University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2007**

Problem Set #1

(due 10/9/07)

1. Consider an economy in which relative producer prices are fixed and a representative household maximizes the following utility function in consumption goods and leisure (l):

$$U(c_1, c_2, l) = \beta_1 \log(c_1 - a_1) + \beta_2 \log(c_2 - a_2) + \beta_3 \log l$$

(where $\beta_1+\beta_2+\beta_3 = 1$), subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 = w(1-l)$$

A. Rewrite the household's utility function and budget constraint in terms of consumption and labor supply, $L = 1\text{-}l$.

B. Derive an explicit expression for the excess burden of taxes on c_1, c_2, and L as a function of original prices, say p_1^0, p_2^0, w^0, distorted prices, p_1^1, p_2^1, w^1, and a fixed utility level.

C. Using the measure you derived in Part B, show that the deadweight loss is zero for a combined consumption tax/wage subsidy policy that raises the prices p_1, p_2, and w by the same proportion.

D. Starting from an initial price vector (p_1, p_2, w), consider the following two experiments:

1. a uniform tax on consumption that makes the final price vector ($p_1/(1\text{-}\theta)$, $p_2/(1\text{-}\theta)$, w)
2. a wage tax that makes the final price vector (p_1, p_2, $w(1\text{-}\theta)$)

Show that, for given utility, the deadweight losses for the two experiments are the same. (Hint: use your answer to part C and the fact that deadweight loss is not path-dependent, i.e., that it is not affected by breaking a given change in prices into two separate changes.)

2. Consider an economy with fixed producer prices and a representative household that maximizes utility that is a function of one consumption good and two types of leisure, perhaps the leisure of two spouses,

$$U(c, l_1, l_2) = c^{\alpha_0} l_1^{\alpha_1} l_2^{\alpha_2}$$

(where $\alpha_0+\alpha_1+\alpha_2 = 1$), subject to the budget constraint:

$$pc = w_1(1-l_1) + w_2(1-l_2)$$

A. As in Problem #1, rewrite the household's problem in terms of consumption and labor supply, $L_1 = (1-l_1)$ and $L_2 = (1-l_2)$.

B. Derive expressions for the compensated supplies of L_1 and L_2.

C. Suppose that the government wishes to raise a fixed amount of revenue from the household using separate proportional taxes on L_1 and L_2. Based on the standard three-good analysis, use your answers to part B to derive a condition in terms of the wage rates and utility function parameters for uniform taxation to be optimal.

D. Derive expressions for the market supplies of L_1 and L_2. (Note that exogenous income is zero and so should disappear from the expressions.)

E. Now, suppose that there are several households, all with the same preferences as those given above but with differing abilities, which may be represented in terms of efficiency units; household h faces wage rates $(w_1 e_1^h, w_2 e_2^h)$. Using this notation, rewrite your expressions for market labor supplies derived in part D. From these new expressions, solve for the household's corresponding supplies of labor in constant efficiency units, $\overline{L}_1^h = L_1^h e_1^h$ and $\overline{L}_2^h = L_2^h e_2^h$.

F. Suppose that the government wishes to use separate proportional taxes on aggregate labor supplies $L_1 = \sum_h \overline{L}_1^h$ and $L_2 = \sum_h \overline{L}_2^h$ to raise a fixed amount of revenue. Unlike in Part C, the optimal policy will now take both efficiency and equity considerations into account. Discuss how equity considerations should affect the relative taxation of the two types of labor if:
   1. $e_1^h = e_2^h \quad \forall h$
   2. $e_2^h = 1 \quad \forall h$

3. In class, we showed that a consumption tax is equivalent to a tax on labor income plus a tax on existing assets. That derivation assumed that assets took the form of homogeneous capital. This question reconsiders the issue for a wider class of assets.

   A. Write down the budget constraint for a household that lives for two periods, works in the first period, has initial assets in the first period, and consumes in both periods. Assume that the household initially faces a uniform tax at rate t on capital income and labor income.

   B. There is no price level in the budget constraint in part A, because there are no nominal magnitudes in the model – only relative prices matter. But suppose now that the household holds two types of assets, real capital and government bonds, each yielding the same rate of interest. Rewrite the budget constraint from part A for this case, letting A_1 be the real quantity of capital and A_2 be the nominal stock of government bonds. Assume that the price level is the same in periods 1 and 2, i.e., that there is no inflation.

C. Now, suppose that, before period 1, the government replaces the income tax with a sales tax on consumption in both periods, and that the real before-tax wage and the real before-tax interest rate remain the same. Assume also that the price level net of the sales tax does not change in either period. Rewrite the budget constraint from part B for this tax system, and show that the consumption tax is equivalent to a tax on labor income and on all existing wealth.

D. Now, change the assumption about the price level in part C. Suppose that, when the sales tax is imposed, the Fed successfully adjusts monetary policy to keep the consumer price index (which includes the sales tax) constant in both periods. How does your answer to part C change?

E. Now, go back to the price level assumption from part C, but assume instead that the government bonds are initially not taxed (as is the case in the United States for bonds issued by state and local governments), and so in capital market equilibrium yield a <u>before-tax</u> rate of return under the income tax of $r(1-t)$, where r is the before-tax return to capital. Suppose also that these bonds are consols, i.e., of infinite duration with constant nominal coupons. How does your answer to part C change?