Preliminary

Marriage Matching, Risk Sharing and Spousal Labor Supplies

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Abstract

The paper integrates marriage matching with a collective model of spousal labor supplies with public goods and full spousal risk sharing. The paper derives testable implications of how changes in marriage market conditions affect spousal labor supplies. The model motivates a sufficient statistic for marriage market tightness that is specific to the marital match and highlights several empirical issues that arise when estimating the effects of marriage market conditions on labor supply. The empirical section of the paper tests for marriage market effects on spousal labor supplies using data from the 2000 US census and on hours in home production from the American Time Use Survey. Changes in marriage market tightness often have large estimated effects on spousal labor supplies and hours in home production in the direction predicted by the theory. Controlling for variation in labor market conditions across marriage markets and for heterogeneity in marital production technologies across different types of matches has substantive implications for the parameter estimates.

1 Introduction

Thirty years ago, Becker (1973; 1974; summarized in his 1991 book) introduced his landmark transferable utilities model of the marriage market. A cornerstone of that model is that resource transfers between spouses are used to clear the marriage market. This model is important for two reasons. First, it recognizes that spouses may have divergent interests. Second, it proposes that the marriage market is a class of general equilibrium models.[1]

The subsequent literature developed in three directions. First, researchers have found empirical evidence that is supportive of Becker's assumption of divergent interests within the family. More specifically with respect to the marriage market, researchers have found that a higher sex ratio (ratio of men to women) will result in more resource transfers from husbands to wives.[2]

Second, Chiappori and his collaborators have developed a framework, the "collective model,"for estimating household members' preferences when members have divergent interests. A key feature of this framework is that it assumes efficient intrahousehold allocations. The intrahousehold allocation is what a social planner will choose if the planner's objective function is the weighted sum of household members' utilities, where the weights reflect the bargaining power of each member. Researchers have also found empirical support for this model. Third, building on earlier research, Choo Siow (2006; hereafter CS) have developed an empirically tractable transferable utilities marriage matching model, where the marginal utility of income is assumed to be constant.

Much of this work has been developed within a labor supply framework, where increases in the sex ratio are predicted to reduce the labor supply of wives and increase the labor supply of husbands via an income effect. The standard female labor supply regression in this literature is:

$$\ln H_{jG}^{r} = \alpha + \beta \ln \frac{m_i^r}{f_j^r} + \gamma d_j^f + \rho d_i^m + u_{jG}^r \tag{1}$$

[1]The equivalence between transferable utilities models of the marriage market and Walrasian models are studied by Ostroy, Zame...

[2]E.g. Angrist 2002; Chiappori, Fortin, and Lacroix 2002; Francis 2005; Grossbard-Schechtman 1993; Seitz 2005, South and Trent.

where d_j^f and d_i^m are controls for the wife's and husband's types, respectively. The parameter β is interpreted as the effect of marriage market conditions on labor supply. An increase in the number of men in region r relative to women typically has a negative effect of the labor supply of women, as better marriage opportunities result in greater transfers to women (resulting in lower labor supply).

In this paper, we make two contributions to the literature. Our first contribution is theoretical in nature. We build a collective model of marriage matching by embedding the collective model within the marriage market. Our collective model of the household builds on the collective model of spousal labor supplies with public goods by Blundell, Chiappori and Meghir (2006; hereafter BCM). We add to that model efficient spousal risk sharing. In the marriage market, individuals choose who to marry or to remain unmarried. The utility weights of husbands relative to their wives in the collective model are used to clear the marriage market. We show the existence of marriage market equilibrium. The transferable utilities marriage market model, e.g. Becker and CS, is a special case of our collective model of marriage matching.

The second contribution of our paper is empirical. Our model motivates a new empirical strategy for estimating the effects of changing marriage market conditions on spousal labor supplies and highlights three empirical difficulties with the standard interpretation of β in the above labor supply regression.

1. **Marital substitution effects.** Our theoretical model highlights the point that the relevant measure of marriage market conditions is not the aggregate sex ratio but a measure of an individual's option value. The model produces a statistic for this option value for a marriage between a type i man and a type j women, the ratio of unmarried men of type i to unmarried women of type j. We call this measure market tightness. Market tightness is a better measure of marriage market conditions than the sex ratio for two reasons. First, the within-region aggregate sex ratio does not capture the notion that matches to spouses of different types are valued differently by the agents. Our measure of market tightness, a measure of the relative supply singles within a match type, directly captures this notion. Second, if the numbers of other types of men and women change, there is no way to predict their effect on labor supply. The problems with simply adding the sex ratio of substitutes are twofold. First, it is not clear to the researcher who are better sub-

stitute spouses. Second, many of the own and 'obvious' substitute sex ratios (such as those from adjacent age groups) are highly collinear and therefore it is difficult to estimate each effect separately. For empirical tractability, researchers have primarily restricted their empirical specifications to own sex ratios. However, since spousal substitutes have been shown to be quantitatively significant (Angrist, *; Brandt, Siow and Vogel, 2007) it is useful to find an empirical proxy for overall market conditions for each marital match. Market tightness provides such a proxy.

