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Maurice Kugler, Thierry Verdier
and Yves Zenou

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Maurice Kugler, Southampton University
Thierry Verdier, DELTA, Paris and CEPR
Yves Zenou, Southampton University and CEPR

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Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

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ABSTRACT

Organized Crime, Corruption and Punishment

We analyse an oligopoly model in which differentiated criminal organizations compete on criminal activities and engage in corruption to avoid punishment. When law enforcers are sufficiently well-paid and difficult to bribe, and corruption detection highly probable, we show that increasing policing or sanctions effectively deters crime. When bribing costs are low – that is badly-paid and dishonest law enforcers working in a weak governance environment – and the rents from criminal activity relative to legal activity are sufficiently high, we find that increasing policing and sanctions can generate higher crime rates. In particular, the relationship between the traditional instruments of deterrence, namely intensification of policing and increment of sanctions, and crime is non-monotonic. Beyond a threshold, increases in expected punishment induce organized crime to corruption, and ensuing impunity leads to higher rather than lower crime.

JEL Classification: K42 and L13
Keywords: corruption, deterrence, oligopoly, organized crime and strategic complements

Maurice Kugler
Department of Economics
Southampton University
Southampton
SO17 1BJ
Tel: (44 23) 8059 3395
Fax: (44 23) 8059 3858
Email: maurice.kugler@soton.ac.uk

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Thierry Verdier
DELTA-ENS
48 Boulevard Jourdan
75014 Paris
FRANCE
Tel: (33 1) 4313 6308
Fax: (33 1) 4313 6310
Email: verdier@delta.ens.fr

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Yves Zenou
Department of Economics
University of Southampton
Southampton
SO17 1BJ
Tel: (44 23) 8059 3264
Fax: (44 23) 8059 3858
Email: y.zenou@soton.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=126027

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1 Introduction

There are occasional examples of successful battles against the corruption perpetrated by criminal organizations to influence law enforcement and politics. For example, in 1931, right after prohibition of alcohol consumption ended in the United States, the conviction of the notorious gang leader Al Capone for tax evasion led to the break up of mobs and rackets built around the distribution of alcohol, and other complementary activities. Yet, failed attempts to curb the influence of organized crime are common place. Recently, in Italy, the investigation *mani pulite* (clean hands) initiated by a courageous group of judges disintegrated after disclosing pervasive corruption by the Mafia, due to a string of assassinations inflicted upon law enforcers and their families. In general, organized crime syndicates are very difficult to eliminate. They are able to protect themselves by a combination of means: (*i*) Physical violence against informants and witnesses, (*ii*) violent threats against prosecutors, judges and members of juries, (*iii*) corruption of law-enforcement officials, (*iv*) Use of lawyers to manipulate the legal system, and (*v*) financial contributions to political campaigns.

The objective of this paper is to better understand the complex relationship between organized crime, corruption and the efficiency of the justice system. We will in fact focus on the evasion from conviction by criminal organizations through bribing law enforcers. However, the relevance of our findings is not confined to the influence on the operation of the legal system exerted through this channel. As long as organized crime can invest to manipulate the incentives faced by the actors involved in making prosecution possible, our results obtain regarding the limited effectiveness of typical crime deterrents in weak governance environments.

Criminal gangs are active and clever in their efforts to bribe policemen. Cooperative police officers are helpful to criminal gangs by passing information to them about police investigations and planned raids, and by making deliberate 'mistakes' in prosecutions. Such technical errors then ensure that the charges against the criminals will not result in guilty verdicts. Corruption of police officers is made easier by the fact that they are modestly paid and, therefore, are subject to temptation. Moreover, like prosecutors and members of juries, law enforcers can be coerced through violence. Also, once a few policemen have been corrupted, they will make strong efforts to ensure that their colleagues are also corrupted. An honest policeman who tries to inform on his corrupt colleagues will come under the most severe pressures from them.

The literature on crime has emphasized the deterrence capacity of the justice system on criminal activities (e.g. Becker 1968, Ehrlich, 1973, Levitt, 1998). Recent evidence for the United States tends to support the hypothesis that the expectations of potential criminals with respect to punishment determine crime rates (see e.g. Levitt, 1997). Yet, expected punishment depends not only on the severity of sentences but also on the probability of conviction once crime is perpetrated. The latter depends on detection by the police, prosecution by attorneys and the deliberation of judges and juries. As long as these three activities are conducted transparently and efficiently, tough sanctions will deliver deterrence of criminal activity. However if, as described above, corruption is pervasive, then the efficiency in law enforcement can be very much reduced.

Since Becker and Stigler (1974) acknowledge that malfeasance by enforcers can diminish the effectiveness of laws and sanctions in controlling crime, the literature on crime has considered the problem of bribed officials.[1] They propose the payment of efficiency wages to prevent bribe taking. Besley and McLaren (1993) and Mookherjee and Png (1995) also propose wage regimes to mitigate the moral hazard problem when rent seekers attempt to co-opt law enforcers. Like Becker and Stigler (1974), Bowles and Garoupa (1997) consider a model in which bribery reduces punishment and thus deterrence. However, the focus is different since it is on the effects of bribery on the optimal allocation of resources (which incorporates the social costs of both crime and corruption) within the public enforcement agency. They show that the maximal fine may not be optimal. Chang et al. (2000) extend Bowles and Garoupa (1997) by introducing psychological costs (or social norms) of caught corrupt officers. They show that, when corruption is widespread, social norms can no longer

[1] For a comprehensive survey on law enforcement, see Polinski and Shavell (2000). Also, for a general survey on corruption and governance, see Bardhan (1997).

take a sufficient sanction against a corrupt officer, and raising fines can in fact result in more crime. Another extension of Bowles and Garoupa (1997) is done by Garoupa and Jellal (2002). They consider the role of asymmetric information on the emergence of collusion between criminals and enforcers. They show that asymmetric information about the private costs of enforcers engaging in collusion might eventually deter corruption and bargaining between the two parties. Finally, Basu et al. (1992) argue that when the possibility of collusion between law enforcing agents and criminals is introduced, control of corruption becomes more difficult than is suggested by the standard Beckerian approach. Marjit and Shi (1998) extend this paper and show that controlling crime becomes difficult, if not impossible, because the probability of detection can be affected by the effort of a corrupt official. Finally, in a recent paper Polinski and Shavell (2001) consider the dilution of deterrence caused by corruption not only due to bribing by criminals but also extortion of the innocent by crooked enforcers. They propose rewards for corruption reports to mitigate the breakdown of deterrence. Our approach differs from the literature in that we focus is on the relationship between organized crime,[2] corruption and punishment in the context of imperfect competition. Hence, we find not only a reduction in deterrence effectiveness due to corruption as in previous models but actually a potential reversal whereby policies usually associated with crime deterrence can become inducements as long as bribery remains unchecked.

In the present paper, *we analyze the role of corruption not only in diluting deterrence but also as a strategic complement to crime and therefore a catalyst to organized crime.* For that, we develop a simple oligopoly model in which n criminal organizations compete with each other on the levels of both criminal activities and corruption. We first show that when the cost of bribing judges or the number of criminal organizations increases, then both crime and corruption decrease whereas when the profitability of crime increases, then both crime and corruption increase. We then show our main results. If corruption is costly, due to law enforcers being well-paid, hard to bribe and easily detected when accepting side payments, relative to the profits from crime, then, as predicted

[2] There is a small theoretical literature on organized crime (without corruption). See in particular Fiorentini and Peltzman (1996), Garoupta (2000) and Mansour et al. (2000).

by the standard literature on crime, it is always effective to reduce crime by intensifying policing or toughening sanctions. However, in the reverse case of low-paid dishonest law enforcers under weak governance and sizable rents from illegal activity relative to the outside lawful options, increasing policing or sanctions may in some cases generate higher crime rates.

This last result is fairly intuitive. As long as the return to legal economic activity is sufficiently low relative to rents from crime, gangs continue pursing crime. When sanctions and policing are toughened, the cost of hiring criminals rises as there is a wage premium to compensate for the risk of conviction if apprehended. This will discourage crime but only up to a point. In particular, if bribing costs are small relative to the rents from crime, there is level of expected punishment beyond which further toughening of sanctions will induce increasingly higher levels of corruption, and of ensuing crime. Indeed, when governance is weak, harsher punishment can be a catalyst for organized crime and may lead to concentration of criminal rents and higher rates of return ex post. For example, in the 1920's during alcohol prohibition in the United States, mob activities were so profitable that organized crime could afford to keep in its payroll government officials at various levels, including elected politicians and law enforcers, to influence the legal system in its favor. Therefore, the potential effectiveness of tough sentencing as an effective policy to stop organized crime and other subsidiary illegal activities is limited. This does not imply that tough sanctioning of crime and policing should be abandoned altogether when institutional checks and balances are underdeveloped. But, rather that unless corruption is curbed, traditional deterrence policies can have the perverse effect of making crime and corruption strategic complements.

