Strategic Interaction between Formal and Informal Lenders in Underdeveloped Credit Markets

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Abstract

This paper studies the strategic interaction between an informal moneylender and an institutional formal lender in underdeveloped credit markets when contracts are non-exclusive. We construct a game-theoretic model with adverse selection, market power, and differences in the cost of lending. It is shown that under general conditions, a co-funding equilibrium will be the Nash outcome of the game, where the game is either a simultaneous- or a sequential-offer game. The model provides an alternative explanation as to why moneylenders in underdeveloped credit markets generally offer small loans at very high rates of interest to relatively safe borrowers. Moreover, the model yields new insight into the role of collateral. We show that the co-funding equilibrium is more easily sustained if the formal lender can require collateral, and that this may actually increase both lenders' profits. In addition, we shed light on the relationship between credit risk and the presence of a co-funding equilibrium. Finally, it is shown that a government subsidy to the formal financial sector may actually lower both lenders' profit. As such, a government subsidy to the formal sector could reduce viability and outreach.

1 Introduction

This paper focuses on the strategic interaction between formal and informal lending institutions in less developed countries. There are several reasons as to why an understanding of this interaction is important. First, despite the recent trend of financial deregulation, government-owned agricultural banks and, more generally, development banks continue to play a significant role in developing-country credit markets. Second, there is the increasing attention given to microfinance institutions by policymakers and NGOs. Both development banks and microfinance institutions compete with informal financial institutions for borrowers, and many of the formal institutions are not financially viable. They have to rely on significant subsidies from donors.[1] Such 'intrusion' on rural credit markets obviously has implications for the operation of informal lenders. And only by attempting to formally model the interaction in credit markets can we hope to gain an understanding of the nature of these implications. This is the aim of the present paper.

The literature on the interaction between formal and informal credit institutions can be divided into two classes according to the type of interplay they model. First, in models of vertical integration borrowers obtain loans exclusively from the informal sector. The informal sector, on the other hand, borrows from the formal sector. Hence, the informal sector serves as middleman, and the effects of changes in the formal sector will depend on the structure of the informal sector, e.g. monopoly, monopolistic competition, or perfect competition. Models of this type are analyzed by Bose (1998), Floro and Ray (1997), and Hoff and Stiglitz (1998).

Second, in models of horizontal integration borrowers can obtain loans in both the formal and the informal sector. Such models have previously been analyzed by Bell (1990), Bell, Srinivasan, and Udry (1997), Kochar (1997), and Jain (1999). This framework is also the focus of the present paper. However, to our knowledge, in all existing models it is assumed that at most one of the lending institutions behave strategically. It is argued in this paper that both the formal and the informal lending agents have incentives to act strategically in the market, and a simple model is developed to capture this strategic interaction.

Bell (1990) constructs a model in which the borrower seeks the cheaper institutional credit first; any recourse to the more expensive private moneylender stems from the borrower experiencing credit rationing at the institutional lender: perhaps due to administered interest ceilings. If profitable,

[1] In 1997, a high-profile group of policymakers, practitioners, and charitable funds initiated a drive to raise over $20 million for new microfinance initiatives (Murdoch, 1999).

the moneylender then offers a loan which is equal in size to the gap between the rationed loan and the borrower's notional demand for credit. Hence, the existence of a private moneylender stems from the regulation-induced disequilibrium in the formal sector. As a comment to his model, Bell notes that: 'it must be ascertained whether the moneylender would have any incentive in preventing the borrower from taking an institutional loan. It seems probable that the moneylender will have no objection. The institutional finance will permit an expansion of the borrower's activities, and if the moneylender is in a position to exercise first claim on the returns produced by the borrower's activities, the institutional loan will, in general, improve the moneylender's expected returns from his loan.' (Bell, 1990 p. 203). We treat this aspect more formally here.

Interestingly, as shown by Jain (1999), non-exclusive contracts may be in the interest of the formal (institutional) lender as well. Thus, co-funding of borrowers by formal and informal lenders need not *per se* be a disequilibrium phenomenon, as suggested by Bell. Jain constructs a model with two types of borrowers: safe and risky. The formal lender is a monopolistic bank, which has a cost advantage *vis-à-vis* the informal lender, but is uninformed with respect to the borrowers' types. The informal lenders are assumed to be fully informed and operate in a perfectly competitive environment.

As in Bell (1990), borrowers first obtain credit at the bank and then turn to the competitive informal sector for residual financing. What Jain shows is that it may sometimes be optimal for the bank to offer two different contracts: a full contract at a high interest rate, and a partial contract at a lower rate. The intuition for this result is that the bank can use the informational advantage of the informal sector to screen borrowers. Since risky borrowers face a higher competitive informal interest rate, they will be more willing to accept a higher formal interest rate in exchange for full formal financing. Thus, in Jain's model, credit rationing and co-funding is an optimal solution by a monopolistic bank to an information problem.[2]

In this paper, we attempt to combine the insights of Bell (1990) with those of Jain (1999). We construct a model of strategic interaction based on the following observations: i) the coexistence of formal and informal lenders, each enjoying considerable market power (monopolies); ii) borrowers are heterogenous with respect to the profitability and risk of their investments; iii) the informal sector has an informational advantage; iv) the formal sector has a lower deposit mobilisation cost; v) in the absence of collateral requirements,

[2] By assuming that the informal sector is perfectly competitive, Jain avoids the complications associated with two-sided strategic interaction. The role of the informal sector is equivalent to that of a collateral requirement, and the problem faced by the formal sector becomes a standard screening problem to which the revelation principle is applicable.

the informal sector has seniority of loans as a consequence of the informational advantage; and vi) both sectors operate under imperfect competition and behave strategically.

This results in a situation where the bank and the moneylender compete for borrowers and hence try to crowd out each other. At the same time, the bank has an interest in exploiting the informational advantage of the informal sector by screening the borrowers through co-funding requirements. The moneylender, on the other hand, also has an interest in co-funding solutions since it can use the cost advantage of the bank to increase its own profits from the lending activity. The outcome is modelled as a Nash equilibrium in both a simultaneous and a sequential game.[3]

The notion of a co-funding equilibrium in underdeveloped credit markets and the incentives described in the previous paragraph are comparable to results from the corporate finance literature on the co-existence of direct lending (securities markets) and intermediated lending (banks) to firms operating in more mature capital markets. In that context, bank loans are a more costly source of finance than corporate securities, but nevertheless firms use both. One reason is that a firm can signal private information through its leverage (debt-equity ratio). Due to the private nature of debts and the role of banks as delegated monitors, a firm can signal its creditworthiness to securities markets by a certain amount of intermediated loans. Moreover, a bank's repayment probability is increasing in the amount of direct lending, so there exists a complementarity between intermediated and direct lending. As such, the notion of a co-funding equilibrium is not a far-fetched idea. Rather, it seems to be a natural outcome in an environment characterised by asymmetric information.[4]

Indeed, the present paper demonstrates that the insights provided by Jain are robust to the above mentioned extensions. The co-funding result is a regular outcome of a simultaneous-offer game between formal and infor-

[3] Nash equilibrium requires that each player's strategy is a best-response to the strategies actually played by his rivals. It thereby imposes a self-enforcing steady-state nature upon the equilibrium: Players act rationally, hold the correct expectations about the other players' behaviour, and no player has any incentive to deviate from the equilibrium. Myerson (1999) notes that the concept should therefore be part of the critical analysis of almost any kind of social institution.

[4] Seward (1990), for instance, constructs a model in which the conjunction of private actions (i.e., unobservable investment allocations) and private information (i.e., unobservable project cash flows) creates an incentive problem that can be resolved through the presence of direct and intermediated lending. The bank acts as a delegated monitor and thereby solves the public goods problem of monitoring in connection with direct lending. Dowd (1996) provides an excellent discussion of theoretical corporate finance issues, and especially of the co-existence of direct and intermediated lending.

mal lenders. Moreover, we find that in a simple sequential environment in which one lender is given a first-offer advantage, the likelihood of observing a co-funding equilibrium increases. The reason is two-fold: First, whenever a co-funding equilibrium exists, it yields a higher payoff to both lenders; second, in a sequential game, the equilibrium is less fragile to deviations. Thus, by explicitly modelling the incentives of both the formal and the informal sector, we provide a complete characterization of the set of conditions required for co-funding to be an equilibrium outcome in an extended environment, characterized by strategic interaction and asymmetric information. This is the main contribution of the paper. In addition to this, the model admits some interesting results that we discuss briefly below.

First, stylized facts on moneylenders in informal credit markets show: that they tend to charge very high interest rates compared to financially viable institutional lenders; that they only lend to relatively safe borrowers; and that they only provide relatively small loans (see Robinson (2001)). The model presented in the present paper is fully consistent with these stylized facts. Yet, the explanation differs from the usual suspects, including high default rates, low geographic mobility, low income, and low education among borrowers, or some sort of market power *per se*. Instead, we show that in a sequential game, where the moneylender has the first-offer advantage, the 'typical' equilibrium is a co-funding equilibrium where the moneylender co-finances the safe types with a very small loan at a very high interest rate. The intuition is that since the bank has a cost advantage, the moneylender has an interest in offering only very small loans, as total rent is decreasing in the moneylender's co-funding share. Moreover, in order for the moneylender to transmit his information on types to the bank, i.e. screen borrowers, a small loan is sufficient.