2. **Labor market conditions and the sex ratio.** As pointed out in several studies in the past (Angrist, CFL), the sex ratio may be determined by labor market conditions and therefore endogenous in the labor supply equation. For example, regions with high relative demand for male labor may have high sex ratios and high male labor supply for reasons unrelated to the marriage market. Failing to control for local labor market conditions will incorrectly attribute the increase in male labor supply in high sex ratio regions to marriage market effects. We might therefore expect the coefficient on tightness to have an upward bias on the male labor supply equation and a downward bias in the female labor supply equation. We account for this issue by directly controlling for local labor market conditions in our regression. In particular, we control for both the state level wage and asset distributions, by gender and type, in all of our labor supply regressions.

3. **Heterogeneity in marital production technologies.** The standard empirical specification assumes that the marital technology is the same across different types of marriages. If there is heterogeneity in the marital production technology across different types of marriages, changes in market tightness will confound changes in supplies in the marriage market and changes in marital technologies. Suppose for instance that the gains to specialization are higher in certain types of marriages than others. Tightness will be higher in marriages where the gains to marriage are higher. Also, women's labor supply will likely be lower when the gains to specialization are greater. In this case, heterogeneity in marital production technologies will induce a positive correlation between women's time at home and market tightness (gains to specialization result in more marriages). Our empirical specification

will consider the implications of ignoring this potential source of bias. Our identification strategy is equivalent to the difference in differences estimation of treatment effects using state and time panel data. Instead of the usual time variation, we use marital match (ij) variations. It is less restrictive than most existing empirical work on the effects of marriage market conditions on spousal labor supplies.[3]

Thus we will investigate the empirical labor supply model for type j wives in ij marriages:

$$\ln H^r_{ijG} = \alpha_{ij} + \beta_{ij} T^r_{ijGk} + z^r_{ij}{}' \beta_1 + \varepsilon^r_{ijG},\ G = 1,.,G^r;\ r = 1,..,R \qquad (2)$$

where H^r_{ijG} is the labor supply of wife G in an ij marriage in society r. $H^r_{ijG} \subset H^r_{jG}$, ε^r_{ijG} is the error term of the regression, z^r_{ij} is a vector of covariates which includes proxies for labor market conditions for type i and type j individuals, and other factors which may affect the marital output of ij marriages. Equation 2 can be directly derived from our theory. The theory will also show that $\beta_{ij} < 0$. That is, when market tightness increases, and the bargaining power of wives in ij marriages increases, their labor supplies fall.

We estimate the effect of market tightness on two elements of household behavior. First, we estimate models of household labor supply using the 2000 Census. Second, we estimate models of hours spent in home production using the American Time Use Survey (ATUS). A summary of the empirical results are as follows. After controlling for labor market conditions, state effects, individual characteristics, marriage market tightness is negatively (positively) correlated with wives' (husbands') labor supplies and hours in home production. The magnitudes of the responses differ by race and gender. Often, the responses are quantitatively large. A one standard deviation increase in marriage market tightness often leads to more than a one quarter standard deviation decrease in wives' labor supplies in all dimensions. Non-white spousal responses are larger than white spousal responses. Thus changes in marriage market tightness have quantitatively significant effects on intrahousehold reallocations in the direction predicted by our theory. Controlling for labor market conditions and heterogeneity in match-specific production changes the parameters estimates substantially.

[3]For example, Angrist uses individual types variation alone and CFL use across state variation alone to identify their sex ratio effects.

The methodological objective of this paper is to provide a unified framework for interpreting reduced form estimates of marriage market conditions on spousal labor supplies. We do not establish identification of the structural parameters of our collective marriage matching model nor do we estimate any structural parameters. Our companion paper, CSSa, studies identification of our collective marriage matching model.

Often, empirical applications of the static collective model of spousal labor supplies ignore spousal risk sharing and public goods. We do not take a stand on how important these two concerns are. As will be discussed below, our reduced form results do not shed light on whether is full spousal risk sharing in marriage or not. Rather we include risk sharing in our model to show that the reduced form implications that we test in this paper are robust to spousal risk sharing or otherwise. Similarly, we include public goods to show that our results are also robust to the extent of public goods in marriage. Thus we do not restrict our analysis to childless couples as usually done in the empirical static collective model literature. This difference is due primarily to the fact that we are estimating a reduced form relationship rather than the structural parameters that the empirical static collective model literature usually do. In CSSa, we will take a stand on these issues when we also estimate structural parameters.

Because our work is related to a large literature, it is convenient to postpone discussion of the literature until the end of the paper.

2 The model

Consider a society in which there are I types of men, $i = 1, .., I$, and J types of women, $j = 1, .., J$. All type i men have the same preferences and ex-ante opportunities; and all type j women also have the same preferences and ex-ante opportunities. That is, the type of an individual is defined by his or her preferences and ex-ante opportunities.