After this introduction, Section 2 sets up the model by describing the problem of the criminal organization. Section 3 characterizes the corruption market. In Section 4, the interaction between crime and corruption is analyzed and the main propositions are presented. Finally, Section 6 concludes the paper by discussing some implications of the results obtained.

2 The model

There are n criminal organizations in the economy. These organizations compete with each other on two aspects: crime and corruption. On the crime market (think for example of drug cartels), there is a pie to be shared and Cournot competition takes place. On the corruption market, there is a continuum of judges to bribe for the n differentiated criminal organizations and spatial competition prevails.

Let us first describe the profit function. For each criminal organization, the revenue from criminal activities depends on the number of crimes and the size of the booty per crime. The cost is given by the wage bill accruing the criminals and the bribes paid to avoid conviction when crimes are detected. For the criminal organization $i = 1, ..., n$, profits are given by:

$$\pi(C, C_i, \alpha) = B(C)\, C_i - w_i L_i - T_i \tag{1}$$

where

$$C = \sum_{j=1}^{j=n} C_j$$

is the total number of crimes perpetrated in the economy , C_i denotes the number of crimes committed by organization i, $B(C)$ is the booty per crime for all criminal organizations, with $B'(C) < 0$ (the booty per crime $B(C)$ is assumed to decrease as the number of crimes increases), w_i is the wage paid by each criminal organization i, determined below, to their L_i employed criminals , and T_i are the total costs to bribe judges borne by the criminal organization i. To be explicitly determined below. For simplicity, we assume about crime profitability and technology that $B(C) = B - C$ and $C_i = L_i$.

Let us determine the wage w_i. The participation constraint for a given criminal working in organization i is given by:

$$\phi\left[w_i - (\alpha_i.0 + (1 - \alpha_i)S)\right] + (1 - \phi)w_i \geq w_0 \tag{2}$$

where $0 < \phi < 1$ is the probability of detection of a crime, α_i denotes the probability that a judge is corrupted by organization i, $S > 0$ is the sanction when punishment of detected crime is enforced and $w_0 > 0$ is the outside wage if the individual has a regular job and is not a criminal. Take equation (2).

The left hand side gives the expected gain of a criminal. Indeed, if he/she is not caught (with probability $1 - \phi$), he/she gets w_i. If he/she is caught (with probability ϕ), he/she still obtains w_i (we assume that criminals get their wage even when they are caught); if the judge is corrupted by organization i (with probability α_i), the criminal has no sanction whereas if the judge is not corrupted by organization i (with probability $1 - \alpha_i$), the criminal has a sanction S (for example number of years in prison). This is key incentive for a criminal to work for an organization since, apart from w_i, he/she benefits from protection (especially corrupted judges).

In equilibrium, this constraint is bidding since there is no incentive for the criminal group to pay more than the outside wage. Therefore, the reservation wage for which workers accept to commit crime for organization i is equal to:

$$w_i = \phi S(1 - \alpha_i) + w_0 \tag{3}$$

Interestingly, in equilibrium, this wage will be determined by the level of corruption α_i in each organization since the higher the level of corruption, the lower this wage. Indeed, if the risk to be prosecuted for a criminal is low, then, as long as w_i is greater w_0 (which is always the case; see (3)), there is no need to pay a high wage.

3 Corruption

The interaction between criminal organizations and judges is modeled here by means of a monopsonistic competitive market inspired by Salop (1979). For that, consider a market with n criminal organizations and a continuum of judges uniformly distributed on the circumference of a circle which has length 1; the density is constant and equal to 1. Organization i's $(= 1, ...n)$ location is denoted by x_i. The space in which both criminal organizations and judges are located is interpreted as the "similarity" space with transaction costs. For tractability, we assume symmetry among criminal organizations so that the distance between two adjacent organizations is equal to $1/n$ in the location space.

Contrary to the standard spatial model (Salop, 1979), the horizontal differentiation of judges is from the point of view of criminal organizations. In other words, the latter are paying all the transaction costs needed to bribe a judge. From the judge's point of view, there is no differentiation since they will accept a bribe if and only if their expected gain is greater than their current wage. As a result, the "distance" of a judge to a criminal organization reflects the transaction cost necessary to agree on a bribe. If we take for example the case of Italy, it is clear that it is easier for a criminal organization located in Sicily to bribe a judge located in Palermo than in Milan because it has more contacts with local people and also speak the same dialect.

Judges' location types are denoted by x. The higher the distance, the higher is the (transaction) cost to bribe a judge. The transaction cost function between a criminal organization x_i and a judge x is $t\left|x-x_i\right|$, where t expresses the transaction cost per unit of distance in the location space. We assume that the outside option of a judge is w_b, i.e. the latter is the current wage of the judge.

In this paper, we focus on non-covered (corruption) markets, i.e. markets in which some of the judges do not accept bribes and thus do not participate in the market activity. We believe it is much more realistic than a covered market in which all judges will be corrupted in equilibrium. This means that each criminal organization acts as a (local) monopsony on the corruption market whereas they will compete a la Cournot on the crime market. Denote by $\overline{x}$ the boundary of the area of each monopsonist, which implies that each criminal organization will bribe $2\overline{x}$ judges in equilibrium. We have of course to check that $\overline{x}<1/2n$ so that, in equilibrium, the corruption market is not covered.

The participation constraint for a judge who is bribed by a criminal organization i located at a distance $\overline{x}_i$ is given by

$$(1-q)(f+w_b)\geq w_b$$

where q is the probability that corruption is caught (quite naturally, we assume that if a judge is caught, he/she loses his/her wage w_b) and f is the bribe given to the judge. Observe that f is not indexed by i since on the corruption market each criminal organization has total monopsony power and thus fixed a bribe that just binds the judge's participation constraint; the latter only depends on q and w_b. Once again, the left hand side gives the expected benefit from corruption whereas the right hand side describes the gain from no-corruption. The sanction for corruption is the loss of the job and the bribe is lost as criminals receive no protection. As a result, for each organization $i = 1, ..., n$ the bribe necessary to corrupt a judge is given by

$$f = \frac{q}{1-q} w_b \tag{4}$$

As stated above, all judges are identical so that at f they will always accept a bribe (we could have assumed that the bribe is $f + \varepsilon$, where ε is very small but positive; this would obviously not change our results so whenever judges are indifferent they accept to be bribed). However, from the criminal organization's point of view each judge is not located at the same "distance" so that the transaction cost to bribe a judge is different from one judge to another. Since $\overline{x}_i$ is the maximum "distance" acceptable for each criminal organization i (i.e. beyond $\overline{x}_i$ the transaction cost of bribing a judge is too high), then the total transaction costs for each criminal organization i is given by:

$$T_i = \int_0^{\overline{x}_i} (f + t) x dx = (f + t) \overline{x}_i^2 / 2$$

In this context, since the length of the circumference of the circle is normalized to 1, the probability α_i (the fraction of law enforcers that will be bribed in equilibrium by paying to each of them a bribe f_i) is given by $\alpha_i = 2\overline{x}_i/1 = 2\overline{x}_i$. In other words, when a criminal belonging to organization i commits a crime at a 'distance' less than $\overline{x}_i$ from organization i, then if he/she is caught and convicted, he/she is sure not to be condemned by the (corrupted) judge. On the contrary, if he commits a crime at a 'distance' larger than $\overline{x}_i$, then he/she will not be judged by someone who is corrupted by organization i. If for example one interprets 'distance' as geographical distance, then this means that crime committed within the area of the criminal organization's location benefits from corrupted judges. If one has a broader interpretation of 'distance', then this implies that judges relatively close (in terms of networks, family, language...) to organization i are more likely to be corrupted than others.

Taking into account all the elements (in particular the participation constraint of the criminal (3) and the participation constraint of each judge), and using (1), the profit function of a criminal group can be written as:

$$\pi(C, C_i, f) = \left(B - \sum_{j=1}^{j=n} C_j\right) C_i - [\phi S (1 - 2\overline{x}_i) + w_0] C_i - (f + t)\frac{\overline{x}_i^2}{2} \qquad (5)$$

This profit function of each criminal organization is divided in three parts. The first one is the proceeds from crime, which depends on the competition in the crime market between the different crime organizations. The second corresponds to the salary costs of hiring criminals while the third part denotes the costs of bribing judges or policemen.