Second, an often pursued policy in developing countries is to give subsidies to the formal sector. In our model, a fall in formal sector banking costs, say as a result of a subsidy, will, ceteris paribus, increase bank profit. This effect is largest in a pooling equilibrium, where the bank fully finances all agents. Moreover, lower formal sector banking costs will tend to reduce the scope for co-funding equilibria as the bank's deviating incentive becomes stronger. However, since a shift from a co-funding to a pooling equilibrium might imply strictly lower profits to the bank, it may well be that the bank will become less viable as a consequence of the subsidy. Borrowers, on the other hand, will benefit from this since they both earn a positive rent in a pooling equilibrium.

Third, we show that the co-funding equilibrium is only attainable in the presence of considerable risk. A mean-preserving reduction in project yields raises the incentives for the bank to opt for a pooling solution. When risk decreases, the bank is able to earn more from risky types at a given rate of interest: that is, adverse selection is less important. Consequently, a co-funding equilibrium should be less prevalent in more mature rural credit markets with less risk. This insight serves to further underscore that a co-funding equilibrium is closely related to information imperfections.

Finally, the model provides an additional explanation of collateral requirements by the formal sector. Debt seniority raises the willingness of the moneylender to co-finance borrowers, since it increases the moneylender's return in the default state. This effect is largest with respect to risky types since they have a higher default probability. Seniority thereby limits the scope for co-funding contracts to safe borrowers. Collateral requirements by the formal lender may circumvent this problem by eroding the debt seniority of the informal lender. Thus, collateral makes it easier for an uninformed lender, i.e. the bank, to screen borrowers, simply because it has a differential effect on the moneylender's incentives to provide co-funding. Since profits are always higher in the co-funding equilibrium, both lenders may actually have an interest in formal sector seniority or, what amounts to the same thing, greater scope for formal sector collateral. The effect on borrowers of increased collateral opportunities is less clear.

The rest of the paper is structured as follows. Section 2 sets out the basic model. In Section 3, the different Nash equilibria in the simultaneous setting are derived and their intuition is explained. In Section 4, a sequential context is analysed and subgame-perfect Nash equilibria are derived. Section 5 analyses comparative statics. In particular, we study changes in the cost of formal funding, changes in risk, and changes in seniority. Section 6 concludes. Proofs are relegated to the Appendix.

2 The Basic Set-up

There are two types of potential borrowers, indexed by $i = a, b$. Each borrower is endowed with a project that requires a fixed investment of size K. Borrowers have no wealth. In order to undertake the project, they have to borrow the amount K. Project outcomes have a good state with a high payoff $\overline{X}_i$, and a failure state with low payoff $\underline{X}$. Type a borrowers are assumed to have a higher success probability than type b, $p_a > p_b > 0$. Moreover, it is assumed that the expected outcome from type a borrowers undertaking a project is higher than for type b, $p_a \left(\overline{X}_a - \underline{X}\right) > p_b \left(\overline{X}_b - \underline{X}\right)$. It is also assumed that $K > \underline{X} > 0$, which implies that failure is associated with default as the borrower cannot meet his obligations at a nonnegative (real) interest rate. The proportion of type a borrowers in the model is given by $\gamma \in (0, 1)$, and the reservation utility, $\overline{U} > 0$, is assumed constant across types.

Borrowers in need of funding for their projects have two sources of funds: A monopolistic formal-sector bank and a monopolistic informal-sector moneylender. Only the moneylender is able to distinguish between the two types of agents, whereas the bank cannot observe the type. Thus, the moneylender is assumed to have an *informational advantage.* In the default state, the information advantage also gives the moneylender first-move in the seizure of $\underline{X}$. That is, the moneylender has *seniority.* The cost of mobilising deposits in the formal sector is c, whereas the cost of mobilising deposits in the informal sector is m. It is assumed that the bank has a *cost advantage* in mobilising deposits *vis-à-vis* the moneylender, $c < m$.[5] Finally, both lenders and borrowers are assumed to be risk neutral.[6]

Let $\bar{r}_i$ be defined as the reservation interest rate of type i which solves $p_i \left[\overline{X}_i - (1+\bar{r}_i)K\right] = \overline{U}$. That is, $\bar{r}_i$ ensures that the expected return to a borrower of type i is equal to his reservation utility when offered the full contract, $(L, r) = (K, \bar{r}_i)$. Hence:

$$\bar{r}_i = \frac{\overline{X}_i - \overline{U}/p_i}{K} - 1, \ i = a, b \tag{1}$$

Moreover, define r_{i0}^B as the interest rate that ensures the bank zero profit from offering the full contract $(L, r) = \left(K, r_{i0}^B\right)$ to type i. I.e., r_{i0}^B solves $p_i(1 + r_{i0}^B)K + (1 - p_i)\underline{X} - (1 + c)\,K = 0$. Defining r_{i0}^M in the same way for the moneylender results in:

$$r_{i0}^B = \frac{(1+c)\,K - (1-p_i)\underline{X}}{p_i K} - 1, \ i = a, b \tag{2}$$

$$r_{i0}^M = \frac{(1+m)\,K - (1-p_i)\underline{X}}{p_i K} - 1, \ i = a, b \tag{3}$$

[5] The difference, $m - c$, might in part be interpreted as the cost of gathering information on behalf of the moneylender. If the moneylender acquires information freely, say through a sharecropping arrangement, another interpretation would be that the scale-effects of the institutional lender outweighs the informational advantage of the moneylender.

[6] Some comments on the assumptions of the model are suitable. First, the neglect of physical collateral in the model is reasonable to the extent that: i) many borrowers do not have any assets that can serve as collateral; ii) many formal lenders do not require the borrower to put up any physical collateral. Second, the neglect of enforcement and moral hazard problems in the model can be justified by the use of dynamic contracts relying on progressive lending and the threat of cutting-off future lending, etc. Third, the debt seniority of the moneylender might be justified by the fact that the moneylender, in contrast to the institutional lender, can use the threat of violence to secure repayment. In addition, as pointed out by Bell (1990), the moneylender often works as a trader buying farmers' crops, and when buying the crops, he can easily exercise first-claim. Bell adds that the moneylender-cum-trader often cooperates with other traders in securing their dues.

Throughout the paper, it is assumed that the following inequalities are satisfied:

$$r_{a0}^{B} < r_{a0}^{M} < \bar{r}_{a} < r_{b0}^{B} < \bar{r}_{b} < r_{b0}^{M}. \tag{4}$$

Several things should be noted: First, $r_{a0}^{B} < r_{a0}^{M}$ and $r_{b0}^{B} < r_{b0}^{M}$ simply reflect the cost advantage of the bank. Second, $\bar{r}_{b} < r_{b0}^{M}$ implies that it is too expensive for the moneylender to offer type b full financing, whereas the bank can earn a profit on type b since $r_{b0}^{B} < \bar{r}_{b}$. Third, since $r_{a0}^{B} < r_{a0}^{M} < \bar{r}_{a}$, both lenders can earn a profit on type a. Fourth, the assumption of $\bar{r}_{a} < \bar{r}_{b}$ implies together with $p_{a}\left(\overline{X}_{a} - \underline{X}\right) > p_{b}\left(\overline{X}_{b} - \underline{X}\right)$ that $\overline{X}_{a}$ must be smaller than $\overline{X}_{b}$. Hence, at an interest rate above $\bar{r}_{a}$, only the risky borrowers will choose to borrow. Finally, $\bar{r}_{a} < r_{b0}^{B}$ implies that at the reservation rate of type a, bank profit from type b is negative.

As a final point, we establish some notation on profits. Specifically, let $\pi^{j}\left[(L_{a}^{B}, r_{a}^{B}), (L_{b}^{B}, r_{b}^{B}), (L_{a}^{M}, r_{a}^{M}), (L_{b}^{M}, r_{b}^{M})\right]$ denote the expected profit to lender j, $j = B, M$, when the bank offers the contracts (L_{a}^{B}, r_{a}^{B}) and (L_{b}^{B}, r_{b}^{B}), and the moneylender offers (L_{a}^{M}, r_{a}^{M}) to type a and (L_{b}^{M}, r_{b}^{M}) to type b. Moreover, use of subscript $i = a, b$ on π_{i}^{j} denotes expected profit to lender j from type i only. When no confusion is possible, some of the arguments in π_{i}^{j} will be suppressed.

2.1 The bank's problem in the absence of an informal sector

In order to set the stage for the subsequent analysis, consider first the bank's problem in the absence of a moneylender. In this case, the timing of events is as follows. First, the bank decides on the menu of contracts to offer, and then the borrowers choose among the contracts offered.