Let m_i be the number of type i men and f_j be the number of type j women. M and F are the vectors of the numbers of each type of men and women respectively.

The model is a two period model. In the first period, individuals choose whether to marry and who to marry if they marry. An $\{i, j\}$ marriage is a marriage between a type i man and a type j woman. At the time of their marital choices, wages and non-labor income for each marital choice are random variables.

After their marital choices, and in the second period, intrahousehold allocations are chosen after wages and non-labor income for each household are realized. We consider a static model of private and public consumption, and labor supply choices. The rationale for including public good consumption within marriage is to capture resources allocated to children, if any, in the marriage.[4]

For expositional simplicity, all individuals have positive hours of work. As will become clear in the development, it is straightforward to extend the model to allow other kinds of marriages such as ones where the wife does not work, or cohabitation rather than marriage.[5]

Let C_{ijgG} be the own consumption of wife G of type j matched to a type i husband g. K_{ijgG} is the amount of public good each of them consumes. H_{ijgG} is her labor supply.We normalize the total amount of time for each individual to 1. Her utility function is:

$$U_{ij}(C_{ijgG}, 1-H_{ijgG}, K_{ijgG}, \varepsilon_{ijG}) = \widehat{Q}_{ij}(C_{ijgG}, 1-H_{ijgG}, K_{ijgG}) + \Gamma_{ij} + \varepsilon_{ijG} \quad (3)$$

$\widehat{Q}_{ij}(.)$, her felicity function, depend on i, j which allows for differences in home production technologies across different marital matches. We will impose restrictions on $\widehat{Q}_{ij}(.)$ later. The invariant gain to an $\{i, j\}$ marriage for the woman, Γ_{ij}, shifts her utility according to the type of marriage and allows the model to fit the observed marriage matching patterns in the data. It may vary across different types of marriages within a society.[6] The important restriction is that Γ_{ij} does not affect her marginal utilities from consumption or labor supply.

Finally, we assume ε_{ijG} is a random variable that is realized before marital decisions are made. ε_{ijG} is independent of C_{ijgG}, H_{ijgG}, K_{ijgG} and also g. That is, it does not depend on the specific identity of the type i male. The independent realizations of this random variable across different women of type j in the same society will produce different marital choices for different type j women in period one. If a woman chooses not to marry, then $i = 0$.

The specification of a representative man's problem is similar to that of women. Let c_{ijgG} be the own consumption of man g of type i matched to a type j woman G. K_{ijgG} is his public good consumption. Denote his labor

[4]We will not formally model children in this paper.

[5]Choo and Siow 2006a extends CS to include cohabitation.

[6]In the empirical work, we allow $\widehat{Q}_{ij}(.)$ and Γ_{ij} to differ across societies as well.

supply by h_{ijgG}. If he chooses not to marry, then $j = 0$. The utility function for males is described by:

$$u_{ij}(c_{ijgG}, 1 - h_{ijgG}, K_{ijgG}, \varepsilon_{ijg}) = \widehat{q}_{ij}(c_{ijgG}, 1 - h_{ijgG}, K_{ijgG}) + \gamma_{ij} + \varepsilon_{ijg}, \quad (4)$$

$\widehat{q}_{ij}(.)$, his felicity function, depends on i, j will allow the model to fit observed labor supply behavior for different types of marriages. We will impose restrictions on $\widehat{q}_{ij}(.)$ later. The invariant gain to an i, j marriage for the man, γ_{ij}, shifts his utility by i, j and allows the model to fit the observed marriage matching patterns in the data. It may vary across different types of marriages. The important restriction is that γ_{ij} does not affect his marginal utilities from consumption and labor supply.

Finally, we assume ε_{ijg} is a random variable that is realized before marital decisions are made. ε_{ijg} is independent of c_{ijgG}, h_{ijgG}, K_{ijgG} and G. The independent realizations of this random variable across different men of type i in the same society will produce different marital choices for different type i men in period one.

2.1 The collective model with efficient risk sharing

The objective of this section is to derive two results, both of which are relevant to the empirical work. First, we will show how efficient risk sharing affects the expected felicities of the spouses as bargaining power within the household changes. Second, we will impose restrictions such that the wife will on average work more and the husband will on average work less as the bargaining power of the husband increases.

We start first with intrahousehold allocation after the marriage decision has been made. Consider a particular husband g and his wife G in an $\{i, j\}$ marriage. Total non-labor family income is A_{ijgG} which is a random variable. The wage for the wife is also a random variable W_{ijgG}. The male's wage is another random variable w_{ijgG}. A_{ijgG}, W_{ijgG} and w_{ijgG} are realized in the second period, after the marriage decision.

The family budget constraint is:

$$c_{ijgG} + C_{ijgG} + K_{ijgG} \leq A_{ijgG} + W_{ijgG}H_{ijgG} + w_{ijgG}h_{ijgG} \quad (5)$$

Because wages and non-labor income, W_{ijgG}, w_{ijgG}, and A_{ijgG}, are random variables whose values are realized after marriage. In the second period, the spouses can share income risk in the first period.