4 Crime and corruption

As stated above, criminal organizations compete on both crime and corruption. On the crime market each criminal organization i competes a la Cournot by determining the optimal C_i. On the corruption market, each acts as local monopsonist by determining the optimal $\overline{x}_i$ (indeed, they have to determine the maximum distance $\overline{x}_i$ beyond which it is not profitable corrupting a judge). As a result, they have to simultaneously determine $\overline{x}_i$ (observe that there is a one-to-one relationship between $\overline{x}_i$ and α_i) and C_i that maximize the profit (5). First order conditions with respect to C_i and $\overline{x}$ yield:

$$B - \sum_{j=1}^{j=n} C_j - C_i - [\phi S (1 - 2\overline{x}_i) + w_0] = 0 \qquad (6)$$

$$2\phi S C_i - (f + t)\overline{x}_i = 0 \qquad (7)$$

Using the Hessian matrix, it is easy to verify that the profit function (5) is strictly concave (implying a unique maximum) if and only if:

$$f + t > 2(\phi S)^2 \qquad (8)$$

Let us now focus on a symmetric equilibrium in which $C_i = C_j = C^*$ and $\overline{x}_i = \overline{x}_j = \overline{x}^*$. These two first order conditions are now given by:

$$B - (n + 1)C^* = \phi S (1 - 2\overline{x}^*) + w_0 \qquad (9)$$

$$2\phi S C^* = (f + t)\overline{x}^* \qquad (10)$$

Now, from (9) we obtain

$$C^* = \frac{B - w_0 - \phi S\left(1 - 2\overline{x}^*\right)}{n+1} \tag{11}$$

Plugging (11) into (10) yields

$$\overline{x}^* = \frac{2\phi S\left(B - w_0 - \phi S\right)}{(f+t)(n+1) - 4(\phi S)^2} \tag{12}$$

Then, by plugging (12) into (11), we have

$$C^* = \frac{(f+t)\left(B - w_0 - \phi S\right)}{(f+t)(n+1) - 4(\phi S)^2} \tag{13}$$

We have finally the following result.

Proposition 1 *Assume*

$$\phi S < \min\left[\sqrt{(f+t)/2}, B - w_0\right] \tag{14}$$

and

$$B - w_0 < \phi S\left(\frac{n-1}{n}\right) + \frac{(f+t)(n+1)}{4n\phi S} \tag{15}$$

Then, the equilibrium number of crime per criminal organization C^* is given by (13) and the equilibrium number of corrupted judges per criminal organization $\alpha^* = 2\overline{x}^*$ by (12), both of them are strictly positive and $1 - n\alpha^*$judges are not corrupted in equilibrium. Moreover, the equilibrium profit of each criminal organization is given by

$$\pi^*(n) = \frac{(f+t)\left(B - w_0 - \phi S\right)^2}{\left[(f+t)(n+1) - 4(\phi S)^2\right]^2}\left[f + t - 2(\phi S)^2\right] > 0 \tag{16}$$

and the wage paid to each criminal is equal to

$$w^*(n) = \phi S\frac{(f+t)(n+1) - 4\phi S(B - w_0)}{(f+t)(n+1) - 4(\phi S)^2} + w_0 > w_0 \tag{17}$$

Proof. See the Appendix.

The following comments are in order. First, condition (14) guarantees that both C^* and $\overline{x}^*$ are strictly positive and that the solution of the maximization problem is unique. Condition (15) ensures that, in equilibrium, some judges or policemen are not corrupted (i.e. $\overline{x}^* < 1/2n$). Indeed, the difference between the booty B and the wage of an individual having a regular job (i.e. working in the "legal" sector) has to be large enough to induce criminal organizations to hire criminals and to bribe judges but at the same time its has to be bounded above otherwise all judges will be corrupted because the profit of each organization would be too large. Second, when choosing C^* the optimal number of criminals to hire, each criminal organization faces two opposite effects. When it increases C, the proceeds from crime is higher (*positive loot effect*) but the competition will be fiercer (*negative competition effect*) and the salary costs higher (*negative salary effect*). As a result, choosing the optimal C^* results of a trade-off between the first positive effect and the second and third negative effects. This trade-off is reflected in the first order condition (9). Finally, when choosing $\overline{x}^*$ the level of corruption, each criminal organization only faces two effects (there is no competition since each criminal organization acts as a monopsonist in the corruption market). Indeed, when it increases $\overline{x}$, each criminal's salary becomes less costly (*positive salary effect*) since criminals have less chance to be sentenced but the costs of bribing judges or policemen increase (*negative bribe effect*). This trade-off is reflected in the first order condition (10).

It is now interesting to analyze the properties of the equilibrium. We have a first simple result.

Proposition 2 *Assume that (14) and (15) hold. Then,*

(*i*) *When f the cost of bribing judges, t the unit transaction cost of bribing judges or n the number of criminal organizations increases, then both crime and corruption decrease.*

(*ii*) *When the net proceeds of crime $B - w_0$ increases, then both crime and corruption increase.*

Proof. See the Appendix.

Not surprisingly, increasing the costs of bribing judges or policemen (f and t) or giving higher wages to judges leads to less crime and to less corruption. Moreover, raising the number of criminal organizations n also decreases crime and corruption because competition in the crime market becomes fiercer and it feeds back to the corruption market. Lastly, when the proceeds from crime increase then obviously crime and corruption increase.

Let us go further in the analysis. The following proposition gives our main results.[3,4]

Proposition 3 *Assume* $\phi S < \min\left[\sqrt{(f+t)/2}, B - w_0, (\phi S)_1^{NC}\right]$*. Then,*

(*i*) *if* $(B - w_0)^2 < (f + t)(n^2 - 1)/n^2$*, for small values of* ϕS*, increasing sanctions increases corruption. But values of* ϕS *larger than a threshold, increasing sanctions decreases corruption. However, increasing sanctions always reduces crime.*

(*ii*) *If* $(B-w_0)^2 > (f+t)(n+1)/3$*, increasing sanctions always increases corruption. However, for small values of* ϕS*, increasing sanctions reduces crime. But values of* ϕS *larger than a threshold, increasing sanctions increases crime.*

Proof. See the Appendix.

Using Figures 1 and 2 that illustrate Proposition 3 we can give the intuition of the main results. When $(B-w_0)^2 < (f+t)(n^2-1)/n^2$, the labor productivity w_0 is high, the proceeds from crime B is quite low, the probability to be caught for a corrupted judge q and his/her wage w_b are quite high (see (4)) and the transaction costs t to corrupt a judge are quite large. If we think of two contrasting regions of the same country, say Italy, then this case could represent the "North". If we think instead of two contrasting countries, say the United States and Colombia, then this would obviously correspond to the United States. Using Figure 1, it is easy to see that, in this case, it is always efficient to reduce crime by increasing ϕ the probability to be caught as a criminal (e.g. frequency of crime detection by policemen in the region) and S the sanctions (e.g. loss due to imprisonment prison).

However, the corruption can in fact increase for low values of ϕS and decrease for high values of ϕS. The intuition runs as follows. When $B - w_0$ is quite low compared to f and t, the productivity of workers is high (implying

[3] $(\phi S)_1^{NC}$ is defined in Lemma 1 in the Appendix.

[4] It is easy to see that there are some parameter values (i.e. when $(f+t)(n^2-1)/n^2 < (B-w_0)^2 < (f+t)(n+1)/3$) for which the signs of $\partial C^*/\partial(\phi S)$ and $\partial \overline{x}^*/\partial(\phi S)$ are not determined. In fact, the complete characterization of the comparative statics of C^* and $\overline{x}^*$ with respect to ϕS is given in Propositions 5 and 6 in the Appendix.

high wages to induce them to become criminal) and the proceeds from crime is low compared to the high costs of bribing judges or policemen. Moreover, it is easy to see that the *negative competition effect and the positive loot effect* are not affected by a variation of ϕS whereas the *negative salary effect* is affected since it becomes even more costly to hire criminal (they have a higher chance to be caught). So, when ϕS increase, each criminal organization finds it optimal to reduce crime (or more exactly the number of criminals hired) because the costs of hiring criminals become too large compared to the benefits of crime. However, this is not true on the corruption market. Indeed, when ϕS varies, the *positive salary effect* is affected since it becomes more costly to hire a criminal whereas the *negative bribe effect* is not affected since the cost of bribing judges or policemen does not depend on ϕS. This can easily be seen in (10) since the right hand side corresponds to the salary effect (which depends on ϕS) and the left hand side to the bribe effect (which does not depends on ϕS). In fact, differentiating the left hand side of (10) with respect to ϕS yields: $C^* + (\phi S)\partial C^*/\partial(\phi S)$. The first effect C^* is positive (i.e. for a given level of crime, when ϕS increase, each criminal organization increases the level of corruption to induce people to become criminal) whereas the second one $(\phi S)\partial C^*/\partial(\phi S)$ is negative (i.e. when ϕS increase, there is less crime and thus there is less need to corrupt judges or policemen so that corruption decreases). As a result, for low values of ϕS, crime C^* is quite high so when ϕS increases, the first effect dominates the second effect so that corruption increases. For high values of ϕS, when ϕS increases, the second effect dominates the first one because the crime level C is quite low and it is not optimal to increase corruption.

