Department of Economics | **Spring 2002**
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Economics 202A
MIDTERM EXAM

Instructions:

1. Be sure to write your name on the cover of each Blue Book.

2. The questions differ in difficulty but count equally. Each question counts 20 points for a total of 80 for the whole exam.

3. Answer all parts to all questions.

1. Show formally from Friedman's model of consumption that there is a positive correlation between transitory and current income.

2. Suppose that $D_t = \alpha\varepsilon_{t-5} + \varepsilon_t$ where the ε_t's are *i.i.d.* $N(0, \sigma_\varepsilon^2)$.
What is $E_t(D_{t+3})$?

3. Show that in Mankiw's model (in the absence of a menu cost *z*) the loss from failure to change price after a constant shift in demand is equal to $2C$ (where C is the area of the small triangle in Mankiw's key diagram).

[Definitions and hints: If (q^m, p^m) are the maximizing quantity and price and (q^n, p^n) are the non-maximizing quantity and price, C is by definition $\frac{1}{2}(q^n - q^m)(p^m - p^n)$. To answer this question you must specify Mankiw's model, recall what C is, and show that it is equal to $\frac{1}{2}$ the loss in profits.]

4. Consider a firm *i* that minimizes at time *t:*

$$E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j \left[\left(p_{i,t+j} - p^*_{i,t+j} \right)^2 + c\left(p_{i,t+j} - p_{i,t+j-1} \right)^2 \right] \quad c > 0,$$

where $p_{i,t+j}$ is the log of the nominal price of firm *i* in period *t+j*, and $p^*_{i,t+j}$ is the log of the nominal price that firm *i* would choose in period *t+j* in the absence of adjustment costs. Costs of changing nominal prices are captured by the second term in the objective function. The information set at time *t* includes current and lagged $p_{i,t}$ and $p^*_{i,t}$.

a. Derive the first order condition of the above minimization problem, giving the current price $p_{i,t}$ as a function of itself lagged, of its expectation at *t+1*, and of the current optimal price.

b. Rewrite the first order condition using the lag operator. Solve by factorization to derive the following expression:

$$p_{i,t} = \lambda_1 p_{i,t-1} + \frac{1}{\lambda_2} \frac{1+r}{c} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2} \right)^j E_t p^*_{i,t+j},$$

where λ_1 and λ_2 are the reciprocals of the roots of $(1+r)x^2 - \left(\frac{1+r}{c} + 1 + r + 1 \right)x + 1 = 0$ and λ_1 is the smaller of the two. Interpret your result.

Suggested solutions to the midterm exam

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Econ 202A, Spring 2002

1. Recall that in Friedman's model of consumption, we assume $y_C = y_P + y_T$ and $\rho(y_P, y_T) = 0$ so that:

$$Cov(y_C, y_T) = Cov(y_P + y_T, y_T) = Cov(y_P, y_T) + Cov(y_T, y_T) = 0 + Var(y_T) > 0.$$

Thus, there is positive correlation between current and transitory income.

2. As the ε_t's are white noise, we have:

$$\mathbb{E}_t(D_{t+3}) = \mathbb{E}_t(\alpha\varepsilon_{t+3-5} + \varepsilon_{t+3}) = \mathbb{E}_t(\alpha\varepsilon_{t-2}) + \mathbb{E}_t(\varepsilon_{t+3}) = \alpha\mathbb{E}_t(\varepsilon_{t-2}) + 0 = \alpha\varepsilon_{t-2}$$

3. A monopolist facing linear demand and constant marginal cost maximizes:

$$\pi = (a + g\varepsilon)p - bp^2 - c(a + \varepsilon) + cbp.$$

The FOC with respect to p yields:

$$a + \varepsilon - 2bp + cb = 0$$
$$\Rightarrow p^m = \frac{a + \varepsilon + cb}{2b}.$$

Plugging the above expression into the inverse demand function, we find:

$$q^m = \frac{a + \varepsilon - cb}{2}.$$

If the monopolist does not reoptimize after the shock, it will charge:

$$p^n = p^m(\varepsilon = 0) = \frac{a + cb}{2b},$$

and it will sell:

$$q^n = q(p^n) = \frac{a + 2\varepsilon - cb}{2}.$$

If the monopilist does not reoptimize, she will then loose:

$$\begin{aligned}
LOSS &= \pi^m - \pi^n = ... = \\
&= (p^m - p^n)q^m - (p^n - c)(q^n - q^m) = \\
&= A - B = ... = \\
&= \frac{a\varepsilon + \varepsilon^2 - cb\varepsilon}{4b} - \frac{a\varepsilon - cb\varepsilon}{4b} = \\
&= \frac{\varepsilon^2}{4b}
\end{aligned}$$

By definition C is:

$$\begin{aligned} C &= (p^m - p^n)(q^n - q^m) = ... = \\ &= \frac{1}{2}\frac{\varepsilon^2}{4b}. \end{aligned}$$

Thus, we have shown that $LOSS = 2C$.