The continuous joint distribution of A_{ijgG}, W_{ijgG} and w_{ijgG} with bounded support is characterized by the parameter vector Z. Z is known to individuals before their marriage decisions. Let $S_{ijgG} = \{W_{ijgG}, w_{ijgG}, A_{ijgG}\}$. Let $F(S_{ijgG}|Z)$ denote the cumulative multivariate wages and non-labor income distribution in the society.

Let $\mathbf{E}$ be the expectations operator. Following the collective model with full risk sharing, we pose the efficient risk sharing spousal arrangement as a planner solving the following problem:

$$\max_{\{C,c,H,h\}} \mathbf{E}(\widehat{Q}(C_{ijgG}, 1 - H_{ijgG}, K_{ijgG})|Z) + p_{ij}\mathbf{E}(\widehat{q}(c_{ijgG}, 1 - h_{ijgG}, K_{ijgG})|Z) \tag{P1}$$

subject to (5) for all S_{ijgG}

Problem (P1) is BCM with efficient risk sharing. In problem (P1), the planner chooses family consumption and labor supplies to maximize the weighted sum of the wife's and the husband's expected felicities subject to their family budget constraint. $p_{ij} \in R^+$ is the weight allocated to the husband's expected felicity. If $p_{ij} > 1$, the husband has more weight than the wife and vice versa. As in the collective model literature, p_{ij} depends on Z, marriage market conditions, and other factors affecting the gains to marriage in which the individuals live. Call p_{ij} the husband's power.

How the husband's power is determined in the marriage market is a central focus of this paper. However the determination of p_{ij} is not a concern of the social planner in solving in problem (P1). The planner takes p_{ij} as exogenous. When the intrahousehold allocation is the solution to problem (P1), the intrahousehold allocation is efficient.

Let $C_{ij}(p_{ij}, S_{ijgG})$, $H_{ij}(p_{ij}, S_{ijgG})$, $c_{ij}(p_{ij}, S_{ijgG})$, $h_{ij}(p_{ij}, S_{ijgG})$, $K_{ij}(p_{ij}, S_{ijgG})$ be the optimal intrahousehold allocation when state S_{ijgG} is realized. Let $\mathbf{Q}_{ij}(p_{ij}, Z)$ and $\mathbf{q}_{ij}(p_{ij}, Z)$ be the expected indirect felicities of the wife and the husband respectively before the state S_{ijgG} is realized:

$$\mathbf{Q}_{ij}(p_{ij}, Z) = \mathbf{E}(\widehat{Q}_{ij}(C_{ij}(p_{ij}, S_{ijgG}), 1 - H_{ij}(p_{ij}, S_{ijgG}), K_{ij}(p_{ij}, S_{ijgG}))|Z)$$
$$\mathbf{q}_{ij}(p_{ij}, Z) = \mathbf{E}(\widehat{q}_{ij}(c_{ij}(p_{ij}, S_{ijgG}), 1 - h_{ij}(p_{ij}, S_{ijgG}), K_{ij}(p_{ij}, S_{ijgG}))|Z)$$

Appendix 1 shows that the solution to problem (P1) implies:

Proposition 1 *The changes in spousal expected felicities as the husband's* *power, p_{ij}, increases satisfy:*

$$\frac{\partial \mathbf{Q}_{ij}(p_{ij}, Z)}{\partial p_{ij}} = -p_{ij} \frac{\partial \mathbf{q}_{ij}(p_{ij}, Z)}{\partial p_{ij}} < 0 \tag{6}$$

The wife's expected felicity falls and the husband's expected felicity increases as p_{ij} increases. (6) traces the redistribution of spousal expected felicities as the husband's power increases.

We will now study how spousal labor supplies change as husband's power changes. A necessary condition for solving problem P1 is that given realized wages and non-labor income, i.e. S_{ijgG}, the planner solves problem P2:

$$\max_{C_{ijgG}, c_{ijgG}, H_{ijgG}, h_{ijgG}, K_{ijgG}} \widehat{Q}_{ij}(C_{ijgG}, 1 - H_{ijgG}, K_{ijgG}) + p_{ij} \widehat{q}_{ij}(c_{ijgG}, 1 - h_{ijgG}, K_{ijgG}) \tag{P2}$$

subject to $c_{ijgG} + C_{ijgG} + K_{ijgG} \leq A_{ijgG} + W_{ijgG} H_{ijgG} + w_{ijgG} h_{ijgG}$

Problem P2 is a deterministic static maximization problem. We will assume that the felicity functions are weakly separable, that the objective function in problem P2 can be written as:

$$\widehat{Q}_{ij}(\Omega(C_{ijgG}, 1 - H_{ijgG}), K_{ijgG}) + p_{ij} \widehat{q}_{ij}(\omega(c_{ijgG}, 1 - h_{ijgG}), K_{ijgG}) \tag{7}$$

BCM first analyzed problem P2 in the general and weakly separable case and we build on their results. In general, it is difficult to determine analytically how spousal labor supplies respond to changes in p_{ij}. Appendix 2 shows that in the weakly separable case, by restricting leisure (with suitably defined individual private income) and the public good to be normal goods for each spouse,

Proposition 2 *The wife's labor supply is increasing in p_{ij} whereas the husband's labor supply is decreasing in the husband's power, p_{ij}:*

$$\frac{\partial H_{ijGg}}{\partial p_{ij}} > 0 \; \forall S_{ijgG} \tag{8}$$

$$\frac{\partial h_{ijGg}}{\partial p_{ij}} < 0 \; \forall S_{ijgG} \tag{9}$$

(8) and (9) are expected.

