University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2010**

Problem Set #2

(due 11/16/10)

1. Consider an economy with three local communities of equal population size. Individuals are identical within each community i, with preferences over private goods c and local public goods g governed by the utility function $log(c - a_i) + log(g)$ and endowment y_i of the private good. For simplicity, assume also that each community has one individual. Both private and public goods have a price of 1 and public goods must be purchased separately for each jurisdiction (i.e., there are no spillovers in public goods consumption across jurisdictions).

   A. Suppose first that each community chooses its own level of spending on the public good. Solve for the level of public spending and the level of utility in each community as a function of the parameters a_i and y_i.

   B. Now, suppose that all public spending is centrally financed by a proportional income tax at rate t, where t is the same across communities and each community's level of the public good equals one-third of revenue raised in the whole economy. Thus, once the level of t is determined, private and public goods consumption in each community is also determined. Assume that t is chosen by a simple majority vote.

      i. Solve for each community's preferred level of t and show that preferences over t are single-peaked in each community, so that the level of t chosen by majority vote will be that of the median voter.

      ii. Under what condition will total public spending be greater under central provision than under local provision (case A.)?

      iii. Is the equilibrium Pareto-optimal under central provision?

   C. Now, suppose that before voting on the level of public goods, individuals vote on whether to use local provision or central provision. Also suppose that $\bar{y} < y_i$, where $\bar{y} = (y_1 + y_2 + y_3)/3$ is average community income. Show that community i will vote for local provision. (*Hint*: First compare the outcomes in the case for which community i is the median voter under central provision.)

2. In class, we showed that a consumption tax is equivalent to a tax on labor income plus a tax on existing assets. That derivation assumed that assets took the form of homogeneous capital. This question reconsiders the issue for a wider class of assets.

   A. Write down the budget constraint for a household that lives for two periods, supplies labor L in the first period for wage w, has initial assets in the first period, A and consumes in both periods (c_1 and c_2). Assume that the household initially faces a uniform tax at rate t on capital income and labor income.

B. There is no price level in the budget constraint in part A, because there are no nominal magnitudes in the model – only relative prices matter. But suppose now that the household holds two types of assets, real capital and nominal government bonds, each yielding the same rate of interest. Rewrite the budget constraint from part A for this case, letting A_1 be the *real* quantity of capital and A_2 be the *nominal* stock of government bonds. Assume that the price level, p, is the same in periods 1 and 2, i.e., that there is no inflation.

C. Now, suppose that, before period 1, the government replaces the income tax with a sales tax on consumption in both periods, and that w and r remain the same. Assume also that the price level net of the sales tax does not change in either period, i.e., remains equal to p. Rewrite the budget constraint from part B for this tax system, and show that the consumption tax is equivalent to a tax on labor income and on all existing wealth.

D. Now, change the assumption about the price level in part C. Suppose that, when the sales tax is imposed, the Fed successfully adjusts monetary policy to keep the consumer price index (which includes the sales tax) constant in both periods. How does your answer to part C change?

E. Now, go back to the price level assumption from part C, but assume instead that the government bonds are initially not taxed (as is the case in the United States for bonds issued by state and local governments), and so in capital market equilibrium yield a before-tax rate of return under the income tax of $r(1\text{-}t)$. Suppose also that these bonds are consols, i.e., of infinite duration with constant nominal payments each period. How does your answer to part C change?

3. Suppose that a risk-neutral investor faces a tax rate of c on capital gains, while facing a tax rate of t (i.e., getting a refund at rate t) on long-term capital losses. The investor has an asset originally purchased for P_0 that is now worth $P_1 > P_0$, and must decide whether to (1) sell the asset now, pay a tax on $(P_1 - P_0)$ at rate c, and reinvest the remaining proceeds for one more period, or (2) continue holding the asset for one more period. In either case, the rate of return over the next period is r, which is stochastic. Under choice (1), subsequent gains will be taxed at c and subsequent losses will be taxed at t. Under choice (2), total gains, $(P_1(1+r) - P_0)$ will be taxed at rate c, for we assume that $P_1(1+r) > P_0$ even if r is negative.

A. Derive a necessary and sufficient condition for the investor to realize the capital gain now, expressed in terms of some critical value of the ratio $R = P_1/P_0$, say R^*.

B. Assuming that there is a distribution of values of $R = P_1/P_0$ in the population, a lower value of R^* will means that fewer gains will be realized immediately. Show that $dR^*/dc < 0$, starting from the case in which t and c are initially equal.

C. Also starting from the case in which $t = c$, show that $dR^*/dt > 0$. Explain your result.