Without an informal sector, co-funding as a screening device is ruled out. Thus, the bank must offer full, if any, financing, and therefore a single contract is sufficient. The optimal choices by the bank are summarised in the following proposition:

Proposition 1 *Without informal lending, the optimal contract offered by the monopolistic bank is either: i) a pooling contract of* $\left(L^{B}, r^{B}\right) = (K, \bar{r}_{a})$*, in which case both types accept the contract; or ii) a separating contract of* $\left(L^{B}, r^{B}\right) = (K, \bar{r}_{b})$*, in which case only type b accepts.*

Proof. See Appendix. ■

According to Proposition 1, the bank chooses to offer full financing at either the reservation rate of type a or at the reservation rate of type b. The first contract is a pooling contract, $(L, r) = (K, \bar{r}_a)$, which both types accept. Under this contract, the bank extracts the entire return from type a but earns a negative profit on type b since $\bar{r}_a < r_{b0}^B$. Hence, risky types earn a rent whereas safe types are pushed to their reservation utility.

The second contract is a separating contract,[7] $(L, r) = (K, \bar{r}_b)$, which only type b accepts since $\bar{r}_a < \bar{r}_b$. Under this contract, all rent is extracted from type b whereas type a is left without financing. In choosing the optimal contract, the bank faces a classical adverse selection (hidden information) problem. By increasing the interest rate from $\bar{r}_a$ to $\bar{r}_b$, it turns the negative earnings on b into a surplus, but at the same time adversely selects only the risky types, thereby loosing profits on the safe types.

2.2 The moneylender's problem in the absence of a formal sector

In the absence of a formal sector, the monopolistic moneylender with full information offers individual and independent contracts to each type, and then the borrowers choose whether or not to accept the contracts offered. Requiring that lenders only offer contracts that yield a nonnegative expected return in case they are accepted, the equilibrium outcome can be summarised as:

Proposition 2 *Without formal lending, a fully informed moneylender offers the contracts: i)* $\left(L_a^M, r_a^M\right) = (K, \bar{r}_a)$ *to type a, which a accepts; and ii) any* $\left(L_b^M, r_b^M\right)$ *to type b that b does not wish to accept.*

Proof. See Appendix. ■

Proposition 2 says that the moneylender offers a contract to type a agents which pushes them to their reservation utility. Since $\bar{r}_b < r_{b0}^M$, the moneylender cannot make a profit on type b. As a consequence, the moneylender chooses to offer any (profitable) contract to b that b does not wish to accept.

[7] In the present paper, use of the term separating contract differs from standard usage of the term, which typically refers to a menu of contracts designed to make borrowers self-select according to their characteristica. In the present paper, this latter type of contract menu is coined a co-funding contract.

3 A Model with Simultaneous Offers

In the model of Section 2.1, the bank can only discriminate between the two types of borrowers by raising the interest rate, in which case it adversely selects the less profitable b types. On the other hand, if the bank wishes to keep the safe a borrowers by offering a lower rate, it cannot prevent the risky borrowers from taking the loan as well. Hence, the bank has an incentive to find other means of screening borrowers. In the presence of fully informed informal lenders, the bank can extract the information on types from the informal sector by requiring borrowers to seek co-financing in this sector. More precisely, Jain (1999) argued that if the risky borrowers find it more difficult (expensive) to obtain co-funding in the informal sector, the bank may seek to target cheaper loans at the safe borrowers by offering less than full financing.[8]

In the present model, the above mechanism is complicated by the fact that the informal sector is not simply a competitive sector where safe and risky types face constant but different interest rates. Instead, the informal sector is comprised of a monopolistic moneylender, who also attempts to maximise expected profits from his lending activity. Hence, the formal and informal sector engage in monopolistic competition about the borrowers. However, since the moneylender is assumed to endure higher lending costs, $m > c$, the total available rent from a given borrower's project is increasing in the share that the bank finances. In conjunction with the debt seniority, this also provides the moneylender with an incentive to opt for a co-funding solution.[9]

In sum, we have a situation with monopolistic competition for borrowers, but where both lenders at the same time have an incentive to opt for co-financing solutions. Technically, the strategic game is modelled as follows: In stage one, the bank and the moneylender simultaneously decide on a menu of contracts to offer.[10] That is, the moneylender offers an individual contract for each type, whereas the bank offers one or two contracts available for both borrowers. In the second stage, the borrowers choose among the contracts offered and undertake their investment. For technical reasons, it is assumed that when borrowers are indifferent between two (sets of) contracts,

[8] Jain was motivated by the question that if borrowers who are rejected by the formal sector seek recourse to informal lenders, then why do the formal institutions not incorporate this knowledge into their lending decision?

[9] Although issues of risk aversion are abstracted from in this model, it should be noted that debt seniority provides the moneylender with a smaller risk. Accordingly, having first-claim on borrowers' return in default states would further strengthen the incentive to opt for a co-funding solution for a risk-averse moneylender. For empirical evidence on co-funding, see Jain (1999) and the references cited therein.

[10] The assumption of simultaneous offers is relaxed in Section 4.

they choose the contract(s) with the highest degree of formal financing. In the third stage, loans are repaid by successful investors, and less successful investors have their income seized. The solution concept used is that of a Nash equilibrium. More precisely, the contracts offered must constitute a Nash equilibrium in the simultaneous game between the bank and the moneylender.[11]

The following proposition characterises the possible Nash equilibria of the simultaneous model:

Proposition 3 *If a Nash equilibrium exists, both types of borrowers will always be financed in equilibrium, and the bank always fully finances type b. Hence, a Nash equilibrium will always belong to one of the following three types:*

1. *A pooling equilibrium where the bank fully finances both types;*
2. *A separating equilibrium where the bank fully finances type b, and where the moneylender fully finances type a.*
3. *A co-funding equilibrium where the bank fully finances type b, and where the bank and the moneylender co-finance type a.*

In addition, a type 1 (pooling) equilibrium and a type 2 (separating) equilibrium cannot co-exist.

Proof. See Appendix. ■

Proposition 3 reveals that the model admits three types of Nash equilibria: a pooling equilibrium in which the bank finances both types; an equilibrium in which the bank finances type b, and the moneylender finances type a; and a co-funding equilibrium where the bank provides full funding of type b, and where type a is co-funded by the bank and the moneylender. In the following subsections, the three Nash equilibria will be characterised in more detail.

3.1 Pooling equilibrium

The following proposition characterises the pooling equilibrium of the simultaneous game:

[11] Furthermore, contracts offered, but not accepted in equilibrium, are required to involve a nonnegative expected profit for the (money)lender in case they are accepted. Clearly, this is just a standard Nash-perfection assumption.

Proposition 4 (Type 1 Nash Equilibrium) *A pooling Nash equilibrium exists if and only if:*

$$\pi^B(K, r_{a0}^M) \geq \pi^B(K, \bar{r}_b). \tag{5}$$

In equilibrium, the bank offers a single contract, $(L^B, r^B) = (K, r_{a0}^M)$*, whereas the moneylender offers the contract* $(L_a^M, r_a^M) = (K, r_{a0}^M)$ *to type a, and any contract* (L_b^M, r_b^M) *to type b such that* $\pi_b^M \left(L_b^M, r_b^M\right) \geq 0$*. In equilibrium, both types choose the contract offered by the bank and earn a strictly positive rent.*

Proof. See Appendix. ■

In the pooling equilibrium, the bank fully finances both types of borrowers. The condition in (5) ensures that the bank prefers the pooling solution to the case where it only lends to the risky types at the interest rate $\bar{r}_b$. The intuition underlying this condition can be illustrated as in Figure 1, where the solid line gives the bank's profit as a function of the interest rate charged by the bank, given that the moneylender offers the contract (K, r_{a0}^M) to type a. The profit is increasing in r^B for rates below r_{a0}^M. If the bank increases r^B above r_{a0}^M, type a borrowers will switch to the moneylender's contract. Hence, profit drops at r_{a0}^M. For rates above $\bar{r}_b$, type b will also choose not to accept the contract offered by the bank, and expected profits to the bank are thus zero. Point A in the Figure represents the profit in a type 1 Nash equilibrium. For the equilibrium strategy to be optimal for the bank, given the contract offered by the moneylender, it must be the case that by increasing r^B to $\bar{r}_b$, i.e. moving to point B, it cannot obtain a higher profit.[12] This is precisely the condition in (5). For the moneylender, any equilibrium strategy which is rejected by borrowers is optimal, since it cannot possibly make a profit when the bank offers full financing at the rate of r_{a0}^M.

[Figure 1 about here]

Note that the interest rate charged by the bank, r_{a0}^M, is lower than in the pooling solution of Proposition 1. The reason is the presence of the moneylender, which requires the bank to charge the competitive rate, r_{a0}^M, in a pooling solution. Otherwise, the moneylender will have an incentive to undercut the bank and lend to type a. This also explains why a pooling equilibrium can only exist at the rate r_{a0}^M, where both types of borrowers earn a positive rent by the assumption in (4).