Let us now interpret the case when $(B-w_0)^2 > (f+t)(n+1)/3$, were labor productivity is low, the probability to be corrupted high and the proceeds of crime large. Using the above interpretation, this case would be either "Southern" Italy or Colombia. Let us use Figure 2 to understand the results. In this case, when ϕS increase, it is always optimal to increase corruption because the resulting gain in the reduction of criminals' wages with the fact that the net proceeds from crime $B - w_0$ are high are always greater than the increasing cost of bribing judges (which is not affected by ϕS). In the crime market, this is not always true. Indeed, as stated above, only the salary effect is affected by ϕS. Take equation (11). It is easy to see the sign of $\partial C^*/\partial(\phi S)$ depends on $-(1-2\overline{x}^*) + 2\phi S \partial\overline{x}^*/\partial(\phi S)$. When $(B - w_0)^2 > (f+t)(n+1)/3$, the first effect $-(1-2\overline{x}^*)$ (i.e. for a given level of corruption, when ϕS increase, it becomes more costly to hire criminals) is negative whereas the second one $2(\phi S)\partial\overline{x}^*/\partial(\phi S)$ (i.e. when ϕS increases, there is more corruption and it becomes less costly to hire criminals since their probability to be sentenced if caught is lower) is positive. As a result, for low values of ϕS, when ϕS increases, the first effect dominates the second one because the corruption is still quite low so that it becomes more costly to pay criminals and thus crime is reduced. However, for high values of ϕS, the second effect dominates the first one since the level corruption is quite high and thus quite effective so that crime increases.

This is our main result. In a country where crime is profitable relative to legal economic opportunities, judges are badly-paid and easy to corrupt, then for crimes that involve large sanctions (drug dealing, murders, ...), increasing the crime detection probability or the severity of the sanctions results in more rather than less crime. This is due to the fact that, when sanctions increase, the optimal response of criminal organizations is to increase corruption to counteract the rise in sanctions. This implies that, in countries with weak governance, the policy implications of the standard crime model may not hold and instead, as our model suggests, deterrence can only be effective ensuing a substantial cut down in corruption. Basically, the issue is that a rise in ϕS can take the model into a set of the parameter space where crime and corruption are strategic complements, as long a the equilibrium bribe is bounded.

$[Insert\ Figures\ 1\ and\ 2\ here]$

We can analyze further the latter effect by investigating case (*ii*) in Proposition 3. We have the following result.

Proposition 4 *Assume* $(B - w_0)^2 > (f + t)(n + 1)/3$ *and* $\phi S < (\phi S)_1^{NC}$. *Then* (i) *the lower the labor productivity* w_0 *in the legal sector,* (ii) *the higher the booty* B *per crime,* (iii) *the easier it is to bribe law enforcers (i.e. the lower the reservation bribe* f *and associated transaction cost* t*), and/or* (iv) *the weaker is the competition between criminal organizations (i.e. the lower is* n*), the lower is the threshold of* ϕS *above which crime and corruption become strategic complements, i.e. the more likely that an increase in policing or sanctions leads to an increase in crime.*

Proof. See the Appendix.

This proposition complements our previous results. It explains why in some countries deterrence works, even if diluted by corruption, while in others it can have perverse effects. The proposition establishes that where productivity is quite low so that legal jobs are not very attractive, bribing is pervasive, and criminal organizations have high market power, then increasing policing and sanctions is more likely to trigger strategic complementarity among corruption and crime resulting in a perverse effect of deterrence.

This result contrasts with the literature that has posited optimal maximal sanctions. First, Polinski and Shavell (1979) show that if fine collection is costless and monitoring of criminal activity is costly, the optimal magnitude of fines corresponds to the maximum payable by criminals. When this maximum falls well short of the booty from crime, nonmonetary sanctions are required for deterrence. Since it is not only costly to apprehend criminals but also to punish them, Shavell (1987) proves that it is optimal for sanctions to be imposed with low frequency. Hence, in the case that the courts' information is imperfect, deterrence requires sufficiently large sanctions. The standard result is that under risk neutrality fines should be maximal. If the optimal fine is not maximal, due to risk aversion,[5] the presence of corruption in Polinski

[5] Polinski and Shavell (2000) present the standard case with risk neutrality (p.50) and then discuss other reasons why maximal fines may not be optimal (p. 62-64). First, marginal deterrance may dictate heterogenous fines across criminal acts harmful in different degrees. Second, the potential for general enforcement investments yields economies of scope in monitoring inducing apprehension probabilities consistent with deterrance for sanction magnitudes below the maximal level.

and Shavell (2001) dictates higher sanctions to counter the deterrence-diluting effects of corruption. In contrast, in our model, until bribery can be eradicated, the rising of sanctions worsens the corruption and crime problems.

Finally, one may wonder what happens to the model if we allow for free entry. In particular, could we still have local monopolies (i.e. non-covered markets) in the corruption market. In fact, when there is free entry with fixed costs so that the number of criminal organizations becomes endogenous, the number of active organizations will remain finite and, for sufficiently large fixed costs, monopoly (and not all judges will be corrupted in equilibrium) and will prevail in the conditions described in Proposition 1.[6]

To be more precise, if G denote the fixed costs, then the two conditions that guarantee that the market will not be covered with free entry are as follows:

$$\pi^*(n) - G > 0 \text{ with } \overline{x}^*(n) < 1/2n$$

$$\pi^*(n+1) - G < 0$$

where $\pi^*(n)$ is defined by (16). The first equation is defined for n local monopoly criminal organizations whereas the second equation is defined for $n+1$ criminal organizations that can be local monopolies or not as long as the profit net of fixed costs is negative.

5 Conclusion

This paper has spelled out the role of corruption and imperfect competition in preventing the justice system to work efficiently. Indeed, in a model where criminal organizations compete a la Cournot on the crime market and act as local monopsonists on the corruption market, we have showed that when bribing costs are small relative to crime profitability, beyond a threshold further sanctions lead to higher rather than lower crime.

We agree with Becker (1968), Ehrlich (1973), Polinski and Shavell (1979) and Levitt (1997, 1998) that enhancing enforcement efficiency and sanction

[6] This can be shown by re-labelling the analysis of Steinmetz and Zenou (2001) who proved this result in the product market.

severity in order to increase expected punishment, thereby reducing criminal activity, is important. However, when dealing with organized crime that engages in corruption to manipulate conviction probabilities, complementary measures, such as crack down on corruption or the institutionalization of checks and balances, are warranted to control the problem. Our model delivers stark conclusions with respect to the relationship between crime and corruption and as to why the standard "crime and punishment" framework may fail for some countries. Further efforts to inflict tougher sentences on criminals will just raise the rents to organized crime, when corruption is pervasive. More generally the enforcement of property rights at large can break down once the police force and courts stop functioning properly. Beyond a threshold of corruption in the justice system, increasing returns in various types of crime may take off. This observation may explain crime dynamics in some countries (e.g. Colombia and Russia) or regions within countries (e.g. Sicily in Italy). Once this process starts, the best policy may be to contain diffusion of corruption by organized crime to neighboring jurisdictions. Before it starts, the best policy may be to try to suppress organized crime rents.

Given the complementarity between crime and corruption, and since building the required institutions for a transparent legal system can take a long time to achieve, tolerating some degree of illegality (or of a harmful activity which is legalized) can be desirable if it helps to destroy the rents of organized crime. It is interesting to observe that, in the 1920's, during prohibition in the United States, organized crime did have police, judges and politicians in its payroll. In this period of time, more monitoring and investigation of alcohol distribution only increased the rents of the business for both traffickers and corrupt "enforcers". On the one hand, in some sense, severe sanctions on alcohol consumption sowed the seeds for a powerful cartel that came to be known as the mob. On the other hand, the destruction of rents through legalization had a lasting effect in weakening the influence of organized crime on the legal system, which had facilitated all kinds of illegal subsidiary operations by the Mafia, including gambling, prostitution and racketeering.

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APPENDIX

Proof of Proposition 1

First, by assuming that $f + t > 2(\phi S)^2$ (see (14)), we guarantee that: (i) the second order condition (8) is always true, (ii) $(f + t)(n + 1) > 4(\phi S)^2$ (since $n > 1$). As a result, (ii) implies that the denominator of C^* and $\overline{x}^*$ are both strictly positive.

Second, using (12) and (13), it is easy to see that $C^* > 0$ and $\overline{x}^* > 0$ is equivalent to $B - w_0 > \phi S$. This is guaranteed by (14).

Third, because we consider the case of local monopsonists, we have to check that in equilibrium some judges will not be corrupted (i.e. the market is not covered). The market is not covered iff $\overline{x}^* < 1/2n$. Using (12), this writes:

$$\frac{2\phi S\,(B - w_0 - \phi S)}{(f + t)(n + 1) - 4(\phi S)^2} < \frac{1}{2n}$$

which is equivalent to (15).

Finally, to calculate the equilibrium profit and the equilibrium criminal's wage, it suffices to plug (12) and (13) in (5) and in (3). ■

Proof of Proposition 2

(i) By differentiating (12) and (13), it is easy to see that

$$\frac{\partial C^*}{\partial f} < 0 \qquad \text{and} \qquad \frac{\partial \overline{x}^*}{\partial f} < 0$$

$$\frac{\partial C^*}{\partial t} < 0 \qquad \text{and} \qquad \frac{\partial \overline{x}^*}{\partial t} < 0$$

$$\frac{\partial C^*}{\partial n} < 0 \qquad \text{and} \qquad \frac{\partial \overline{x}^*}{\partial n} < 0$$

(ii) By differentiating (12) and (13), it is easy to see that

$$\frac{\partial C^*}{\partial (B - w_0)} > 0 \qquad \text{and} \qquad \frac{\partial \overline{x}^*}{\partial (B - w_0)} > 0$$

Proof of Proposition 3

Before proving the result of this proposition, we need to study the condition for the market not to be covered, i.e. (15). The following lemma states this result.[7]

Lemma 1

(i) *When* $(B - w_0)^2 < (f+t)(n^2-1)/n^2$, *the market is always non-covered whatever the value of* ϕS.

