Problem Set #2

(due 11/20/07)

1. In Harberger's model, capital bears all of the corporate (sector X) income tax if labor's share of *before-tax* income is unchanged as T_{KX} changes, that is, if the following ratio stays fixed:

$$\frac{wL}{wL + rK + (T_{KX} - 1)rK_X}$$

   A. For K and L fixed and T_{KX} initially equal to 1, show that this implies that the following relationship holds for the relative changes in w and r, where $\lambda_{KX} = K_X/K$:

$$(*) \quad \hat{w} - \hat{r} = \lambda_{KX}\hat{T}_{KX}$$

   B. Using the expression derived in class relating $\hat{w} - \hat{r}$ to $\hat{T}_{KX}$, show that (*) holds if X and Y have the same initial factor proportions and production elasticities of substitution.

   C. Now suppose that X and Y have the same initial factor proportions but that the production elasticity of substitution in sector Y is zero. What fraction of the tax does capital bear?

2. Consider an economy in which relative producer prices are fixed and there are H identical households, each with the following utility function in household consumption of goods 1 and 2, c_1 and c_2, household leisure, l, and aggregate consumption of good 1, $C_1 = Hc_1$:

$$u(c_1, c_2, l, C_1) = c_1^{\alpha_1} c_2^{\alpha_2} l^{1-\alpha_1-\alpha_2} C_1^{-\beta}$$

   Each household maximizes this utility function subject to the budget constraint:

$$p_1c_1 + p_2c_2 + wl = y$$

   where y equals the value of the household's labor endowment, $w\overline{L}$, less any lump-sum taxes paid to the government. In its optimization process, the household ignores the effect of its own consumption of good 1 on C_1, i.e., it treats C_1 as fixed when choosing c_1.

   A. Solve for the household's indirect utility function, conditional on the value of C_1, $V(p_1, p_2, w, y; C_1)$. Use the household's demand function for c_1 and the fact that all households are identical to express C_1 in terms of income and prices, and substitute this expression for C_1 into the indirect utility function to obtain an expression for individual utility that is solely dependent on prices and income, $\widetilde{V}(p_1, p_2, w, y)$. Letting the social welfare function be the sum of the utilities of the H identical households, use your expression for $\widetilde{V}(\cdot)$ to obtain a solution for social welfare in terms of prices and aggregate income $Y = Hy$, i.e., $W(p_1, p_2, w, Y)$.

B. Let labor be the numeraire ($w = 1$) and let *producer* prices for goods 1 and 2 be q_1 and q_2. Suppose that the government raises revenue R for public expenditures (which don't affect utility directly) with uniform lump-sum taxes and taxes on goods 1 and 2. Let θ_i be the proportional tax on good i, i.e., $\theta_i = (p_i - q_i)/p_i$ or $p_i = q_i/(1-\theta_i)$. Solve for the optimal values of θ_1 and θ_2, showing that the tax on good 2 is zero and that the tax on good 1 is β/α_1. (*Hint*: use the definition of Y to express it in terms of R, θ_1 and θ_2, insert the result into your expression for $W(\cdot)$ and maximize welfare directly with respect to the taxes.)

C. Now, suppose that the government must raise R without lump-sum taxes. Solve for the optimal taxes, θ_1^* and θ_2^*, and show that the ratio of consumer prices should be the same as in part B, i.e., that $\dfrac{q_1/(1-\theta_1^*)}{q_2/(1-\theta_2^*)} = \dfrac{q_1/(1-\theta_1^p)}{q_2}$, where $\theta_1^p = \beta/\alpha_1$ is the Pigouvian tax.

3. Consider an economy in which the government may provide two public goods, X and Y, each with unit cost 1, and household $h \in H$ has a utility function of the form $U^h(X,Y,M^h) = g^h(X,Y) + Z^h$, where Z^h is the household's consumption of private goods, equal to exogenous income M^h less the household's assessment for public goods. The functions $g^h(\cdot)$ may vary over h.

A. Suppose that $g^h(X,Y) = \alpha^h \log(X+Y)$ and that household h is assigned a budget share β^h of the overall cost of each of the public goods. Show that preferences are single peaked with respect to the total level of public goods, $X+Y$.

B. Now, suppose that $H = 3$, that $g^1(X,Y) = X \cdot Y$; $g^2(X,Y) = \max(X, Y)$; $g^3(X,Y) = 0$, and that $\beta^1 = \beta^2 = \beta^3 = 1/3$. The government has three spending options, $(X,Y) = (0,0)$, $(0,1)$, and $(1,1)$. Are preferences over public expenditures single-peaked?

4. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:

(1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;

(2) how to distribute these assets between a taxable account and a tax-sheltered account.

Interest on bonds held in the taxable account (*TA*) is taxed at rate τ, while equity returns are taxed at rate $\alpha\tau$, where $0 < \alpha < 1$. Assets placed in the tax-sheltered account (*TSA*) are tax-exempt. An amount up to $V < W$ may be placed initially in the tax-sheltered account.

A. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account, for an individual who is risk neutral and for one who is infinitely averse to risk.

B. Show that, regardless of the individual's risk aversion, it will never be optimal to hold equity in the *TSA* and bonds in the *TA* at the same time. (Hint: by considering a portfolio shift, prove that such an initial allocation would permit the investor to achieve a higher after-tax return on debt for a given after-tax distribution of returns on equity.)