[12] Recall, that by assumption, the profit in point B is positive since $\bar{r}_b > r_{b0}^B$.

3.2 Separating equilibrium

Proposition 5 characterises the separating equilibrium:

Proposition 5 (Type 2 Nash Equilibrium) *A separating Nash equilibrium exists if and only if:*

$$\pi^B(K, \bar{r}_b) \geq \pi^B(K, \bar{r}_a). \tag{6}$$

In equilibrium, the bank offers one contract, $(L^B, \bar{r}_b)$, whereas the moneylender offers the contract $(L_a^M, r_a^M) = (K, \bar{r}_a)$ to type a, and any contract (L_b^M, r_b^M) to type b such that $\pi_b^M \left(L_b^M, r_b^M\right) \geq 0$. In equilibrium, type a chooses the contract offered by the moneylender, and type b chooses the contract offered by the bank. Both types are pushed to their reservation utilities.

Proof. See Appendix. ■

In the separating equilibrium, the bank fully finances the risky types, and the moneylender finances the safe types. The condition in (6) says that the bank must prefer this to the pooling solution where it fully finances both types at the interest rate $\bar{r}_a$. In Figure 1, this means that profits in point B must exceed profits in point C, where the broken line gives profit to the bank, provided that the moneylender offers the contract $(K, \bar{r}_a)$ to type a. The moneylender, on the other hand, has no attractive alternatives. By offering $(K, \bar{r}_a)$ to type a, he pushes type a to his reservation utility, and the moneylender has no possibilities of making a profit on type b. In this equilibrium, all rent is extracted from both types.

Note the asymmetry caused by the simultaneous game. In a type 1 equilibrium, the relevant alternative for the bank is the profit in a type 2 equilibrium. In a type 2 equilibrium, however, the relevant alternative is not the type 1 equilibrium, but a pooling contract at rate $\bar{r}_a$. At $\bar{r}_a$ the bank can attract both types of borrowers when the moneylender offers $(K, \bar{r}_a)$ to type a. This also explains why a Nash equilibrium might not exist in some situations.

3.3 Co-funding equilibrium

The most involved type of equilibrium in the model is a co-funding equilibrium in which contracts are designed to make types self-select according to their type. The following proposition characterises the co-funding equilibrium:

Proposition 6 (Type 3 Nash Equilibrium) *A co-funding equilibrium exists if and only if the set $\Gamma\left(L_a^B, r_a^B\right)$ given by:*

$$\Gamma\left(L_a^B, r_a^B\right) = \left\{\left(L_a^B, r_a^B\right) \in \mathbb{R}_+^2 : \begin{array}{l} \pi_a^M\left(K - L_a^B, r_a^M\right) \geq \pi_a^M(K, \bar{r}_a), \\ \pi^B\left[\left(L_a^B, r_a^B\right), \left(K, r_b^B\right)\right] \geq \max\left\{\pi^B(K, \bar{r}_b), \pi^B(K, \bar{r}_a)\right\} \end{array}\right\}, \tag{7}$$

where:

$$r_a^M = \frac{\overline{X}_a - \frac{\overline{U}}{p_a} - (1 + r_a^B)L_a^B}{(K - L_a^B)} - 1 \tag{8}$$

$$r_b^B = \min\left\{\bar{r}_b, \frac{r_{b,low}^M\left(K - L_a^B\right) + r_a^B L_a^B}{K}\right\} \tag{9}$$

$$r_{b,low}^M = \max\left\{m, \frac{(1+m)\left(K - L_a^B\right) - (1 - p_b)\underline{X}}{(K - L_a^B)\, p_b} - 1\right\}, \tag{10}$$

is non-empty. A given pair, $(L_a^B, r_a^B) \in \Gamma\left(L_a^B, r_a^B\right)$, then corresponds to a specific type 3 equilibrium where the bank offers the contracts (L_a^B, r_a^B) and $\left(K, r_b^B\right)$ with r_b^B given by (9), and the moneylender offers $\left(K - L_a^B, r_a^M\right)$ to type a, where r_a^M is given by (8), and:

a. If $r_b^B = \bar{r}_b$, the moneylender offers any $\left(L_b^M, r_b^M\right)$ to type b such that $\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$.

b. If $r_b^B < \bar{r}_b$, the moneylender offers $\left(K - L_a^B, r_{b,low}^M\right)$ with $r_{b,low}^M$ given by (10) to type b.

In equilibrium, type b is financed by the bank, and type a is financed both by the bank and the moneylender. In a type 3a equilibrium, both types are pushed to their reservation utilities, whereas in a type 3b equilibrium, type b earns a positive rent.

Proof. See Appendix. ■

In a co-funding equilibrium, the bank fully finances type b, whereas the bank and the moneylender co-finance type a. Proposition 6 gives the set of partial bank contracts $\Gamma(L_a^B, r_a^B)$ which are consistent with a co-funding equilibrium, and it also specifies the associated contracts offered by the bank and the moneylender in a given co-funding equilibrium. The intuition underlying Proposition 6 is the following:

In a co-funding equilibrium, the bank offers (L_a^B, r_a^B) with $L_a^B < K$ to type a, and the moneylender therefore provides $L_a^M = K - L_a^B$ to type a while extracting all remaining rent from type a by setting r_a^M according to (8), i.e. as a function of L_a^B and r_a^B. For this to be optimal for the moneylender, it must be because he cannot get a higher profit from type a by offering him the full contract $(K, \bar{r}_a)$. This is the first constraint on Γ in (7), and the condition is illustrated by the line M_1 in Figure 2. If r_a^B is too high, it leaves less rent to be extracted by the moneylender, and the moneylender will thus prefer to fully finance type a himself.[13]

[Figure 2 about here]

Furthermore, in a co-funding equilibrium, the moneylender must not find it optimal to offer a partial contract to type b, which type b then accepts in connection with the partial contract, $\left(L_a^B, r_a^B\right)$, offered by the bank. Given $\left(L_a^B, r_a^B\right)$, $r_{b,low}^M$ in (10) is therefore defined as the lowest rate that the moneylender can possibly bear on a co-financing loan to type b of size $K - L_a^B$ without incurring negative profits. For a co-funding equilibrium to exist, the bank must offer a rate to type b, r_b^B, which causes b to prefer the full bank loan, even when the moneylender offers $r_{b,low}^M$. This rate is given by (9). If $r_{b,low}^M$ is sufficiently low, the bank is forced to charge an r_b^B below $\bar{r}_b$, which is the case in a type 3b equilibrium. On the other hand, if $r_{b,low}^M$ is high, it poses no threat to the bank, and the bank extracts all rent from type b by setting $\bar{r}_b$ as in a type 3a equilibrium. Thus, $\left(L_a^B, r_a^B\right)$ indirectly determines the rent that the bank can charge type b in a co-funding equilibrium.

Graphically, the upward sloping line M_2 in Figure 2 gives those combinations of L_a^B and r_a^B where the two arguments in (9) are equal. To the left of this line, the second argument in (9) is highest. The bank can therefore set r_b^B equal to $\bar{r}_b$, whereas to the right, the second argument is smaller so the bank must reduce r_b^B accordingly. Intuitively, the moneylender will find it less profitable to co-finance type b if either the bank share, L_a^B, is low or the rate charged by the bank, r_a^B, is high, in which case there is less rent left for the moneylender to extract. Hence, type 3a equilibria are to be found to the left of this line, and type 3b to the right.

Finally, for a co-funding equilibrium to exist, it must be associated with a higher expected profit to the bank than offering either a single pooling contract $(K, \bar{r}_a)$ or a single separating contract $(K, \bar{r}_b)$ to type b. That is, the

[13] Note that as L_a^B gets high, the maximum rate r_a^B by the bank that the moneylender will accept in a co-funding equilibrium becomes lower. This is because of seniority. As L_a^B is increased, the bank gets a higher share in the bad state outcome. This leaves less rent for the moneylender at a given r_a^B.

bank must prefer the co-funding equilibrium to both the (type 2) separating equilibrium and the improved (type 1) pooling equilibrium. These two contracts are outside options of the bank, and they give rise to the constraint(s) in the bottom line of (7).

