University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2010**

Problem Set #1

(due 10/5/10)

1. Consider an economy with fixed producer prices and a representative household that maximizes utility, which is a function of one consumption good and two types of leisure, perhaps the leisure of two spouses,

$$U(c,l_1,l_2) = c^{\alpha_0} l_1^{\alpha_1} l_2^{\alpha_2}$$

(where $\alpha_0+\alpha_1+\alpha_2 = 1$), subject to the budget constraint:

$$pc = w_1(1-l_1) + w_2(1-l_2)$$

A. In order to set the problem up in terms of transactions between households and firms, rewrite the household's utility function and budget constraint in terms of consumption and labor supply, $L_1 = (1\text{-}l_1)$ and $L_2 = (1\text{-}l_2)$.

B. Derive expressions for the compensated demand for c and compensated supplies of L_1 and L_2.

C. Suppose that the government wishes to raise a fixed amount of revenue from the household using separate proportional taxes on L_1 and L_2 Based on the standard three-good analysis, use your answers to part B to derive a condition in terms of the wage rates and utility function parameters for uniform taxation to be optimal.

D. Derive expressions for the market (uncompensated) supplies of L_1 and L_2. (Note that exogenous income, y, is zero in this model.)

E. Now, suppose that there are several households, all with the same preferences as those given above but with differing abilities, which may be represented in terms of efficiency units; household h faces wage rates $(w_1e_1^h, w_2e_2^h)$. Using this notation, rewrite your expressions for uncompensated labor supplies derived in part D. From these new expressions, solve for the household's corresponding supplies of labor in constant efficiency units, $\overline{L}_1^h = L_1^h e_1^h$ and $\overline{L}_2^h = L_2^h e_2^h$.

F. Suppose that the government wishes to use separate proportional taxes on aggregate labor supplies $L_1 = \sum_h \overline{L}_1^h$ and $L_2 = \sum_h \overline{L}_2^h$ to raise a fixed amount of revenue. Unlike in Part C, the optimal policy will now take both efficiency and equity considerations into account. <u>Without</u> deriving an expression for the optimal tax rates, discuss how equity considerations should affect the relative taxation of the two types of labor if:

   1. $e_1^h = e_2^h \quad \forall h$
   2. $e_2^h = 1 \quad \forall h$

(Hint: discuss how the ratio of $\overline{L}_2^h$ to $\overline{L}_1^h$ varies as e_1^h rises.)

2. In the Harberger two-sector model, with overall supplies of labor and capital fixed and earning rates of return w and r, respectively, labor bears a fraction ψ of an incremental tax burden Δ if the ratio

$$R = \frac{wL + \psi\Delta}{wL + rK + \Delta}$$

is unchanged as Δ increases from its initial value of 0; that is, $dR/\mathrm{d}\Delta = 0$.

A. Show that labor's share of the burden, ψ, equals it share of initial income, $wL/(wL+rK)$, if there is no change in the ratio w/r (i.e., $\hat{w} - \hat{r} = 0$) as the tax is introduced.

B. Suppose that the tax introduced is on capital income in sector X, so that $\Delta = \tau_{KX} rK_X$. Derive a condition for $\hat{w} - \hat{r} = 0$, using the expression for $\hat{w} - \hat{r}$ derived in class for this tax experiment.

C. Now suppose that $\sigma_D = \sigma_X$ and that sector X uses both capital and labor in production. Show that the condition you derived in part B cannot be satisfied, and hence that $\hat{w} - \hat{r} > 0$: capital's relative share of the tax must be higher than labor's. (You will need to use the fact that $a_X = \lambda_{LX}\theta_{KX} + \lambda_{KX}\theta_{LX}$).

3. Consider an economy with overlapping generations, each with a single agent who lives for two periods. (Let generation t be the generation that is young in period t.) The world interest rate is fixed at r. The timing convention is that government debt is issued at the beginning of the period, and taxes, transfer payments and government purchases occur at the end of the period. Initially, in year t, the government has no national debt outstanding, and operates a social security system that transfers 1 unit of output to the older individual from the younger individual in each period.

A. Write down the government's intertemporal budget constraint (GIBC) in year t in terms of national debt, government purchases and government net taxes (taxes less transfers), and show that the government's policy satisfies the GIBC.

B. Now write down the GIBC in its alternative formulation, in terms of the initial level of debt, government purchases, and the generational accounts for all existing and future generations. Solve for the generational account for each generation, and show that this version of the GIBC is also satisfied under current government policy.

C. Suppose that, at the end of the current period, t (i.e., at the beginning of period t+1), the government eliminates the social security system by issuing bonds to pay for the current elderly agent's benefit. Assuming that government services the debt using equal taxes on each future elderly generation, solve for the tax needed to satisfy the GIBC. Solve for all generational accounts after this policy change, and show that they are the same as for the original social security system in part B.