Problem P2 is a unitary model of the family faced with wages W_{ijgG}, w_{ijgG}, and non-labor income A_{ijgG}. Thus we cannot reject a unitary model of the family for $\{i, j\}$ couples in the same society, by observing their spousal labor supplies behavior if they share risk efficiently.[7] For example, spousal labor supplies will satisfy Slutsky symmetry.

For notational convenience, if woman G of type j remains unmarried, denote her expected indirect utility as $\mathbf{Q}_{0j}(p_{0j}, Z)$ where $p_{0j} = 0$ and $\widehat{q}_{0j} \equiv 0$. Similarly, if man g of type i remains unmarried, denote his expected indirect utility as $\mathbf{q}_{i0}(p_{i0}, Z)$ where $p_{i0} = 1$ and $\widehat{Q}_{i0} \equiv 0$.

3 Marriage decisions in the first period

In the first period, agents decide whether to marry and who to marry if they choose to marry. We will use the additive random utility model to model this choice.

Consider a particular woman G of type j. Recall that she can choose between I types of men and whether or not to marry. She can choose between $I + 1$ choices. Let $p_{0j} = 0$. Her expected utility in an $\{i, j\}$ marriage is:

$$\overline{V}(i, j, p_{ij}, \varepsilon_{ijG}) = \mathbf{Q}_{ij}(p_{ij}, Z) + \Gamma_{ij} + \varepsilon_{ijG},\ i = 0, ..I \tag{10}$$

Given the realizations of all the ε_{ijG}, she will choose the marital choice which maximizes her expected utility. Let $\underline{\varepsilon_{jG}} = [\varepsilon_{0jG}, .., \varepsilon_{ijG}, .., \varepsilon_{IjG}]$ and $\Omega(\underline{\varepsilon_{jG}})$ denote the joint density of $\underline{\varepsilon_{jG}}$. The expected utility from her optimal choice will satisfy:

$$V^*(\underline{\varepsilon_{jG}}) = \max[\overline{V}(0, j, p_{0j}, \varepsilon_{0jG}), .., \overline{V}(i, j, p_{ij}, \varepsilon_{ijG}), ..] \tag{11}$$

The problem facing men in the first stage is analogous to that of women. Let $p_{i0} = 0$. A man g of type i in an $\{i, j\}$ marriage, with ε_{ijg}, attains an expected utility of:

$$\overline{v}(i, j, p_{ij}, \varepsilon_{ijg}) = \mathbf{q}_{ij}(p_{ij}, Z) + \gamma_{ij} + \varepsilon_{ijg},\ j = 0, .., J \tag{12}$$

Given the realizations of all the ε_{ijg}, he will choose the marital choice which maximizes his expected utility. He can choose between $J + 1$ choices.

[7] This point is well known. Hayashi, Altonji and Kotlikoff, Lich Tyler, Mazzacco, Ogaki.

Let $\underline{\varepsilon_{ig}} = [\varepsilon_{i0g}, .., \varepsilon_{ijg}, ..]$ and $\omega(\underline{\varepsilon_{ig}})$ denote the joint density of $\underline{\varepsilon_{ig}}$. The expected utility from his optimal choice will satisfy:

$$v^*(\underline{\varepsilon_{ig}}) = \max[\overline{v}(i, 0, p_{i0}, \varepsilon_{i0g}), ..\overline{v}(i, j, p_{ij}, \varepsilon_{ijg})..] \tag{13}$$

4 The Marriage Market

Let p be the matrix of husband's powers where a typical element is p_{ij} for $i, j \geq 1$. Assume that the random vectors $\underline{\varepsilon_{jG}}$ and $\underline{\varepsilon_{ig}}$ are independent of p and Z. Let $\Phi_{ij}(p)$ denote the probability that a woman of type j will choose a spouse of type i, $i = 0, ..I$.