In Figure 2, the two constraints are illustrated by the lines B_1 and B_2, respectively. If $\left(L_a^B, r_a^B\right)$ lies below the B_2 line, the bank prefers the separating solution at $\bar{r}_b$ to a co-funding solution involving $\left(L_a^B, r_a^B\right)$. Intuitively, this happens when r_a^B is small, in which case the bank does not earn 'much' on a in a co-funding equilibrium. Note that the slope of the B_2 line increases significantly after crossing the upward sloping M_2 line. The reason is that to the right of this line, only type 3b equilibria can exist where the bank earns less on type b than in a type 2 equilibrium. Hence, for the bank to prefer a type 3b to a type 2 equilibrium, its earnings on type a must be correspondingly higher, i.e. r_a^B must be higher. Similarly, if $\left(L_a^B, r_a^B\right)$ lies below the B_1 line, the bank prefers the pooling solution at $\bar{r}_a$. This also happens if r_a^B is too low in the co-funding situation. In Figure 2, it also happens if L_a^B is too low, because then profits from type a will also be small in a co-funding equilibrium compared to a pooling equilibrium.[14]

In sum, in Figure 2, the area 3a defines the possible type 3a equilibria, whereas area 3b defines type 3b equilibria. Note that the bank share, L_a^B, in a type 3b equilibrium is typically larger. By providing type b with a positive rent, it gives the bank some additional leverage which enables it to finance a larger share of type a. Since the bank has a cost advantage, $c < m$, this increases the bank's profit on type a. Hence, in some circumstances it will prove optimal for the bank to give type b some rent in order to earn more on type a. Note also that in a type 3a equilibrium, type b's participation constraint is binding, whereas in a type 3b equilibrium, type b is indifferent between taking the full loan offered by the bank and taking the partial bank loan intended for type a in combination with co-funding from the moneylender, i.e. type b's incentive constraint is binding.

Proposition 6 allows two corollaries stated below:

Corollary 7 *Whenever a type 3 Nash equilibrium exists, it has a higher equilibrium payoff to both the bank and the moneylender than any co-existing Nash equilibrium of type 1 or 2.*

This follows because both the bank and the moneylender have their profits in type 1 and 2 equilibria as outside options in a type 3 equilibrium. Hence, if a co-funding equilibrium exists both lenders will prefer this.

[14]Note, the relative position of the B_1 and B_2 lines depends on the relative profitability of type 1 and 2 equilibria.

Corollary 8 *Existence of a separating (type 2) Nash equilibrium implies existence of a co-funding (type 3) Nash equilibrium.*

The intuition behind Corollary 8 is straightforward: Since $c < m$, we can always increase total rent, compared to the situation in a type 2 equilibrium, by letting the bank co-finance a small share, ε, of type a. ε should just be small enough to deter the moneylender from providing type b with a partial loan, $L_b^M = K - \varepsilon$. Since $\overline{r}_b < r_{b0}^M$, such an ε can always be found. By splitting the rent between the bank and the moneylender, a co-funding equilibrium is easily constructed.

4 Sequential Offers

As shown above, the simultaneous game has several equilibria and sometimes no equilibrium at all. This indeterminacy can be partly overcome if one is willing to impose a sequential structure on the game, and Corollary 7 and 8 provide some indication as to which type of equilibrium will prevail.

Consider the following sequential structure: In the first stage, the lender with the first move offers a contract, which the other lender observes. In the second stage, the second lender makes an offer in response to the first-stage offer; and in the third stage, borrowers decide among the contracts offered and undertake their investments. In the final stage, loans are repaid by successful investors and defaulters have their income seized.[15]

4.1 Bank makes the first offer

First, consider the extensive game in which the bank makes the first offer. In this case, the bank can always offer full financing at either the rate r_{a0}^M, in which case it gets the same profit as in a type 1 equilibrium in the simultaneous game, or the rate $\bar{r}_b$, in which case it gets a profit as in the type 2 simultaneous equilibrium. At any rate $r^B \in (r_{a0}^M, \bar{r}_a]$, the moneylender will choose to undercut the bank with respect to type a borrowers, and the bank can therefore earn a higher profit by charging $\bar{r}_b$.

However, the bank might prefer to opt for a co-funding solution. Define the set of potential co-funding solutions in this sequential game as those contracts where the moneylender has no incentive to deviate, and which the

[15] As in Section 3, it is assumed that when borrowers are indifferent between two (sets of) contracts, they choose the contract(s) with the highest degree of formal financing. In addition, since this is a extensive game with perfect information, the equilibrium concept used is the subgame-perfect Nash equilibrium.

bank prefers to its two outside options above. This set is equivalent to the set Γ depicted in Figure 2, with the exception that the line B_1 has been shifted downward. In the simultaneous game, a pooling contract at the rate $\bar{r}_a$ was the relevant outside option; in the sequential game, the relevant outside option is a pooling contract at $r_{a0}^M < \bar{r}_a$. Whenever this extended set is non-empty, the bank will choose a co-funding contract in the sequential game. Since the set Γ is extended compared to the simultaneous game, this means that a co-funding solution will exist for a larger set of parameter values in this sequential game. Specifically, whenever $\pi^B(K, \bar{r}_b) \geq \pi^B(K, r_{a0}^M)$, the extended set will be non-empty, and so a co-funding solution will prevail. This leaves only two potential equilibria in this sequential game:

Proposition 9 *In the sequential game where the bank makes the first offer, the subgame-perfect Nash equilibrium is either: i) a co-funding equilibrium; or ii) a pooling equilibrium where the bank charges* r_{a0}^M.

More precisely, in a co-funding solution, the bank will choose a contract on the bold line in Figure 2. The bank will either choose point A on this line, where it extracts all rent from type b and offers the highest interest rate and loan size to type a consistent with a co-funding equilibrium of type 3a, or it will choose a point on the bold line to the right of A, where it extracts less rent from type b but earns more from type a than at point A.

4.2 Moneylender makes the first offer

In a sequential game where the moneylender makes the first offer, the resulting equilibrium is either a co-funding equilibrium, or a separating equilibrium where the moneylender finances type a, and the bank finances type b.

Consider first the co-funding equilibrium, and let $\underline{L}$ denote the smallest feasible loan that the moneylender can offer. In any co-funding equilibrium, the bank must earn at least $\max\{\pi^B(K, \bar{r}_a), \pi^B(K, \bar{r}_b)\}$ in order not to deviate. Due to the cost advantage of the bank, the maximum co-funding investment rent is generated when the bank fully finances type b and the share $K - \underline{L}$ of type a. The maximum rent is then:

$$R^{\max} = \pi_a^B(K - \underline{L}, \bar{r}_a) + \pi_b^B(K, \bar{r}_b) + \pi_a^M(\underline{L}, \bar{r}_a).$$

If the moneylender chooses a co-funding contract, it therefore offers a loan of size $\underline{L}$ to type b at an interest rate which leaves the bank with a residual profit exactly equal to $\max\{\pi^B(K, \bar{r}_a), \pi_b^B(K, \bar{r}_b)\}$. That is, the profit to the moneylender equals:

$$\pi_a^M(\underline{L}, r_a^{M*}) = R^{\max} - \max\{\pi^B(K, \bar{r}_a), \pi^B(K, \bar{r}_b)\}. \tag{11}$$

Clearly, the lower is $\underline{L}$, the higher is the co-funding interest rate, r_a^{M*}, charged by the moneylender.

In some circumstances, however, the moneylender will not find it optimal to offer a co-funding contract. Specifically, when $\pi^B(K, \bar{r}_a) > \pi^B(K, \bar{r}_b) > \pi^B(K, r_{a0}^M)$, the moneylender is able to confine the bank's payoff to $\pi^B(K, \bar{r}_b)$. This is done by offering a full contract $\left(K, r_a^{M**}\right)$ to type a, where r_a^{M**} is defined implicitly by $\pi^B\left(K, r_a^{M**}\right) = \pi_b^B(K, \bar{r}_b)$. Given this contract by the moneylender, the bank offers the full contract $(K, \bar{r}_b)$, which only type b accepts.

This can only be optimal for the moneylender in situations where we have $\pi^B(K, \bar{r}_a) > \pi^B(K, \bar{r}_b) > \pi^B(K, r_{a0}^M)$.[16] In the co-funding solution the bank takes $\pi^B(K, \bar{r}_a)$, whereas under $\left(K, r_a^{M**}\right)$, it gets $\pi^B(K, \bar{r}_b)$. Furthermore, offering $\left(K, r_a^{M**}\right)$ implies that total rent is lower than in a co-funding solution (due to the cost disadvantage of the moneylender), and that type a borrowers get a share of the rent. Hence, $\pi^B(K, \bar{r}_a)$ must be somewhat larger than $\pi^B(K, \bar{r}_b)$ for $\left(K, r_a^{M**}\right)$ to be the optimal contract by the moneylender.

Proposition 10 *In the above extensive game, where the moneylender makes the first offer, the subgame-perfect Nash equilibrium is either: i) a co-funding equilibrium, where the moneylender finances type a with the smallest feasible loan,* $\underline{L}$*, at the interest rate* r_a^{M*}*, defined implicitly by (11); or ii) a separating equilibrium, where the moneylender fully finances type a at* r_a^{M**}*, defined implicitly by* $\pi^B\left(K, r_a^{M**}\right) = \pi_b^B(K, \bar{r}_b)$*, and the bank fully finances type b at* $\bar{r}_b$*. In the latter equilibrium, type a borrowers earn a positive rent.*

Thus, in a sequential game, where the moneylender has the first-offer advantage, the more 'typical' equilibrium is a co-funding equilibrium where the moneylender co-finances the safe types with a very small loan at a very high interest rate. Since the bank has a cost advantage, the moneylender has an interest in offering only very small loans as total rent is decreasing in the moneylender's co-funding share. Moreover, in order for the moneylender to transmit his information on types to the bank, i.e. screen borrowers, a small loan is sufficient. As such, the model is consistent with the stylized facts mentioned in Section 1.