4. The firm solves:

$$\min_{\{p_{i,t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \left[\left(p_{i,t+j} - p^*_{i,t+j}\right)^2 + c\left(p_{i,t+j} - p_{i,t+j-1}\right)^2\right]$$

(a) The FOC with respect to $p_{i,t}$ yields:

$$\begin{aligned} &\Rightarrow \quad \left(\frac{1}{1+r}\right)^0 \left[2\left(p_{i,t} - p^*_{i,t}\right) + 2c\left(p_{i,t} - p_{i,t-1}\right)\right] + \mathbb{E}_t \left(\frac{1}{1+r}\right)^1 \left[-2c\left(p_{i,t+1} - p_{i,t}\right)\right] = 0 \\ &\Leftrightarrow \quad \left(p_{i,t} - p^*_{i,t}\right) + c\left(p_{i,t} - p_{i,t-1}\right) - c\left(\frac{1}{1+r}\right)\left(\mathbb{E}_t\left(p_{i,t+1}\right) - p_{i,t}\right) = 0 \\ &\Leftrightarrow \quad \left(1 + c + \frac{c}{1+r}\right) p_{i,t} - p^*_{i,t} - c p_{i,t-1} - \frac{c}{1+r}\mathbb{E}_t\left(p_{i,t+1}\right) = 0 \end{aligned}$$

(b) Using the lag operator, we can rewrite the FOC as:

$$\begin{aligned} &\Leftrightarrow \quad \left(1 + c + \frac{c}{1+r}\right) p_{i,t} - p^*_{i,t} - cLp_{i,t} - \frac{c}{1+r}L^{-1}p_{i,t} = 0 \\ &\Leftrightarrow \quad \left(\left(1 + c + \frac{c}{1+r}\right) - cL - \frac{c}{1+r}L^{-1}\right) p_{i,t} = p^*_{i,t} \\ &\Leftrightarrow \quad \frac{1+r}{c}L\left(\frac{1+r+c+cr+c}{1+r} - cL - \frac{c}{1+r}L^{-1}\right) p_{i,t} = \frac{1+r}{c}Lp^*_{i,t} \\ &\Leftrightarrow \quad \left(\left(\frac{1+r}{c} + 1 + r + 1\right) L - (1+r)L^2 - 1\right) p_{i,t} = \frac{1+r}{c}Lp^*_{i,t} \end{aligned}$$

We can factor $\left(\frac{1+r}{c} + 1 + r + 1\right)x - (1+r)x^2 - 1$ as $-(1-\lambda_1 x)(1-\lambda_2 x)$ where

$$\begin{aligned} \lambda_1 + \lambda_2 &= \frac{1+r}{c} + 1 + r + 1, \\ \lambda_1\lambda_2 &= 1 + r, \end{aligned}$$

and therefore

$$(1-\lambda_1)(1-\lambda_2) = 1 - (\lambda_1 + \lambda_2) + \lambda_1\lambda_2 = -\frac{1+r}{c}. \tag{1}$$

Since the sum and the product of λ_1 and λ_2 are positive, both roots are positive. Furthermore, equation (1) implies that one of the root is greater than one and while the other is smaller than one. Let λ_1 be the smaller root so that we have $0 < \lambda_1 < 1 < \lambda_2$.

Factorization then yields:

$$\begin{aligned} &\Leftrightarrow \quad -(1-\lambda_1 L)(1-\lambda_2 L)p_{i,t} = \frac{1+r}{c}Lp^*_{i,t} \\ &\Leftrightarrow \quad -(1-\lambda_1 L)\left(\frac{1-\lambda_2 L}{\lambda_2 L}\right) p_{i,t} = \frac{1+r}{c}\frac{L}{\lambda_2 L}p^*_{i,t} \end{aligned}$$

$$
\begin{aligned}
\Leftrightarrow &\quad -(1-\lambda_1 L)\left(\frac{1}{\lambda_2}L^{-1}-1\right)p_{i,t} = \frac{1+r}{c}\frac{1}{\lambda_2}p^*_{i,t} \\
\Leftrightarrow &\quad (1-\lambda_1 L)\left(1-\frac{1}{\lambda_2}L^{-1}\right)p_{i,t} = \frac{1+r}{c}\frac{1}{\lambda_2}p^*_{i,t} \\
\Leftrightarrow &\quad (1-\lambda_1 L)p_{i,t} = \frac{1+r}{c}\frac{1}{\lambda_2}\frac{1}{1-\frac{1}{\lambda_2}L^{-1}}p^*_{i,t} \\
\Leftrightarrow &\quad (1-\lambda_1 L)p_{i,t} = \frac{1+r}{c}\frac{1}{\lambda_2}\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}L^{-1}\right)^j p^*_{i,t+j},
\end{aligned}
$$

where in the last line we used the fact that $\frac{1}{\lambda_2}$ is inside the unit cirle. At this point, we just need to use the lag operator to get the expression we are looking for:

$$
\Leftrightarrow p_{i,t} = \lambda_1 p_{i,t-1} + \frac{1+r}{c}\frac{1}{\lambda_2}\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^j \mathbb{E}_t\left(p^*_{i,t+j}\right).
$$

Therefore, the current price charged firm is a weighted average between the price in the past period and its expectation of the optimal price in the present and the future periods. (This problem was adapted from of a paper by Julio Rotember published in the Review of Economic Studies in 1982.)