Since each woman of type j is solving the same spousal choice problem (11),

$$\Phi_{ij}(p) = \Pr(\varepsilon_{i'jG} - \varepsilon_{ijG} < \mathbf{Q}_{ij}(p_{ij}, Z) + \Gamma_{ij} - \mathbf{Q}_{i'j}(p_{i'j}, Z) - \Gamma_{i'j}\ \forall i' \neq i) \tag{14}$$

$$= \int_{\varepsilon_{ijG}=-\infty}^{\infty} \int_{\varepsilon_{0jG}=-\infty}^{\mathbf{R}(0,i,j,G,p,Z,)} .. \int_{\varepsilon_{IjG}=-\infty}^{\mathbf{R}(I,i,j,G,p,Z,)} \Omega(\underline{\varepsilon_{jG}}) d\varepsilon_{ijG} d\underline{\varepsilon_{\neq i,jG}}$$

where $\mathbf{R}(i', i, j, G, p, Z,) \equiv \mathbf{Q}_{ij}(p_{ij}, Z) + \Gamma_{ij} - \mathbf{Q}_{i'j}(p_{i'j}, Z) - \Gamma_{i'j} + \varepsilon_{ijG}$

When there are f_j number of type j women, the number of type j women who want to choose type i spouses, $i = 0, .., I$ is approximated by $\overline{\mu}_{ij}(p, f_j) = \Phi_{ij}(p) f_j$.

Using (14), for $i \geq 1$,

$$\frac{\partial \overline{\mu}_{ij}(p, f_j)}{\partial p_{i'j}} = f_j \frac{\partial \Phi_{ij}(p)}{\partial p_{i'j}} = \begin{cases} \leq 0, & i' = i \\ \geq 0, & i' \neq i \end{cases} \tag{15}$$

$\overline{\mu}_{ij}(p, f_j)$ is the demand function by type j women for type i husbands. (15) says that the demand function satisfies the weak gross substitute assumption. That is, the demand by type j women for type i husbands, $i \geq 1$, is weakly decreasing in p_{ij} and weakly increasing in $p_{i'j}$, $i' \neq i$. Such a result is expected. All other types of potential spouses, $i' \neq i$, are substitutes for type i spouses. When the bargaining power of type i spouses increase, demand for that type of spouse is expected to weakly fall and the demand for other types of spouses is expected to weakly increase.

Similarly, let $\phi_{ij}(p)$ denote the probability that a man of type i will choose a spouse of type j, $j = 0, ..J$. Since each man of type i is solving the same spousal choice problem (13),

$$\phi_{ij}(p) = \Pr(\varepsilon_{ij'g} - \varepsilon_{ijg} < \mathbf{q}_{ij}(p_{ij}, Z) + \gamma_{ij} - \mathbf{q}_{ij'}(p_{ij'}, Z) - \gamma_{ij'} \ \forall j' \neq j) \tag{16}$$
$$= \int_{\varepsilon_{ijg}=-\infty}^{\infty} \int_{\varepsilon_{i0G}=-\infty}^{\mathbf{r}(0,i,j,g,p,Z)} .. \int_{\varepsilon_{iJG}=-\infty}^{\mathbf{r}(J,i,j,g,p,Z)} \omega(\underline{\varepsilon_{ig}}) d\varepsilon_{ijg} d\underline{\varepsilon_{i,\neq jg}}$$
where $\mathbf{r}(j', i, j, g, p, Z) \equiv \mathbf{q}_{ij}(p_{ij}, Z) + \gamma_{ij} - \mathbf{q}_{ij'}(p_{ij'}, Z) - \gamma_{ij'} + \varepsilon_{ijg}$

When there are m_i number of type i men, the number of type i men who want to choose type j spouses, $j = 0, .., J$ is approximated by $\underline{\mu}_{ij}(p, m_i) = \phi_{ji}(p) m_i$.

Using (16), for $j \geq 1$,

$$\frac{\partial \underline{\mu}_{ij}(p, f_j)}{\partial p_{ij'}} = m_i \frac{\partial \phi_{ij}(p)}{\partial p_{ij'}} = \begin{cases} \geq 0, \ j' = j \\ \leq 0, \ j' \neq j \end{cases} \tag{17}$$

$\underline{\mu}_{ij}(p, m_i)$ is the demand function by type i men for type j wives. (17) says that the demand function satisfies the weak gross substitute assumption. The explanation is the same as that given above for the demand for husbands.

Marriage market clearing requires the supply of wives (husbands) to be equal to the demand (husbands) for wives for each type of marriage:

$$\underline{\mu}_{ij} = \overline{\mu}_{ij} = \mu_{ij} \ \forall \ \{i > 0, \ j > 0\} \tag{18}$$

There are feasibility constraints that the stocks of married and single agents of each gender and type cannot exceed the aggregate stocks of agents of each gender in the society:

$$f_j = \mu_{0j} + \sum_i \mu_{ij} \tag{19}$$

$$m_i = \mu_{i0} + \sum_j \mu_{ij} \tag{20}$$

We can now define a rational expectations equilibrium. There are two parts to the equilibrium, corresponding to the two stages at which decisions are made by the agents. The first corresponds to decisions made in the marriage market; the second to the intra-household allocation. In equilibrium, agents make marital status decisions optimally, the sharing rules clear each marriage market, and conditional on the sharing rules, agents choose consumption and labor supply optimally. Formally:

Definition 3 *A rational expectations equilibrium consists of a distribution of males and females across individual type, marital status, and type of marriage* $\{\hat{\mu}_{0j}, \hat{\mu}_{i0}, \hat{\mu}_{ij}\}$*, a set of decision rules for marriage, a set of decision rules for spousal consumption, leisure and public goods* $\{\hat{C}_{ijgG}, \hat{c}_{ijgG}, \hat{L}_{ijgG}, \hat{l}_{ijgG}, \widehat{K}_{ijgG}\}$*, and a matrix of husbands' powers* $\widehat{p}$ *such that:*

1. *Marriage decisions solve (11) and (13), obtaining* $\{V^*(\underline{\varepsilon_{jG}}), v^*(\underline{\varepsilon_{ig}})\}$*.*

2. *All marriage markets clear implying (18), (19), (20) hold;*

3. *For an* $\{i, j\}$ *marriage, the decision rules* $\{\hat{C}_{ijgG}, \hat{c}_{ijgG}, \hat{L}_{ijgG}, \hat{l}_{ijgG}, \widehat{K}_{ijgG}\}$ *solve (P1).*

Theorem 4 *A rational expectations equilibrium exists.*

Sketch of proof: We have already demonstrated (1) and (3). So what needs to be done is to show that there is a matrix of husbands' powers, $\widehat{p}$ which clears the marriage market. Consider a matrix of admissible husband's powers p. For every marriage market $\{i, j\}$ excluding $i = 0$ or $j = 0$, define the excess demand function for marriages by men:

$$E_{ij}(p) = \underline{\mu}_{ij}(p) - \overline{\mu}_{ij}(p) \tag{21}$$

The demand and supply functions, $\underline{\mu}_{ij}(p)$ and $\overline{\mu}_{ij}(p)$, for every marriage market $\{i, j\}$, satisfy the weak gross substitute property, (15) and (17). So the excess demand functions also satisfy the weak gross substitute property. Mas-Colell, Winston and Green (1995: p. 646, exercise 17.F.16^C) provide a proof of existence of market equilibrium when the excess demand functions satisfy the weak gross substitute property. For convenience, we reproduce their proof in our context in Appendix 3. Kelso and Crawford (1982) were the first to use the gross substitute property to demonstrate existence in matching models.

Our collective model of marriage matching shows that the transferable utilities model of the marriage market can be generalized to non-transferable utilities where the marginal utilities of consumption is not constant.

5 The logit spousal choice model

The rest of the paper concerns some empirical implications of the above model.

¿From here on, we will assume the logit random utility model, that ε_{ijG} and ε_{ijg} are i.i.d. extreme value random variables. In this case, McFadden (1974) showed that for every type of woman j, the relative demand for type i husbands is:

$$\ln \overline{\mu}_{ij} - \ln \mu_{0j} = (\Gamma_{ij} - \Gamma_{0j}) + \mathbf{Q}_{ij}(p_{ij}, Z) - \mathbf{Q}_{0j}(Z) \ , \quad i = 1, .., I \tag{22}$$

where $\overline{\mu}_{ij}$ is the number of $\{i, j\}$ marriages demanded by j type females and μ_{0j} is the number of type j females who choose to remain unmarried.

Similarly, for every type of man i, the relative demand for type j wives is:

$$\ln \underline{\mu}_{ij} - \ln \mu_{i0} = (\gamma_{ij} - \gamma_{i0}) + \overline{q}_{ij}(p_{ij}, Z) - \overline{q}_{i0}(Z), \quad j = 1, .., J, \tag{23}$$

where $\underline{\mu}_{ij}$ is the number of $\{i, j\}$ marriages supplied by j type males and μ_{i0} is the number of type i males who choose to remain unmarried.

6 One period marriage without uncertainty

Most of literature on the collective model deals with a static model of intrahousehold allocations without uncertainty. That is, wages and non-labor income are known as of the time the individuals enter into the marriage. Our marriage matching framework can accommodate this case and our structural labor supply paper, CSSa, studies this case.

Let observed wages, non-labor income and labor supplies be equal to true wages, non-labor income and labor supplies plus measurement error:

$$\widetilde{W}_{ij} = W_{ij} + \varepsilon^{W\pi}_{ijgG} \tag{24}$$

$$\widetilde{w}_{ij} = w_{ij} + \varepsilon^{w\pi}_{ijgG} \tag{25}$$

$$\widetilde{A}_{ij} = A_{ij} + \varepsilon^{A\pi}_{ijgG} \tag{26}$$

$$\widetilde{H}_{ij} = H_{ij} + \varepsilon^{L\pi}_{ijgG} \tag{27}$$

$$\widetilde{h}_{ij} = h_{ij} + \varepsilon^{l\pi}_{ijgG} \tag{28}$$

where $\widetilde{X}_{ij}$ is the observed values of X_{ij}. $\varepsilon^{W\pi}_{ijgG}$, $\varepsilon^{w\pi}_{ijgG}$, $\varepsilon^{L\pi}_{ijgG}$, $\varepsilon^{l\pi}_{ijgG}$and $\varepsilon^{A\pi}_{ijgG}$ are measurment errors which are uncorrelated with the true values. Marriages are still identified by $\{i, j, \pi\}$. Thus we can still use p_{ij}, the husband's power, to clear the marriage market. Given p_{ij}, instead of problem P1, the planner will now solve:

$$\begin{aligned}&\max_{\{C_{ij},c_{ij},H_{ij},h_{ij}\}} \widehat{Q}(C_{ij}, 1 - H_{ij}, K_{ij}) + p_{ij}\widehat{q}(c_{ij}, 1 - h_{ij}, K_{ij}) \\ &\text{subject to } C_{ij} + c_{ij} + K_{ij} \leq A_{ij} + W_{ij}H_{ij} + w_{ij}h_{ij} \; \forall \; S_{ij}\end{aligned} \tag{P1a}$$