5 Comparative Statics

In the previous Sections, the model has been outlined and the equilibria under the different assumptions about the strategic interaction have been identified.

[16] That is, in situations where neither a type 1 nor a type 2 equilibrium exist in the simultaneous game.

In this Section, we elaborate and provide results from comparative statics.

To begin, recall that the model gave rise to three types of equilibria: a pooling equilibrium where the moneylender is crowded out by the bank; a separating equilibrium where the bank fully finances type b, and where the moneylender fully finances type a; and a co-funding equilibrium where the bank fully finances type b, and where the two lenders co-finance type a.

In the model, co-funding equilibria was found to more likely in sequential games than in the simultaneous one, because: i) co-funding is more profitable to both lenders, so it will be chosen in sequential games when it exists in the simultaneous game; ii) when the moneylender has a first-mover advantage, his incentive to co-finance type b is eliminated, since he can credibly commit to not offering type b a contract; and iii) when the bank is first mover, its incentive to switch to a pooling contract at $\bar{r}_a$ is eliminated.

In the simultaneous game, the bank always earns a profit, whereas the moneylender does not earn a profit in the pooling equilibrium. With respect to borrowers, both types get a rent in the pooling solution. In addition, type b can get a rent in a type 3b equilibrium. In the sequential game, where the bank is first mover, payoffs are similar with the bank getting the highest possible payoff in co-funding solutions. When the moneylender moves first, the bank is pushed to its reservation payoff in the co-funding equilibrium. Contrary to the other games, only type a agents can get a rent in this game.

It follows that the structure of the strategic interaction is very important for the division of payoffs, and for the type of equilibria that prevail.[17] As an example, there may be parameter values which do not allow a Nash equilibrium in the simultaneous game, but only in sequential games. This would be even more important in situations where loans to type b agents are socially inoptimal, i.e. if $r_{b0}^{B} > \bar{r}_b$. In this case, a type 2 equilibrium will not exist, and type 1 equilibria are associated with socially inoptimal financing of type b agents. Hence, society would have an interest in changing the structure of the game to a sequential one, where a co-funding equilibrium is more easily sustainable. This would also be in the interest of both lenders.

5.1 Lower formal costs

Analysing what happens when the bank's mobilisation costs, c, fall may yield some interesting insights into the consequences of a government subsidy to the formal sector; a policy that has been pursued in many developing countries in connection with the forming of microfinance institutions and development banks.

[17] To be sure, this is a very common feature of game-theoretic models (see Hellwig, 1987).

Figure 3 shows an example of the existence of different equilibria and associated profits to the bank as a function of c in the simultaneous game. For given equilibrium strategies, a fall in c increases bank profits. Since the effect of a fall in c depends on the amount which the bank lends, the effect is largest (i.e. the profit line is steepest) in a type 1 (pooling) equilibrium and smallest in a type 2 (separating) equilibrium. Figure 3 also shows that a fall in c will eventually cause a co-funding equilibrium to break down. To the left of point A, the bank's ex-post incentive to deviate from a co-funding equilibrium by offering a pooling contract at $\bar{r}_a$ becomes too attractive.[18] Hence, the only prevailing equilibrium to the left of A is a type 1 equilibrium at the rate r_{a0}^M. This implies a discrete drop in bank profits as the economy shifts from a co-funding to a pooling equilibrium.

[Figure 3 about here]

In the sequential game, on the other hand, the bank is able to maintain the co-funding solution for lower c, because it can credibly commit to the co-funding strategy. Hence, the negative effect on profits is avoided.

What about the moneylender? As the economy moves from a type 3 to a type 1 equilibrium, the moneylender is crowded out. Hence, his profits must be reduced. Borrowers, however, might gain from this, since in a type 1 equilibrium, they are offered the cheapest loans.

In sum, subsidies to formal lending institutions may decrease their viability in settings characterised by monopolistic competition between a formal and an informal lender, simply because it worsens the bank's credibility in committing to a co-funding outcome.

5.2 The role of risk

In this section, we investigate how the relative size of $\underline{X}$ and $\overline{X}_i$ matter for the outcome of the (simultaneous) model.[19] That is, we analyse the effect of increasing $\underline{X}$, while keeping expected returns, $p_i\overline{X}_i + (1 - p_i)\underline{X}$, constant.[20] One might expect that a higher value of $\underline{X}$ would make it more attractive for the moneylender to co-finance borrowers at a given loan $\left(L_a^B, r_a^B\right)$ since it increases the moneylender's bad state return due to his seniority. While

[18] In terms of Figure 2, the B_1 line shifts upwards as c is decreased.

[19] Note that $\underline{X}$ can be interpreted as the return on the investment in the bad state, or more broadly, as the (liquid) value of the borrower in the bad state. As a consequence, the moneylender's right to $\underline{X}$ can be interpreted as a first-mover advantage in seizing the returns, or as a claim on collateral put forward by the borrower.

[20] This corresponds to a mean-preserving reduction in variances.

this is true if the moneylender is risk averse, there is no such effect when the moneylender is risk neutral. All that matters to the moneylender, when deciding on whether to co-finance type i, given the partial loan (L_a^B, r_a^B) by the bank, is the residual rent left to extract. The residual rent is given as total rent less the profit extracted by the bank. The total rent at a bank loan of size L_a^B is:

$$R_i\left(L_a^B\right) = p_i\overline{X}_i + (1-p_i)\underline{X} - \bar{U} - (1+c)L_a^B - (1+m)\left(K - L_a^B\right),$$

which clearly is unchanged when $p_i\overline{X}_i + (1-p_i)\underline{X}$ is kept constant. The profit to the bank depends only on $\left(L_a^B, r_a^B\right)$, as long as the bank gets no return in the bad state, and is therefore also unchanged. Thus, residual rent is constant, and the moneylender's co-funding incentives therefore unaffected.[21]

However, an increase in $\underline{X}$, while keeping expected returns constant, raises the incentive for the bank to opt for a pooling solution. This happens because the earnings from type b at a given interest rate become higher.[22] In other words, there is less adverse selection. Thus, changes in $\underline{X}$ do affect the equilibrium outcome by making a co-financing solution harder to sustain, so co-funding outcomes require the presence of some risk.[23] If risk aversion was a concern for the moneylender, there would be an additional effect through this channel.

5.3 Seniority

What would happen if the bank had seniority in the bad state such that the bank could seize $\underline{X}$? Recall that seniority by the bank can be interpreted as collateral in connection with the formal loans.

[21] In the case where $\underline{X}$ exceeds the outstanding debt to the moneylender, further increases in $\underline{X}$ would actually decrease the residual rent at a given $\left(L_a^B, r_a^B\right)$, since the moneylender would get no more in the bad state, but less in the good state. In sum, a mean-preserving increase in $\underline{X}$ can only make it less attractive for the moneylender to co-finance type i at a given $\left(L_a^B, r_a^B\right)$.

[22] Actually, earnings become higher from both agents, but they increase more from type b. Technically, $\bar{r}_b$ drops more than $\bar{r}_a$ and r_{a0}^M when $\underline{X}$ is increased.

[23] The effects in the sequential model where the bank moves first are similar, with the pooling solution at r_{a0}^M becoming more attractive to the bank. In the model where the moneylender has the first move, $\pi^B(K, \bar{r}_a)$ is increased which decreases the rent left for the moneylender. In some situations, this might cause a shift to the equilibrium where the moneylender fully finances type a, i.e. co-funding becomes suboptimal. The reason is that the moneylender has to give up too much rent to the bank in a co-funding equilibrium, because the bank's outside option, the pooling solution, becomes more attractive. If $\underline{X}$ was just increased, the total rent of both types would increase, but most for type b. Hence, in this case, a given co-funding contract, $\left(L_a^B, r_a^B\right)$, would be harder to sustain.

Note first that a change in seniority will not affect profits on full loans since seniority is not an issue here. That is, it will not affect profits in type 1 and 2 equilibria. Consider instead what happens at a given partial loan, $\left(L_a^B, r_a^B\right)$. Without seniority, residual rent to the moneylender is:

$$R_i\left(L_a^B\right) - p_i\left(1 + r_a^B\right) L_a^B,$$

if $\underline{X}$ can be seized completely by the moneylender. Changing to a situation where the bank has seniority, the residual rent at the same loan size changes to:

$$R_i\left(L_a^B\right) - p_i\left(1 + r_a^B\right) L_a^B - (1 - p_i)\underline{X},$$

provided that $\left(1 + r_a^B\right) L_a^B \geq \underline{X}$. Hence, a change in seniority decreases the residual rent to be extracted by the moneylender. More importantly, it decreases the residual rent more in connection with the b types. It thus makes it less attractive for the moneylender to co-finance type a, but it makes it even less attractive to co-finance type b. This increases the scope for co-funding solutions. In terms of Figure 2, the horizontal lines are shifted downwards, and the upward sloping lines are shifted rightward. That is, co-funding solutions involving higher loans by banks become feasible. On the other hand, the rate r_a^B must be lower since the change in seniority makes full financing of type a more attractive to the moneylender (the M_1 line shifts downwards). The higher formal share implies higher total rent and therefore a possibility for higher profits to both lenders.

