{(58.24)(0.5)+(54.08)(0.5)}{1.08} = 52$

(b) You will try (incorrectly as shown in part (c)) $60 = 52 + \frac{E_t(d_{t+1})}{1.08}$, then $d_{t+1} = 8.64$

(c) Expanding the formula one more period forward, you have $p_t = \frac{E_t(d_{t+1})}{1+\rho} + \frac{E_t(d_{t+2})}{(1+\rho)^2} + \frac{E_t(p_{t+2})}{(1+\rho)^2}$. Then since you didn't plan to give out any dividend next period, so that $E_t(d_{t+1}) = 0$. Since $\rho = 0.08$, old price of GM (before the dividend announcement) is $p_t^{OLD} = \frac{E_t(d_{t+2})}{1.08^2} + \frac{E_t(p_{t+2})}{1.08^2}$. When you give out dividend from borrowing at t+1, then you have to pay back with interest one period later, which will lower dividend as much in that period. Therefore under this plan, dividend will increase by 8.64 at t+1, but will decrease by 8.64×1.08 at t+2. Then, Wall Street does not react to your announcement as you expected since they will figure out the new price of GM stock under your plan as follows;

$p_t^{NEW} = \frac{8.64}{1.08} + \left(\frac{E_t(d_{t+2})}{1.08^2} - \frac{(8.64)(1.08)}{1.08^2} \right) + \frac{E_t(p_{t+2})}{1.08^2}$, which implies that $p_t^{OLD} = p_t^{NEW}$. Perhaps a more sophisticated way to think about this is to recognize that the company is really just offering to rearrange the pattern of cash flows that the shareholder can expect from the company. Ask yourself if this is something that the shareholder would value? It is quite possible that they will not, especially if they can do this rearrangement for themselves, say by borrowing from the bank directly. This result is the famous *Modigliani-Miller theorem*, which says that the capital structure of the firm—i.e. the way it finances itself—is irrelevant to the firm's value.

3. Let y_n and y_{n+1} be n year and n+1 year bond. Suppose $E_n(r_{n+1})$ is the expected one year spot rate at time n, maturing at time n+1. Expectation hypothesis says investing n+1 year bond should give the same return as when you buy n year bond and 1 year bond sequentially, you have

$(1+y_{n+1})^{n+1} = (1+y_n)^n(1+E_n(r_{n+1}))$. Then $E_n(r_{n+1}) = \frac{(1+y_{n+1})^{n+1}}{(1+y_n)^n} - 1$ or if you use logarithmic approximation, then $E_n(r_{n+1}) \approx (n+1)(y_{n+1}) - (n)(y_n)$.

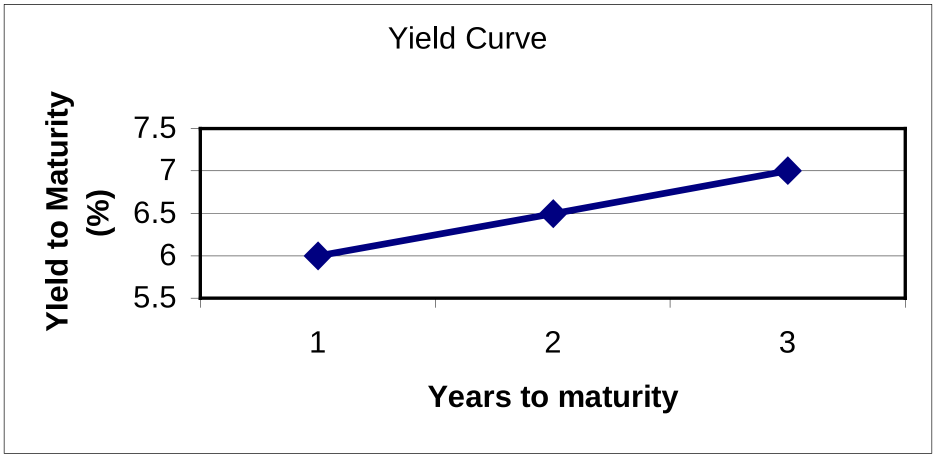
(a) $E_1(r_2) = \frac{1.06^2}{1.05} - 1 = 0.07$

(b) $E_2(r_3) = \frac{1.07^3}{1.06^2} - 1 = 0.09$

(c) $E_3(r_4) = \frac{1.06^4}{1.07^3} - 1 = 0.03$

4. (a) In order to construct a yield curve first we have to compute the yields.

y_1=6%
$y_2=(1.06*1.07)^{1/2}$=1.065 i.e. 6.5%
$y_3=(1.06*1.07*1.08)^{1/3}$=1.07 i.e. 7%



(b) $E(r_{3\text{-}4})_{t=2}$=1.07*1.08=1.156

(c) Yield curve no longer conveys much information. An upward slopping yield curve can result if short-term rates are expected to rise, stay the same, or even decrease if a positive liquidity premium is attached to a long-maturity bond. If the premium is constant then shifts in the yield curve or changes in its slope can imply changes in expectations of future rates. Otherwise not much can be said about expected interest rates.