Economics 250c
Fall 2008, Lecture 6
Two-sided selection

We have seen that when selection into one of two choices is determined by a comparison of the indirect utilities of the two choices, the "selection bias" in the unobserved component of one of the choices is a function of the probability that choice is taken. This result depends critically on the single agent model of selection. Many situations in labor economics are actually 2-sided selection problems. For example, a match is observed in the labor market if the worker prefers the job to other alternatives and if the firm prefers the worker to other available workers. Under two-sided selection, the degree of selection bias is not necessarily the same for any two observations with the same probability of selection. We illustrate some of the issues using a model (again, from the ice age) of union-nonunion sectoral choice and wage determination.

Assume that worker i has observed characteristics X_i and unobserved ability θ_i that together determine productivity p_i. In the nonunion sector the expected (log) wage for the worker is equal to his or her expected (log) productivity:

$$E[w_i|X_i, \theta_i, nonunion] = E[p_i|X_i, \theta_i] = X_i\beta + \theta_i.$$

In the union sector, the wage structure is "flatter" in the sense that

$$E[w_i|X_i, \theta_i, union] = \delta_0 + \delta_1(X_i\beta) + \lambda\theta_i$$

where $(0 < \delta_1 < 1)$ and $(0 < \lambda < 1)$. These assumptions imply that returns to observed and unobserved ability are lower in the union sector. Effectively, the union wage structure redistributes wages from high productivity to lower productivity workers. A worker compares the expected wages in the sectors and chooses a union job if

$$\begin{aligned} E[w_i|X_i, \theta_i, union] &\geq E[w_i|X_i, \theta_i, nonunion] + \tau_i \\ &\Leftrightarrow \delta_0 + \delta_1(X_i\beta) + \lambda\theta_i \geq X_i\beta + \theta_i + \tau_i \\ &\Leftrightarrow \tau_i + \theta_i(1-\lambda) \leq \delta_0 + (\delta_1 - 1)(X_i\beta), \end{aligned}$$

where τ_i is a "taste shock" (or error component of some more general description). A given union employer will be willing to hire worker i if

$$\begin{aligned} E[p_i|X_i, \theta_i] + \rho_i &> E[w_i|X_i, \theta_i, union] \\ &\Leftrightarrow X_i\beta + \theta_i + \rho_i > \delta_0 + \delta_1(X_i\beta) + \lambda\theta_i \\ &\Leftrightarrow \rho_i + \theta_i(1-\lambda) > \delta_0 + (\delta_1 - 1)(X_i\beta), \end{aligned}$$

where ρ_i is a "productivity shock." For a worker with observed characteristics X_i and unobserved ability θ_i to be observed working in a union job ($U_i = 1$) two things have to be true:

$$\tau_i + \theta_i(1-\lambda) \leq \delta_0 + (\delta_1 - 1)(X_i\beta) = k_i \quad (1)$$

$$\rho_i + \theta_i(1-\lambda) > \delta_0 + (\delta_1 - 1)(X_i\beta) = k_i \quad (2)$$

Consider a worker with a very high value of $X_i\beta$: for this worker k_i is a large negative number and equation (1) is more likely to be a constraint than equation (2). Intuitively, highly skilled workers are not so likely to want to work in the "flat" sector, but firms will be quite willing to hire them. For such workers, the combination $\tau_i + \theta_i(1-\lambda)$ must be relatively low, which means that on average union workers with high observed skills are negatively selected (θ_i is low).

Now consider the reverse situation of a worker with a very low value of $X_i\beta$: for this worker k_i is a large positive number and equation (2) is more likely to be a constraint than equation (1). Intuitively, low skilled workers are likely to want to work in the "flat" sector, but firms will be unwilling to hire them. For such workers, the combination $\tau_i + \theta_i(1-\lambda)$ must be relatively high, which means that on average union workers with low observed skills are positively selected (θ_i is high).

To see how this would look, I simulated the model assuming $X_i\beta$ is uniformly distributed between -1 and $+1$, that $\theta_i = \alpha X_i\beta + v_i$, where $v_i\tilde{}N(0, 0.2)$, and that τ_i and ρ_i are both normally distributed with mean 0 and standard deviation 0.2, and $(v_i, \tau_i, \rho_i, X_i\beta)$ are independent. I set $\delta_0 = 0.1$ and $\lambda = \delta_1 = 0.2$. These values give rise to a pattern of union densities, relative gaps between union and nonunion wages (from an OLS regression $w_i = (X_i\beta)\pi_x + U_i\pi_u + error$), and "corrected" relative wage gaps (from an OLS regression $w_i = (X_i\beta)\pi_x + U_i\pi_u + \theta_i\pi_\theta + error$) across the "observed skill distribution" that look a lot like what we see in real data. See the attached tables.

Note that in this model union membership rate is U-shaped w.r.t. skill: people with very low $X_i\beta$ and very high $X_i\beta$ are unlikely to be in the union sector (as is true in the 'real world'). So people from the ends of the skill distribution can have the same probability of unionization, but will have very different "selection bias" in their wages.

More formally:

$$P[U_i = 1|X_i\beta] = P[\tau_i + \theta_i(1-\lambda) \leq k_i, \quad \rho_i + \theta_i(1-\lambda) > k_i]$$

If $\tau_i\tilde{}F_\tau$ and $\rho_i\tilde{}F_\rho$, independent of each other, then

$$P[U_i = 1|X_i\beta] = \int_{\theta_i} F_\tau(k_i - \theta_i(1-\lambda)) \times (1 - F_\rho(k_i - \theta_i(1-\lambda)))dF_{\theta_i}.$$