In other word, the model emphasizes an alternative reason as to why formal institutions require collateral. Collateral makes it easier for an uninformed lender to screen borrowers, because it has a differential effect on the possibilities for borrowers to obtain the needed co-financing. Furthermore, since co-financing is typically associated with higher profits to both lenders, they might both find it worthwhile to give the formal lender seniority.

6 Concluding Remarks

In this paper, we have studied the strategic interaction in underdeveloped credit markets between an informal moneylender and an institutional formal lender when contracts are non-exclusive. It was shown that under very general conditions, a co-funding equilibrium would be the outcome of the game, where the game could be either a simultaneous- or a sequential-offer game. In a sequential setting, the model provided an alternative explanation as to why moneylenders in underdeveloped credit markets generally offer small loans at very high rates of interest to relatively safe borrowers. In addition, the model yielded insights into the role of collateral, and into the relationship between risk and co-funding. Finally, it was shown that a government subsidy to the formal financial sector may lower both lenders' profit, and, as such, it could possibly lower both viability and outreach.

A Proofs of Propositions

This Appendix contains proofs of the Propositions in the text.

Proof of Proposition 1. First, $r_{b0}^B < \bar{r}_b$ implies that $\pi^B(K, \bar{r}_b) > 0$. Hence, offering no contract is never optimal.

Second, since $\bar{U} > 0$, borrowers will never accept a contract with $L^B < K$. Similarly, raising L^B above K implies a higher cost, cL^B, to the bank without increasing the total rent to be extracted from borrowers. Hence, the bank can always do at least as well by offering only $L^B = K$.

Third, given $L^B = K$, the expected profit to the bank, as a function of r^B, can be expressed as:

$$\pi^B\left(r^B\right) = \begin{cases} 0, & r^B > \bar{r}_b \\ (1-\gamma)\left[p_b\left(1+r^B\right)K + (1-p_b)\underline{X} - (1+c)K\right], & \bar{r}_a < r^B \leq \bar{r}_b \\ \gamma\left[p_a\left(1+r^B\right)K + (1-p_a)\underline{X} - (1+c)K\right] + & \\ (1-\gamma)\left[p_b\left(1+r^B\right)K + (1-p_b)\underline{X} - (1+c)K\right], & r^B \leq \bar{r}_a \end{cases} \tag{12}$$

It follows from (12) that $\pi^B\left(r^B\right)$ is increasing in r^B for $r^B \leq \bar{r}_a$ and for $r^B \in (\bar{r}_a, \bar{r}_b]$, but might drop discontinuously at $r^B = \bar{r}_a$. Hence, only $\bar{r}_a$ and $\bar{r}_b$ can be candidates for a maximum. At $\bar{r}_a$, both types accept the contract, and at $\bar{r}_b$, only type b accepts. ■

Proof of Proposition 2. First, by assumption, the moneylender can only offer contracts which are profitable to him if accepted by the borrower. Hence, $\left(L_b^M, r_b^M\right)$ must satisfy $\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$ and involves $L_b^M \geq K$. Since $\bar{r}_b < r_{b0}^M$, such a contract will never be accepted by type b.

Second, $r_{a0}^M < \bar{r}_a$ ensures that the moneylender can earn a positive profit on a. As in the proof of Proposition 1, L_a^M should then equal K, and the profit to the moneylender from type a as a function of r_a^M and given $L_a^M = K$ is:

$$\pi_a^M\left(r_a^M\right) = \begin{cases} 0, & r_a^M > \bar{r}_a \\ \gamma\left[p_b\left(1+r_a^M\right)K + (1-p_b)\underline{X} - (1+m)K\right], & r_a^M \leq \bar{r}_a \end{cases}$$

which is increasing in r_a^M for $r_a^M \leq \bar{r}_a$. Thus, $r_a^M = \bar{r}_a$ is the profit maximizing interest rate. At $\bar{r}_a$, type a accepts the contract. ■

Proof of Proposition 3. First, since $r_{a0}^B < r_{a0}^M < \bar{r}_a$, type a will always be financed in equilibrium. Suppose not, then the moneylender could always increase its profit from type a by offering the contract $\left(L_a^M, r_a^M\right) = (K, \bar{r}_a)$ to type a, which a would accept. Note also, that since $\bar{r}_a < \bar{r}_b$, the bank can never fully finance type a without also fully financing type b.

Second, the bank will always fully finance type b in equilibrium. Suppose not, then either: i) b is not financed at all; or ii) b is only partially financed by the bank in equilibrium. In case of i), since $r_{b0}^B < \bar{r}_b$, the bank could increase its profit by offering the contract $(K, \bar{r}_b)$ which type b, and only type b, would accept. In case of ii), the moneylender finances a share of b's project, and since $c < m$, overall mobilisation costs must be higher than when the bank finances b alone. This implies that the total surplus from b's project to be divided between the moneylender, the bank, and the borrower will be smaller. Since, the moneylender must earn a non-negative profit on b in equilibrium, i.e. he takes a non-negative share of the surplus, the bank could get a higher profit from b by offering a full contract to b which gives b the same rent as under the co-funding contract. Hence, in a Nash equilibrium, b will always be fully financed by the bank.

Third, type 1 and type 2 equilibria are mutually exclusive. In a type 1 equilibrium, the bank fully finances both types. Hence, it must charge a rate which the moneylender has no incentive to undercut, i.e. r_{a0}^M. In a type 2 equilibrium, the bank only finances type 2 and hence charges the highest interest rate consistent with not loosing type b clients, $\bar{r}_b$. Now, in a type 1 equilibrium, the bank always has the option to switch to the contract $(K, \bar{r}_b)$ which gives the same profit as in a type 2 equilibrium. Similarly, in a type 2 equilibrium, the bank can always switch to the contract $\left(K, r_{a0}^M\right)$ which gives the same profit as in a type 1 equilibrium. Thus, if a type 2 equilibrium exists, type 1 can not exist, and vice versa. ■

Proof of Proposition 4. It must be shown that: 1) the contracts: $(L^B, r^B) = (K, r_{a0}^M)$, $(L_a^M, r_a^M) = (K, r_{a0}^M)$, and (L_b^M, r_b^M), where $\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$, constitute a Nash equilibrium if and only if:

$$\pi^B(K, r_{a0}^M) \geq \pi^B(K, \bar{r}_b) \tag{13}$$

holds; and 2) that no other pooling equilibrium exists.

Ad. 1: First, given the contract $(L^B, r^B) = (K, r_{a0}^M)$ by the bank, the moneylender can never earn profits in the market. Hence, $(L_a^M, r_a^M) =$

(K, r_{a0}^M) and (L_b^M, r_b^M) with $\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$ are automatically best responses by the moneylender.

Second, given $(L_a^M, r_a^M) = (K, r_{a0}^M)$ and (L_b^M, r_b^M) by the moneylender, $(L^B, r^B) = (K, r_{a0}^M)$ is a best response by the bank if and only if the profit associated with this contract, $\pi^B(K, r_{a0}^M)$, is positive and greater than the profit associated with: i) offering a full contract at another interest rate; ii) offering only a partial contract; and iii) offering a full contract and a partial contract. Obviously, with only two types of agents, offering more than two contracts can never increase profits to the bank, and if it offers two contracts, at least one of them should be partial.

Ad. i). The best alternative interest rate is $\bar{r}_b$. Raising the rate above r_{a0}^M means that type a borrowers shift to the moneylender, and profits from type b are maximised at $\bar{r}_b$, cf. Proposition 1. Hence, the relevant condition becomes exactly (13). Since $\bar{r}_b > r_{b0}^B$, this ensures that profits to the bank are positive in equilibrium.

Ad. ii). If the bank is to offer only a partial contract, it must be aimed at co-funding type b together with the loan from the moneylender, (L_b^M, r_b^M). However, since total surplus from b's project becomes smaller with co-financing (due to $m > c$), and since the moneylender takes part of the surplus ($\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$), the bank will necessarily get a smaller profit than by fully financing b himself at the rate $\bar{r}_b$. Hence, this strategy is dominated by the strategy in i).

Ad. iii). If offering a full and a partial contract should be better than i) and ii), the interest rate on the full contract must be less than or equal to r_{a0}^M. This implies that for the partial contract to be accepted, it should leave b with a higher rent than under (K, r_{a0}^M). The total surplus from b's project becomes smaller with co-financing, and the moneylender takes part of it, so the profit to the bank will be smaller than under one pooling contract at r_{a0}^M.

