University of California, Berkeley **Economics 230a**
Department of Economics **Fall 2011**

Problem Set #2

(due 11/22/11)

1. Consider an economy in which relative producer prices are fixed and there are H identical households, each with the following utility function in household consumption of goods 1 and 2, c_1 and c_2, household leisure, l, and aggregate consumption of good 1, $C_1 = Hc_1$:

$$U(c_1, c_2, l, C_1) = c_1^{\alpha_1} c_2^{\alpha_2} l^{1-\alpha_1-\alpha_2} C_1^{-\beta}$$

Each household maximizes this utility function subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 + wl = y$$

where y equals the value of the household's labor endowment, $w\bar{L}$, less any lump-sum taxes paid to the government. In its optimization process, the household ignores the effect of its own consumption of good 1 on C_1, i.e., it treats C_1 as fixed when choosing c_1.

A. Solve for the household's indirect utility function, conditional on the value of C_1, $V(p_1, p_2, w, y; C_1)$. Use the household's demand function for c_1 and the fact that all households are identical to express C_1 in terms of income and prices, and substitute this expression for C_1 into the indirect utility function to obtain an expression for individual utility that is solely dependent on prices and income, $\tilde{V}(p_1, p_2, w, y)$. Letting the social welfare function be the sum of the utilities of the H identical households, use your expression for $\tilde{V}(\cdot)$ to obtain a solution for social welfare in terms of prices and aggregate income $Y = Hy$, i.e., $W(p_1, p_2, w, Y)$.

B. Let labor be the numeraire (w =1) and let *producer* prices for goods 1 and 2 be q_1 and q_2. Suppose that the government raises revenue R for public expenditures (which don't affect utility directly) with uniform lump-sum taxes and taxes on goods 1 and 2. Let θ_i be the proportional tax on good i, i.e., $\theta_i = (p_i - q_i)/p_i$ or $p_i = q_i/(1\text{-}\theta_i)$. Solve for the optimal values of θ_1 and θ_2, showing that the tax on good 2 is zero and that the tax on good 1 is β/α_1. (*Hint*: use the definition of Y to express it in terms of R, θ_1 and θ_2, insert the result into your expression for $W(\cdot)$ and maximize welfare directly with respect to the taxes.)

C. Now, suppose that the government must raise R without lump-sum taxes. Show that the ratio of consumer prices should be the same as in part B, i.e., that $\dfrac{q_1/(1-\theta_1^*)}{q_2/(1-\theta_2^*)} = \dfrac{q_1/(1-\theta_1^p)}{q_2}$, where $\theta_1^p = \beta/\alpha_1$ is the Pigouvian tax. (*Hint*: one can show this by comparing first-order conditions; it is not necessary to solve completely for taxes.)

2. Suppose that an economy has two goods, education and housing, and that every family has preferences over the two goods defined by the common utility function, $U(E,H) = E^{\alpha} H^{1-\alpha}$. Households differ only with respect to income level, with household i's exogenous income

equal to y^i. Housing is produced subject to constant unit cost $p_H = 1$, and may be purchased in any quantity. Education may be produced using one of two technologies: by the private sector as a regular private good with unit cost $p_E = 1$ per family, and by the public sector as a *pure public good* with unit cost q. The common level of public education, G, is financed by a proportional tax at rate τ on income, and no individual household may use public and private education at the same time.

A. For fixed values of the tax rate, τ, and the level of public education, G, show that there exists a critical level of income, $\hat{y}$, above which households choose private school, and below which households choose public school. Show that $\hat{y}$ is increasing in G, given τ.

B. Start with your solution for $\hat{y}$ as a function of G and τ from part A. Letting Y equal aggregate income in the economy, substitute for τ using the government's budget constraint that relates τ to G and Y. Calculate the full effect of G on $\hat{y}$, i.e., the effect taking into account the impact of G on τ. Show that this effect is larger than the partial effect you solved for in part A, and explain why.

C. Show that, among individuals who choose public education, there single level of public education, say G^*, that is most preferred by all. If the existing level of public education is initially at G^*, under what condition would a majority of the overall population vote for a small decrease in spending on public education spending? (*Hint*: relate $\hat{y}$ at G^* to the income of the median voter.)

3. Consider an individual who wishes to invest initial wealth, W, to maximize the utility of terminal wealth one period hence. The investor's problem consists of two decisions:

   (1) how much of this wealth to place in bonds, which yield a certain return, $i > 0$, and how much to invest in stocks, which yield a stochastic return $r \in [0, R]$, $E(r) = \bar{r} > i$;

   (2) how to allocate these assets between a taxable account and a tax-sheltered account.

   Interest on bonds held in the taxable account (TA) is taxed at rate τ $(0 < \tau < 1)$, while equity returns are taxed at rate $\lambda\tau$ $(0 < \lambda < 1)$. Assets placed in the tax-sheltered account (TSA) are tax-exempt. An amount up to $V < W$ may be placed in the tax-sheltered account.

A. Derive the optimal portfolio, in terms of the amounts of debt and equity held in each account, for an individual who is risk neutral; perform the same exercise for an individual who is infinitely risk averse.

B. Show that, regardless of an individual's degree of risk aversion, it will never be optimal for the individual to hold equity in the TSA and bonds in the TA at the same time. (Hint: starting with such an initial allocation, show that a portfolio shift would permit the investor to achieve higher safe after-tax returns on holdings of debt for a given distribution of after-tax returns on holdings of equity.)