(ii) *When* $(B - w_0)^2 > (f+t)(n^2-1)/n^2$, *the market is non-covered if and only if*

$$\phi S < (\phi S)_1^{NC} \equiv \frac{B - w_0 - \sqrt{(B-w_0)^2 - (f+t)(n^2-1)/n^2}}{2(n-1)/n}$$

Proof. We can write (15) as:

$$\Omega^{NC}(\phi S) \equiv \left(\frac{n-1}{n}\right)(\phi S)^2 - (B - w_0)\phi S + \frac{(f+t)(n+1)}{4n} > 0$$

The discriminant of $\Omega^{NC}(\phi S)$ is given by: $\Delta^{NC} = (B - w_0)^2 - (f+t)(n^2-1)/n^2$. Thus,

• If $(B - w_0)^2 < (f+t)(n^2-1)/n^2$ (i.e. $\Delta^{NC} < 0$), then $\Omega^{NC}(\phi S) > 0$ is always true since the graph of $\Omega^{NC}(\phi S)$ is situated in the positive orthant. This demonstrates (i).

• If $(B - w_0)^2 > (f+t)(n^2-1)/n^2$ (i.e. $\Delta^{NC} > 0$), then we have to study $\Omega^{NC}(\phi S)$. The two roots are given by

$$(\phi S)_1^{NC} = \frac{B - w_0 - \sqrt{(B-w_0)^2 - (f+t)(n^2-1)/n^2}}{2(n-1)/n}$$

$$(\phi S)_2^{NC} = \frac{B - w_0 + \sqrt{(B-w_0)^2 - (f+t)(n^2-1)/n^2}}{2(n-1)/n}$$

Let us show that

$$(\phi S)_1^{NC} < B - w_0 < (\phi S)_2^{NC}$$

[7] The superscript NC stands for non-covered.

First, $B - w_0 > (\phi S)_1^{NC}$. This is equivalent to:

$$B - w_0 > \frac{B - w_0 - \sqrt{(B - w_0)^2 - (f+t)(n^2-1)/n^2}}{2(n-1)/n}$$

By doing simple calculations, this inequality can be written as:

$$4(B - w_0)^2 > -(f+t)(n+1)$$

which is always true.

Second, $(\phi S)_2^{NC} > B - w_0$. This is equivalent to:

$$\frac{B - w_0 + \sqrt{(B - w_0)^2 - (f+t)(n^2-1)/n^2}}{2(n-1)/n} > B - w_0$$

By doing simple calculations, this inequality can be written as:

$$4(B - w_0)^2 > (n+1)(f+t)$$

which is always true by using the fact that $\Delta^{NC} > 0$.

Since $\Omega^{NC}(\phi S)$ is a quadratic function and the coefficient of $(\phi S)^2$, $(n-1)/n$, is positive, $\Omega^{NC}(\phi S)$ is a convex function that intersects the vertical axe twice. Thus, $\Omega^{NC}(\phi S)$ is positive if and only if $\phi S < (\phi S)_1^{NC}$or $\phi S > (\phi S)_2^{NC}$. However, for Proposition 1 to be viable, (14) has to hold, i.e. ϕS has to be less than $B - w_0$. Since we have shown that $(\phi S)_1^{NC} < B - w_0 < (\phi S)_2^{NC}$, this implies that ϕS cannot be greater than $(\phi S)_2^{NC}$. This demonstrates (ii). ■

Let us know first study the comparative statics of C^* with respect to ϕS. The following Lemma states a first result.

Lemma 2

(i) When $(B - w_0)^2 < (f+t)(n+1)/4$, $\partial C^*/\partial(\phi S) < 0$ whatever the value of ϕS.

(ii) When $(B - w_0)^2 > (f+t)(n+1)/4$, $\partial C^*/\partial(\phi S) < 0$ if and only if

$$\phi S < (\phi S)_1^{CSC} \equiv B - w_0 - \sqrt{(B - w_0)^2 - \frac{(f+t)(n+1)}{4}}$$

and $\partial C^*/\partial(\phi S) > 0$ if and only if $(\phi S)_1^{CSC} < \phi S < B - w_0$.

Proof. By differentiating (13), we obtain:

$$\frac{\partial C^*}{\partial(\phi S)} = (f+t)\frac{-4(\phi S)^2 + 8(\phi S)(B-w_0) - (f+t)(n+1)}{\left[(f+t)(n+1) - 4(\phi S)^2\right]^2}$$

In order to study the sign of $\partial C^*/\partial(\phi S)$, we have to study[8]

$$\Omega^{CSC}(\phi S) \equiv -4(\phi S)^2 + 8(\phi S)(B-w_0) - (f+t)(n+1)$$

The discriminant is given by: $\Delta^{CSC} = 16\left[4(B-w_0)^2 - (f+t)(n+1)\right]$. Two cases arise.

• If $(B-w_0)^2 < (f+t)(n+1)/4$ (i.e. $\Delta^{CSC} < 0$), then $\Omega^{CSC}(\phi S) < 0$ is always true since the graph of $\Omega^{CSC}(\phi S)$ is situated in the negative orthant. This implies that $\partial C^*/\partial(\phi S) < 0$. This proves (i).

• If $(B-w_0)^2 > (f+t)(n+1)/4$ (i.e. $\Delta^{CSC} > 0$), then we have to study $\Omega^{CSC}(\phi S)$. The two roots are given by

$$(\phi S)_1^{CSC} = B - w_0 - \sqrt{(B-w_0)^2 - \frac{(f+t)(n+1)}{4}}$$

$$(\phi S)_2^{CSC} = B - w_0 + \sqrt{(B-w_0)^2 - \frac{(f+t)(n+1)}{4}}$$

which again implies that $(\phi S)_1^{CSC} < B - w_0 < (\phi S)_2^{CSC}$. As a result, since $\Omega^{CSC}(\phi S)$ is a quadratic function and the coefficient of $(\phi S)^2$, -4, is negative, $\Omega^{CSC}(\phi S)$ is a concave function that intersects the vertical axe twice. Thus, $\Omega^{CSC}(\phi S)$ is negative if and only if $\phi S < (\phi S)_1^{CSC}$or $\phi S > (\phi S)_2^{CSC}$. However, for Proposition 1 to be viable, (14) has to hold, i.e. ϕS has to be less than $B - w_0$. Since $(\phi S)_1^{CSC} < B - w_0 < (\phi S)_2^{CSC}$, this implies that ϕS cannot be greater than $(\phi S)_2^{CSC}$. Therefore, $\partial C^*/\partial(\phi S) < 0$, if $\phi S < (\phi S)_1^{CSC}$ and $\partial C^*/\partial(\phi S) > 0$, if $(\phi S)_1^{CSC} < \phi S < B - w_0$. This proves (ii). ■

Now, we have to check that the comparative statics of C^* with respect to ϕS holds when the market is not covered. In other words, we are only interested in $\partial C^*/\partial(\phi S)$ when the market is non-covered, i.e. we have to check if Lemma 2 is compatible with Lemma 1. Let us formulate first the following result.

[8] The superscript CSC means the comparative statics of C.

Lemma 3 *If* $(B-w_0)^2 > (n+1)(f+t)/3$, *then* $(\phi S)_1^{CSC} < (\phi S)_1^{NC}$.

Proof. $(\phi S)_1^{CSC} < (\phi S)_1^{NC}$ is equivalent to

$$B-w_0-\sqrt{(B-w_0)^2-\frac{(f+t)(n+1)}{4}} < \frac{B-w_0-\sqrt{(B-w_0)^2-(f+t)(n^2-1)/n^2}}{2(n-1)/n}$$

After some manipulations, this can be written as

$$\sqrt{n^2(B-w_0)^2-(f+t)(n^2-1)}+(n-2)(B-w_0) < 2(n-1)\sqrt{4(B-w_0)^2-(f+t)(n+1)}$$

A sufficient condition for this inequality to hold

$$\sqrt{n^2(B-w_0)^2}+(n-2)(B-w_0) < 2(n-1)\sqrt{4(B-w_0)^2-(f+t)(n+1)}$$

which is equivalent to

$$(B-w_0)^2 > \left(\frac{n+1}{3}\right)(f+t)$$

■

We need a final result.

Lemma 4 *If* $(B-w_0)^2 > (f+t)(n+1)/3$, *then* $(\phi S)_1^{NC} \leq \sqrt{(f+t)/2}$.