In sum, the contracts $(L^B, r^B) = (K, r_{a0}^M)$, $(L_a^M, r_a^M) = (K, r_{a0}^M)$, and (L_b^M, r_b^M), where $\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$, constitute a Nash equilibrium if and only if (13) holds. Since $r_{a0}^M < \bar{r}_a < \bar{r}_b$, both types of borrowers earn a strictly positive rent.

Ad. 2: A pooling situation where the bank charges an interest rate above r_{a0}^M can never constitute an equilibrium, since the moneylender will have an incentive to offer a cheaper loan to type a. Similarly, if the moneylender does not offer $(L_a^M, r_a^M) = (K, r_{a0}^M)$ to type a, the bank will have to raise the interest rate charged. Hence, a pooling equilibrium must involve the strategies $(L^B, r^B) = (K, r_{a0}^M)$, $(L_a^M, r_a^M) = (K, r_{a0}^M)$, and (L_b^M, r_b^M) where $\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$. ■

Proof of Proposition 5. It must be shown that: 1) the contracts:

$(L^B, r^B) = (K, \bar{r}_b)$, $(L_a^M, r_a^M) = (K, \bar{r}_a)$, and (L_b^M, r_b^M) where $\pi_b^M \left(L_b^M, r_b^M\right) \geq 0$ constitute a Nash equilibrium if and only if:

$$\pi^B(K, \bar{r}_b) \geq \pi^B(K, \bar{r}_a) \tag{14}$$

holds; and 2) that no other separating equilibrium exists.

Ad. 1: First, consider the moneylender. Given, the full contract by the bank, he cannot possibly earn a profit on type b since $\bar{r}_b < r_{b0}^M$, whereas he is extracting maximum profit from type a by pushing him to his reservation rate, $\bar{r}_a$. Hence, $(L_a^M, r_a^M) = (K, \bar{r}_a)$ and (L_b^M, r_b^M) with $\pi_b^M \left(L_b^M, r_b^M\right) \geq 0$ are best responses by the moneylender.

Second, as in the proof for Proposition 4, given $(L_a^M, r_a^M) = (K, \bar{r}_a)$ and (L_b^M, r_b^M) by the moneylender, $(L^B, r^B) = (K, \bar{r}_b)$ is a best response by the bank if and only if the profit associated with this contract, $\pi^B(K, r_b^M)$, is positive and greater than the profit associated with: i) offering a full contract at another interest rate; ii) offering only a partial contract; and iii) offering a full contract and a partial contract.

Ad. i). The best alternative interest rate is $\bar{r}_a$ where the bank attracts both types of borrowers. Hence, for $(K, \bar{r}_b)$ to be optimal, it must be the case that (14) holds.

Ad. ii). This strategy is dominated by $(K, \bar{r}_b)$ by the same argument as in the proof of Proposition 4.

Ad. iii). This strategy is dominated by the strategy in i), again by an argument analogous to the one used under Proposition 4.

In sum, the contracts $(L^B, r^B) = (K, \bar{r}_b)$, $(L_a^M, r_a^M) = (K, \bar{r}_a)$, and (L_b^M, r_b^M) where $\pi_b^M \left(L_b^M, r_b^M\right) \geq 0$ constitute a Nash equilibrium if and only if (14) holds. It follows that both types of borrowers are pushed to their reservation utilities.

Ad. 2: A separating situation where the bank charges an interest rate below or above $\bar{r}_b$ can never be optimal since profits from type b are maximised at $\bar{r}_b$. Similarly, the moneylender must charge $\bar{r}_a$ to type a to maximise profits from type a. ■

Proof of Proposition 6. In a co-funding equilibrium, the equilibrium contracts, $\left(L_a^B, r_a^B\right)$, (L_b^B, r_b^B), (L_a^M, r_a^M), and (L_b^M, r_b^M), must by definition satisfy: $L_a^B < K$, $L_b^B = K$, and $L_a^M \geq K - L_a^B$. The proof will then proceed by deriving the necessary and sufficient conditions for these contracts to constitute a co-funding equilibrium.

Consider the moneylender: Given $\left(L_a^B, r_a^B\right)$ with $L_a^B < K$ and (L_b^B, r_b^B) with $L_b^B = K$, the optimal contract to offer to type a is either: i) a full contract at $\bar{r}_a$, which yields positive profit by assumption; or ii) a co-funding contract of size $K - L_a^B$, in which case the optimal interest rate, r_a^M, is set so as to extract all rent from type a given $\left(L_a^B, r_a^B\right)$, i.e.:

$$p_a\left[\overline{X}_a - (1+r_a^B)L_a^B - (1+r_a^M)\left(K - L_a^B\right)\right] = \overline{U} \Leftrightarrow$$
$$r_a^M = \frac{\overline{X}_a - \frac{\overline{U}}{p_a} - (1+r_a^B)L_a^B}{(K - L_a^B)} - 1, \tag{15}$$

which is identical to the expression in (8). Note that type a will have no incentive to switch to the contract (L_b^B, r_b^B) in this case since r_b^B must exceed r_{b0}^B (and hence $\bar{r}_a$) in equilibrium. Now, the moneylender prefers the second contract if and only if it yields higher expected profit:

$$\pi_a^M\left(K - L_a^B, r_a^M\right) \geq \pi_a^M\left(K, \bar{r}_a\right), \tag{16}$$

with r_a^M given by (15). (16) is simply the first condition on the set Γ from Proposition 6.

With respect to type b, the moneylender can never earn a positive profit on a full contract. Given $L_a^B < K$ and $L_b^B = K$, the only potentially viable strategy is to offer a partial contract of size $L_b^M = K - L_a^B$. In this case, the lowest profitable interest the moneylender can offer, $r_{b,low}^M$, satisfies:

$$p_b(1+r_{b,low}^M)\left(K - L_a^B\right) + (1-p_b)\min\left\{(1+r_{b,low}^M)\left(K - L_a^B\right), \underline{X}\right\}$$
$$- (1+m)\left(K - L_a^B\right) = 0,$$

which gives:

$$r_{b,low}^M = \max\left\{m, \frac{(1+m)\left(K - L_a^B\right) - (1-p_b)\underline{X}}{(K - L_a^B)\,p_b} - 1\right\}, \tag{17}$$

which is identical to the expression in (10).

In a co-funding equilibrium, the moneylender must not have an incentive to offer a partial contract to type b. Hence, in equilibrium it must hold that type b has no incentive to accept a contract of the form $(L_b^M, r_b^M) = (K - L_a^B, r_{b,low}^M)$ given the contracts offered by the bank. Given $\left(L_a^B, r_a^B\right)$ and $L_b^B = K$, this places a condition on r_b^B:

$$p_b\left[\overline{X}_b - (1+r_b^B)K\right] =$$
$$\max\left\{\overline{U}, p_b\left[\overline{X}_b - (1+r_a^B)L_a^B - (1+r_{b,low}^M)\left(K - L_a^B\right)\right]\right\}$$

which gives:

$$r_b^B = \min\left\{\bar{r}_b, \frac{r_{b,low}^M\left(K - L_a^B\right) + r_a^B L_a^B}{K}\right\} \tag{18}$$

where $\bar{r}_b = \left(\overline{X}_b - \overline{U}/p_b\right)/K - 1$. This is exactly (9) from Proposition 6. Given $\left(L_a^B, r_a^B\right)$ and (9), the moneylender cannot possibly earn a strictly positive profit on type b in equilibrium, and is therefore willing to offer any potentially profitable contract (L_b^M, r_b^M) in equilibrium

Now, for the bank to find the co-funding equilibrium optimal, it must be that it yields higher profits than any of its feasible alternative strategies: i) a separating equilibrium at $\bar{r}_b$; ii) a pooling equilibrium at $\bar{r}_a$; iii) a co-funding strategy with alternative interest rates; and iv) partial contracts to both types.

Ad. i) and ii). They give the last condition(s) on the set Γ from Proposition 6:

$$\pi^B\left[\left(L_a^B, r_a^B\right), \left(K, r_b^B\right)\right] \geq \max\left\{\pi^B(K, \bar{r}_b), \pi^B(K, \bar{r}_a)\right\} \tag{19}$$

where r_b^B is given by (18).

Ad. iii). Raising r_a^B causes type a to drop out, but if (18) implies $r_b^B < \bar{r}_b$, the bank will have an incentive to raise r_b^B in equilibrium unless the moneylender offers the contract $(L_b^M, r_b^M) = (K - L_a^B, r_{b,low}^M)$. This is case b in Proposition 6. If, on the other hand, $r_b^B = \bar{r}_b$, the bank has no incentive to change r_b^B, and the moneylender can offer any potentially profitable contract to type b. This is case a.

Ad. iv). This is only relevant in case a. However, since total surplus from b's project becomes smaller with co-financing (due to $m > c$), and since the moneylender takes part of the surplus ($\pi_b^M\left(L_b^M, r_b^M\right) \geq 0$), the bank will necessarily get a smaller profit than by fully financing b himself at the rate $\bar{r}_b$. Hence, this strategy is not optimal. ■

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