Proof. $(\phi S)_1^{NC} \leq \sqrt{(f+t)/2}$ is equivalent to

$$\frac{B-w_0-\sqrt{(B-w_0)^2-(f+t)(n^2-1)/n^2}}{2(n-1)/n} \leq \sqrt{\frac{f+t}{2}}$$

This can be written as

$$B-w_0 \leq 2\frac{(n-1)}{n}\sqrt{\frac{f+t}{2}}+\sqrt{(B-w_0)^2-(f+t)(n^2-1)/n^2}$$

or equivalently

$$\begin{aligned}(B-w_0)^2 \quad &\leq \quad 2\frac{(n-1)^2}{n^2}(f+t)+(B-w_0)^2-(f+t)(n^2-1)/n^2 \\ &\quad +4\frac{(n-1)}{n}\sqrt{\frac{f+t}{2}}\sqrt{(B-w_0)^2-(f+t)(n^2-1)/n^2}\end{aligned}$$

or

$$4\frac{(n-1)}{n}\sqrt{\frac{f+t}{2}}\sqrt{(B-w_0)^2-(f+t)(n^2-1)/n^2}+(f+t)\frac{(n-1)}{n^2}(n-3) \geq 0$$

This inequality is obviously verified whenever $n \geq 3$. Let us see if it is true for $n = 1$ and $n = 2$. For $n = 1$, the left hand side is equal to zero and thus this inequality is true.

Finally, when $n = 2$, this inequality rewrites

$$2\sqrt{\frac{f+t}{2}}\sqrt{(B-w_0)^2-(f+t)3/4} \geq \frac{f+t}{4}$$

or equivalently

$$(B-w_0)^2 \geq \frac{25}{32}(f+t)$$

Now observe that, for $n = 2$, $(f+t)(n+1)/3 > (f+t)25/32$. As a result, for $n = 2$, if $(B-w_0)^2 > (f+t)(n+1)/3$, then $32(B-w_0)^2 \geq 25(f+t)$ and the inequality above is also true for $n = 2$. Thus, it is true whatever n. ■

We are now able to totally characterize the comparative statics of ϕS on C^*. We have:

Proposition 5

(i) *When* $(B-w_0)^2 \leq (f+t)/2$ *and* $\phi S < B-w_0$*, then* $C^* > 0$*,* $\partial C^*/\partial(\phi S) < 0$*, and the market is non-covered.*

(ii) *When* $(f+t)/2 < (B-w_0)^2 < (f+t)(n^2-1)/n^2$ *and* $\phi S < \sqrt{(f+t)/2}$*, then* $C^* > 0$*,* $\partial C^*/\partial(\phi S) < 0$*, and the market is non-covered.*

(iii) *When* $(f+t)(n^2-1)/n^2 \leq (B-w_0)^2 \leq (f+t)(n+1)/4$ *and* $\phi S < \min\left[\sqrt{(f+t)/2}, (\phi S)_1^{NC}\right]$*, then* $C^* > 0$*,* $\partial C^*/\partial(\phi S) < 0$*, and the market is non-covered.*

(iv) *When* $(f+t)(n+1)/4 < (B-w_0)^2 \leq (f+t)(n+1)/3$ *and* $\phi S < \min\left[\sqrt{(f+t)/2}, (\phi S)_1^{NC}\right]$*,* $C^* > 0$*, the market is non-covered but we cannot determine the sign of* $\partial C^*/\partial(\phi S)$*.*

(v) *When* $(B-w_0)^2 > (f+t)(n+1)/3$*, and* $\phi S < (\phi S)_1^{NC}$*, then* $C^* > 0$*, the market is non-covered and,*

   (va) *if* $\phi S < (\phi S)_1^{CSC}$*,* $\partial C^*/\partial(\phi S) < 0$*.*

   (vb) *if* $(\phi S)_1^{CSC} < \phi S < (\phi S)_1^{NC}$*,* $\partial C^*/\partial(\phi S) > 0$*.*

Proof.

(*i*) Using Proposition 1, $(B-w_0)^2 < (f+t)/2$ implies that condition (14) that guarantees that $C^* > 0$ becomes $\phi S < B - w_0$. Moreover, using Lemma 1, and observing that $(f+t)/2 < (f+t)(n^2-1)/n^2$, the condition $(B-w_0)^2 < (f+t)/2$ guarantees that the market is non-covered. Finally, using Lemma 2, and observing that $(f+t)/2 < (f+t)(n+1)/4$, the condition $(B-w_0)^2 < (f+t)/2$ guarantees that $\partial C^*/\partial(\phi S) < 0$.

(*ii*) Using Proposition 1, $(f+t)/2 < (B-w_0)^2 < (f+t)(n^2-1)/n^2$ implies that condition (14) that guarantees that $C^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Moreover, using Lemma 1, $(B-w_0)^2 < (f+t)(n^2-1)/n^2$ guarantees that the market is non-covered. Finally, using Lemma 2, and observing that $(f+t)(n^2-1)/n^2 < (f+t)(n+1)/4$, the condition $(B-w_0)^2 < (f+t)(n^2-1)/n^2$ guarantees that $\partial C^*/\partial(\phi S) < 0$.

(*iii*) Using Proposition 1, $(f+t)(n^2-1)/n^2 < (B-w_0)^2 < (f+t)(n+1)/4$ implies that condition (14) that guarantees that $C^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Moreover, using Lemma 1, since $(B-w_0)^2 > (f+t)(n^2-1)/n^2$, the condition $\phi S < (\phi S)_1^{NC}$ now guarantees that the market is non-covered. Finally, using Lemma 2, the condition $(B-w_0)^2 < (f+t)(n+1)/4$ guarantees that $\partial C^*/\partial(\phi S) < 0$.

(*iv*) Using Proposition 1, $(f+t)(n+1)/4 < (B-w_0)^2 < (f+t)(n+1)/3$ implies that condition (14) that guarantees that $C^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Moreover, using Lemma 1, since $(B-w_0)^2 > (f+t)(n^2-1)/n^2$, the condition $\phi S < (\phi S)_1^{NC}$ guarantees that the market is non-covered. However, using Lemma 2 when $(B-w_0)^2 > (f+t)(n+1)/4$, we cannot determine the sign of $\partial C^*/\partial(\phi S)$ since it depends on whether $(\phi S)_1^{NC}$ is greater or lower than $(\phi S)_1^{CSC}$. Since in Lemma 3, a sufficient condition for $(\phi S)_1^{CSC} < (\phi S)_1^{NC}$ is $(B-w_0)^2 > (n+1)(f+t)/3$, we cannot in the present case determine which root is greater than the other.

(*v*) Using Proposition 1, $(B-w_0)^2 > (f+t)(n+1)/3$ implies that condition (14) that guarantees that $C^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Using Lemma 4, the condition $(B-w_0)^2 > (f+t)(n+1)/3$ implies that

$(\phi S)_1^{NC} \leq \sqrt{(f+t)/2}$. As a result, $\phi S < (\phi S)_1^{NC}$ guarantees that $C^* > 0$. Furthermore, using Lemma 1 when $(B-w_0)^2 > (f+t)(n^2-1)/n^2$, $\phi S < (\phi S)_1^{NC}$ guarantees that the market is non-covered. Now using Lemma 2, when $(B-w_0)^2 > (n+1)(f+t)/3$, $(\phi S)_1^{CSC} < (\phi S)_1^{NC}$. As a result, Lemma 2 implies that two cases may arise:

(va) If $\phi S < (\phi S)_1^{CSC}$, then $\partial C^*/\partial(\phi S) < 0$.

(vb) Using also Lemma 1 and observing that $(\phi S)_1^{NC} < B - w_0$, we have: If $(\phi S)_1^{CSC} < \phi S < (\phi S)_1^{NC}$, then $\partial C^*/\partial(\phi S) > 0$. ■

Let us now study the comparative statics of $\overline{x}^*$ with respect to ϕS.

Lemma 5 *Define*

$$(\phi S)_1^{CSX} \equiv \frac{(f+t)(n+1) - \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2}}{4(B-w_0)}$$

and

$$(\phi S)_2^{CSX} \equiv \frac{(f+t)(n+1) + \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2}}{4(B-w_0)}$$

Then, when $(B-w_0)^2 < (f+t)(n+1)/4$,

$$(\phi S)_1^{CSX} < B - w_0 < (\phi S)_2^{CSX}$$

and

$$\sqrt{\frac{f+t}{2}} < (\phi S)_2^{CSX}$$

When $(B-w_0)^2 < 2(n+1)^2(f+t)/(n+3)^2$, *then*

$$(\phi S)_1^{CSX} < \sqrt{\frac{f+t}{2}}$$

Proof.

First, $(\phi S)_1^{CSX} < B - w_0$ writes

$$\frac{(f+t)(n+1) - \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2}}{4(B-w_0)} < B - w_0$$

which is equivalent to

$$\sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1)-4(B-w_0)^2} > (f+t)(n+1)-4(B-w_0)^2$$

or

$$(f+t)^2(n+1)^2-4(f+t)(n+1)(B-w_0)^2 > (f+t)^2(n+1)^2+16(B-w_0)^4-8(f+t)(n+1)(B-w_0)^2$$

This is equivalent to

$$(f+t)(n+1) > 4(B-w_0)^2$$

which is always true when $(B-w_0)^2 < (f+t)(n+1)/4$.

Second, $(\phi S)_2^{CSX} > B - w_0$ writes

$$\frac{(f+t)(n+1)+\sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1)-4(B-w_0)^2}}{4(B-w_0)} > B-w_0$$

which is equivalent to

$$(f+t)(n+1)+\sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1)-4(B-w_0)^2} > 4(B-w_0)^2$$

or

$$\sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1)-4(B-w_0)^2} > 4(B-w_0)^2-(f+t)(n+1)$$

This is inequality is obviously always true when $(B-w_0)^2 < (f+t)(n+1)/4$.

Third, $(\phi S)_1^{CSX} < \sqrt{\frac{f+t}{2}}$ writes

$$\frac{(f+t)(n+1)-\sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1)-4(B-w_0)^2}}{4(B-w_0)} < \sqrt{\frac{f+t}{2}}$$

which is equivalent to

$$(f+t)(n+1)-4(B-w_0)\sqrt{\frac{f+t}{2}} < \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1)-4(B-w_0)^2}$$

or

$$8(B-w_0)^2(f+t)-8(f+t)(n+1)(B-w_0)\sqrt{\frac{f+t}{2}} < -4(f+t)(n+1)(B-w_0)^2$$

This is equivalent to

$$(B - w_0) < 2\frac{(n+1)}{(n+3)}\sqrt{\frac{f+t}{2}}$$

or

$$(B - w_0)^2 < 2\frac{(n+1)^2}{(n+3)^2}(f+t)$$

Finally, $(\phi S)_2^{CSX} > \sqrt{\frac{f+t}{2}}$ writes

$$\frac{(f+t)(n+1) + \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2}}{4(B-w_0)} > \sqrt{\frac{f+t}{2}}$$

which is equivalent to

$$\sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2} > 4(B-w_0)\sqrt{\frac{f+t}{2}} - (f+t)(n+1)$$

Let us show that

$$4(B - w_0)\sqrt{\frac{f+t}{2}} < (f+t)(n+1)$$

This is equivalent to

$$(B - w_0)^2 < \frac{(f+t)(n+1)^2}{8}$$

Now, it is easy to verify that, for $n \geq 1$, we have

$$(B - w_0)^2 < \frac{(f+t)(n+1)}{4} \leq (f+t)\frac{(n+1)^2}{8}$$

As a result, $(\phi S)_2^{CSX} > \sqrt{\frac{f+t}{2}}$ is always true. ■

Lemma 6

(*i*) *When* $(B - w_0)^2 < (f+t)(n+1)/4$, $\partial\overline{x}^*/\partial(\phi S) > 0$ *if and only if* $\phi S < (\phi S)_1^{CSX}$ *and* $\partial\overline{x}^*/\partial(\phi S) < 0$ *if and only if* $(\phi S)_1^{CSX} < \phi S < (\phi S)_2^{CSX}$.

(*ii*) *When* $(B - w_0)^2 > (f+t)(n+1)/4$, $\partial\overline{x}^*/\partial(\phi S) > 0$ *whatever the value of* ϕS.

Proof. By differentiating (12), we obtain:

$$\frac{\partial\overline{x}^*}{\partial(\phi S)} = 2\frac{4(\phi S)^2(B-w_0) - 2\phi S(f+t)(n+1) + (f+t)(n+1)(B-w_0)}{\left[(f+t)(n+1) - 4(\phi S)^2\right]^2}$$

In order to study the sign of $\partial\overline{x}^*/\partial(\phi S)$, we have to study[9]

$$\Omega^{CSX}(\phi S) \equiv 4(\phi S)^2(B-w_0) - 2\phi S(f+t)(n+1) + (f+t)(n+1)(B-w_0)$$

The discriminant is given by: $\Delta^{CSX} = 4(f{+}t)(n{+}1)\left[(f+t)(n+1) - 4(B-w_0)^2\right]$. Two cases arise.

• If $(B-w_0)^2 > (f+t)(n+1)/4$ (i.e. $\Delta^{CSX} < 0$), then $\Omega^{CSX}(\phi S) > 0$ is always true since the graph of $\Omega^{CSX}(\phi S)$ is situated in the positive orthant. This implies that $\partial\overline{x}^*/\partial(\phi S) > 0$. This proves (ii).

• If $(B-w_0)^2 < (f+t)(n+1)/4$ (i.e. $\Delta^{CSX} > 0$), then we have to study $\Omega^{CSX}(\phi S)$. The two roots are given by

$$(\phi S)_1^{CSX} = \frac{(f+t)(n+1) - \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2}}{4(B-w_0)}$$

$$(\phi S)_2^{CSX} = \frac{(f+t)(n+1) + \sqrt{(f+t)(n+1)}\sqrt{(f+t)(n+1) - 4(B-w_0)^2}}{4(B-w_0)}$$

Since $\Omega^{CSX}(\phi S)$ is a quadratic function and the coefficient of $(\phi S)^2$, $4(B-w_0)$, is positive, $\Omega^{CSX}(\phi S)$ is a convex function that intersects the vertical axe twice. Thus, $\Omega^{CSX}(\phi S)$ is positive if and only if $\phi S < (\phi S)_1^{CSX}$or $\phi S > (\phi S)_2^{CSX}$. However, we have seen in Lemma 5 that, when $(B-w_0)^2 < (f+t)(n+1)/4$, $(\phi S)_2^{CSX} > B - w_0$ and $(\phi S)_2^{CSX} > \sqrt{\frac{f+t}{2}}$. Because of Proposition 1, this implies that ϕS cannot be greater than $(\phi S)_2^{CSX}$. Therefore, $\partial\overline{x}^*/\partial(\phi S) > 0$, if $\phi S < (\phi S)_1^{CSX}$ and $\partial\overline{x}^*/\partial(\phi S) < 0$, if $(\phi S)_1^{CSC} < \phi S < (\phi S)_2^{CSC}$. This proves (i). ■

[9]The superscript CSX means the comparative statics of $\overline{x}$.

We are now able to completely characterize the comparative statics of $\overline{x}^*$ with respect to ϕS. We have:

Proposition 6

- *(i) When* $(B - w_0)^2 \leq (f + t)/2$ *and* $\phi S < B - w_0$*, then* $\overline{x}^* > 0$*, the market is non-covered, and*

  - *(ia) if* $\phi S < (\phi S)_1^{CSX}$, $\partial\overline{x}^*/\partial(\phi S) > 0$.
  - *(ib) if* $(\phi S)_1^{CSC} < \phi S < B - w_0$, $\partial\overline{x}^*/\partial(\phi S) < 0$.

- *(ii) When* $(f+t)/2 < (B-w_0)^2 < (f+t)(n^2-1)/n^2$ *and* $\phi S < \sqrt{(f+t)/2}$*, then* $\overline{x}^* > 0$*, the market is non-covered, and*

  - *(iia) if* $\phi S < (\phi S)_1^{CSX}$, $\partial\overline{x}^*/\partial(\phi S) > 0$.
  - *(iib) if* $(\phi S)_1^{CSC} < \phi S < \sqrt{(f+t)/2}$, $\partial\overline{x}^*/\partial(\phi S) < 0$.

- *(iii) When* $(f + t)(n^2 - 1)/n^2 \leq (B - w_0)^2 \leq (f + t)(n + 1)/4$ *and* $\phi S < \min\left[\sqrt{(f+t)/2}, (\phi S)_1^{NC}\right]$*, then* $\overline{x}^* > 0$*, the market is non-covered but we cannot determine the sign of* $\partial\overline{x}^*/\partial(\phi S)$.

- *(iv) When* $(B-w_0)^2 > (f+t)(n+1)/4$*, and* $\phi S < \min\left[\sqrt{(f+t)/2}, (\phi S)_1^{NC}\right]$*, then* $\overline{x}^* > 0$*, the market is non-covered and* $\partial\overline{x}^*/\partial(\phi S) > 0$.

Proof.

- *(i)* Using Proposition 1, $(B - w_0)^2 < (f + t)/2$ implies that condition (14) that guarantees that $\overline{x}^* > 0$ becomes $\phi S < B - w_0$. Moreover, using Lemma 1, and observing that $(f+t)/2 < (f+t)(n^2-1)/n^2$, the condition $(B-w_0)^2 < (f+t)/2$ guarantees that the market is non-covered. Finally, using Lemma 6, and observing that $(f + t)/2 < (f + t)(n + 1)/4$, the condition $(B - w_0)^2 < (f + t)/2$ implies that two cases may arise:

  - *(ia)* If $\phi S < (\phi S)_1^{CSX}$, then $\partial\overline{x}^*/\partial(\phi S) > 0$.
  - *(ib)* Observing that $B-w_0 < (\phi S)_2^{CSX}$, then if $(\phi S)_1^{CSX} < \phi S < B-w_0$, $\partial\overline{x}^*/\partial(\phi S) < 0$.

(*ii*) Using Proposition 1, $(f+t)/2 < (B-w_0)^2 < (f+t)(n^2-1)/n^2$ implies that condition (14) that guarantees that $\overline{x}^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Moreover, using Lemma 1, $(B-w_0)^2 < (f+t)(n^2-1)/n^2$ guarantees that the market is non-covered. Finally, using Lemma 6, and observing that $(\phi S)_1^{CSX} < \sqrt{(f+t)/2}$,[10] and $(f+t)(n^2-1)/n^2 < (f+t)(n+1)/4$, the condition $(B-w_0)^2 < (f+t)(n^2-1)/n^2$ implies that two cases may arise:

(*iia*) If $\phi S < (\phi S)_1^{CSX}$, then $\partial\overline{x}^*/\partial(\phi S) > 0$.

(*iib*) then if $(\phi S)_1^{CSX} < \phi S < \sqrt{(f+t)/2}$, $\partial\overline{x}^*/\partial(\phi S) < 0$.

(*iii*) Using Proposition 1, $(f+t)(n^2-1)/n^2 < (B-w_0)^2 < (f+t)(n+1)/4$ implies that condition (14) that guarantees that $\overline{x}^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Moreover, using Lemma 1, since $(B-w_0)^2 > (f+t)(n^2-1)/n^2$, the condition $\phi S < (\phi S)_1^{NC}$ now guarantees that the market is non-covered. Finally, since in this case we cannot compare $(\phi S)_1^{CSX}$ and $\sqrt{(f+t)/2}$, the sign of $\partial\overline{x}^*/\partial(\phi S)$ cannot be determined.

(*iv*) Using Proposition 1, $(B-w_0)^2 > (f+t)(n+1)/4$ implies that condition (14) that guarantees that $\overline{x}^* > 0$ becomes $\phi S < \sqrt{(f+t)/2}$. Moreover, using Lemma 1, since $(B-w_0)^2 > (f+t)(n^2-1)/n^2$, the condition $\phi S < (\phi S)_1^{NC}$ guarantees that the market is non-covered. Finally, since now $(B-w_0)^2 > (f+t)(n+1)/4$, Lemma 6 shows that we always have $\partial\overline{x}^*/\partial(\phi S) > 0$. ■

By combining Propositions 5 and 6, it is now easy to see that Proposition 3 states the comparative statics results for C^* and $\overline{x}^*$ with respect to ϕS only when the signs of $\partial C^*/\partial(\phi S)$ and $\partial\overline{x}^*/\partial(\phi S)$ are both determined. ■

[10] Indeed, according to Lemma 5, $(\phi S)_1^{CSX} < \sqrt{(f+t)/2}$ if and only if

$$B - w_0 < 2\frac{(n+1)}{(n+3)}\sqrt{\frac{f+t}{2}}$$

But since

$$2\frac{(n+1)^2}{(n+3)^2}(f+t) > \frac{(n^2-1)}{n^2}(f+t)$$

the condition $B - w_0 < (n^2-1)(f+t)/n^2$ guarantees that $(\phi S)_1^{CSX} < \sqrt{(f+t)/2}$.

Proof of Proposition 4

(i) Let us start with w_0. By differentiating the threshold

$$(\phi S)_1^{CSC} \equiv B - w_0 - \sqrt{(B - w_0)^2 - \frac{(f+t)(n+1)}{4}}$$

we obtain

$$\frac{\partial (\phi S)_1^{CSC}}{\partial w_0} = -1 + \frac{B - w_0}{\sqrt{(B - w_0)^2 - \frac{(f+t)(n+1)}{4}}} > 0$$

(ii) For the booty B we have:

$$\frac{\partial (\phi S)_1^{CSC}}{\partial w_0} = 1 - \frac{B - w_0}{\sqrt{(B - w_0)^2 - \frac{(f+t)(n+1)}{4}}} < 0$$

(iii) For f and t, by differentiating the threshold, we obtain:

$$\frac{\partial (\phi S)_1^{CSC}}{\partial f} = \frac{\partial (\phi S)_1^{CSC}}{\partial t} = \frac{1}{2} \frac{(n+1)/4}{\sqrt{(B - w_0)^2 - \frac{(f+t)(n+1)}{4}}} > 0$$

(iv) Finally, for n, we have:

$$\frac{\partial (\phi S)_1^{CSC}}{\partial n} = \frac{1}{2} \frac{(f+t)/4}{\sqrt{(B - w_0)^2 - \frac{(f+t)(n+1)}{4}}} > 0$$

■

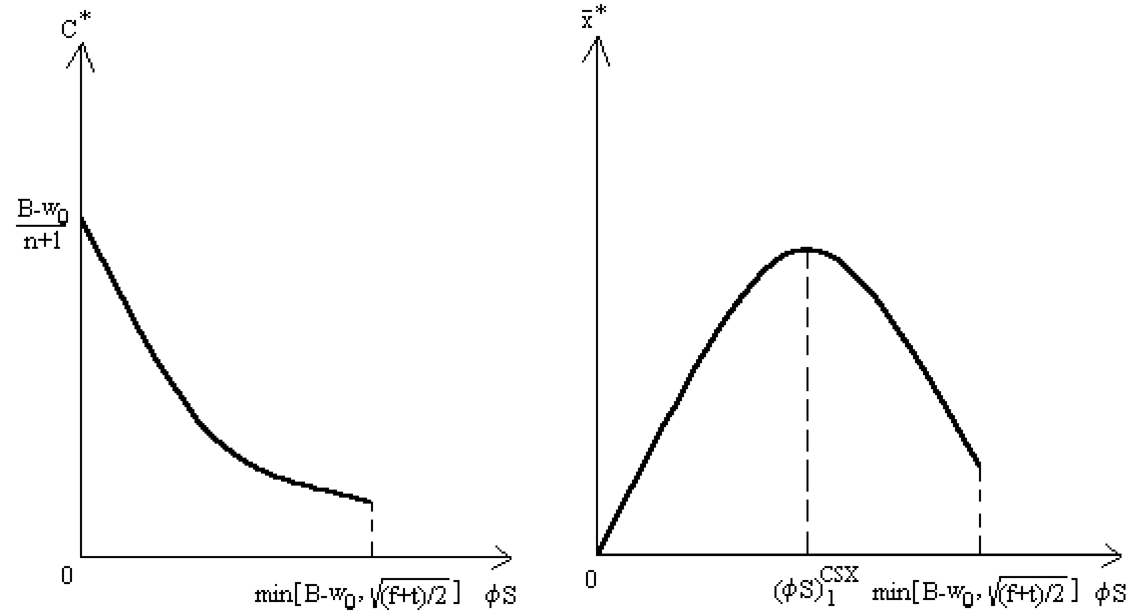

Figure 1: Case when $(B - w_0)^2 < (f + t)(n^2 - 1) / n^2$

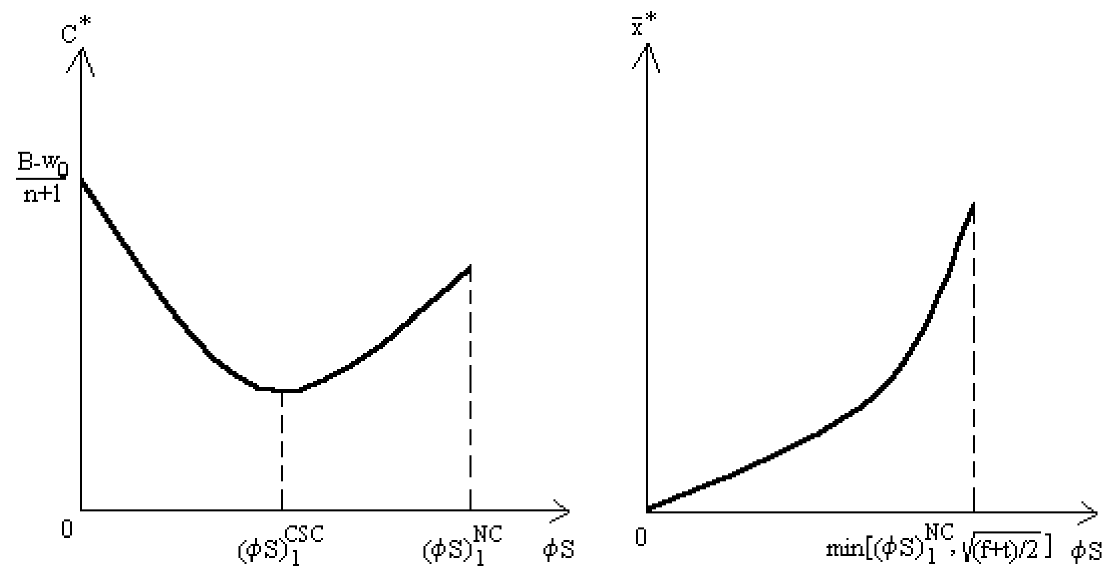

Figure 2: Case when $(B - w_0)^2 > (f + t)(n + 1) / 3$