(6), appropriately reinterpreted, continues to hold which is what is critical for marriage market clearing. Thus as long as we can identify the type of an individual and the marital matches that the individual can enter into, i.e. $\{i, j\}$, the empirical tests that we develop in this paper remain valid. Differences in observed spousal labor supplies across $\{i, j\}$ couples in the same society are interpreted as due to different realizations of measurement errors across these couples.

Thus the empirical results in this paper should be interpreted with care. *Even if our empirical results are consistent with our model predictions, they do not shed light on whether there is efficient risk sharing within the family or not.*

In our reduced form regressions, we do not include individual spousal wages as covariates. For every $\{i, j\}$ match, we observe labor income and labor supplies of multiple couples. Wages can be constructed by dividing labor income by hours of work. But measurement error in labor supplies and idiosyncratic labor supply shocks will induce variation in constructed wages as discussed above. Since risk sharing in marriage, measurement error in labor supplies, and idiosyncratic labor supply shocks are all salient factors in our data, and we do not have instruments for the idioysncratic components of wages, we do not use constructed wages in our reduced form labor supply estimates. Consequently, we do not take a stand on how much risk sharing there is in our data.

Put in another light, the reduced form implications that we test in this paper are independent of whether there is risk sharing or not. Similarly, our results are also independent of whether there are public goods in marriage or not.

6.1 Home Production

In this section, we extend our theoretical model to incorporate home production and a distinction between leisure time and time for work at home.

To be completed

7 Empirical Analysis

In this section, we estimate the reduced form of our structural model relating marriage market tightness to spousal labor supplies in both market and home production. We investigate the empirical relevance of three issues highlighted by our theoretical model of marital matching and intra-household allocations: (i) the role of marriage market substitutes, (ii) the endogeneity of market tightness to labor market conditions, and (iii) heterogeneity in the marital production technology.

7.1 Implications of the theory for reduced form labor supply estimation

In our companion paper (hereafter CSS) we establish formal identification of the structural parameters from observations on family labor supplies and marriage matching patterns in at least two marriage markets. In this paper, we will focus on the implications of our theory for empirical work that aims to measure the reduced form impact of the sex ratio on labor supply for married couples.

Let the equilibrium husband's power be $\{p_{ij}(\Gamma, \gamma, Z, M, F)\}$. Using market clearing, and subtracting relative supply, (22), from relative demand, (23):

$$\begin{aligned} T_{ij}^r = \ln \frac{\mu_{i0}^r}{\mu_{0j}^r} &= (\Gamma_{ij} - \Gamma_{0j}) + \mathbf{Q}_{ij}(p_{ij}^r, Z^r) - \mathbf{Q}_{0j}(Z^r) \\ &- ((\gamma_{ij} - \gamma_{i0}) + \mathbf{q}_{ij}(p_{ij}^r, Z^r) - \mathbf{q}_{i0}(Z^r)) \end{aligned} \tag{29}$$

where T_{ij} is the log of the ratio of the number of unmarried type i men to unmarried type j women. This measure of marriage market tightness, or the net spousal gain of the wife relative to her husband, is used in our empirical analysis in place of the aggregate ratio of men to women. Market tightness for an ij match in market r is determined by two components of the matching environment. The first component is the invariant gains to entering an ij relative to remaining single. The higher the invariant gains to marriage, the greater is market tightness for ij matches. The second component depends on the indirect utility derived from an ij match in society r relative to remaining single. As the relative indirect utility from an ij marriage increases, tightness is predicted to increase.

Equation (29) is a fundamental equilibrium relationship in the marriage market, a direct implication of marriage market clearing. It is the basis of the empirical content of marriage matching on the collective model in this paper and in CSS. To see this, consider a change in an exogenous parameter, ζ, that affects each component of market tightness. Using (6) and (29) we obtain

$$\begin{aligned}\frac{\partial T_{ij}}{\partial \zeta} =& \frac{\partial((\Gamma_{ij}-\Gamma_{0j})-(\gamma_{ij}-\gamma_{i0}))}{\partial \zeta} + (\frac{\partial(\mathbf{Q}_{ij}-\mathbf{Q}_{0j})-(\mathbf{q}_{ij}-\mathbf{q}_{i0})}{\partial Z})\frac{\partial Z}{\partial \zeta} \\ &-(1+p_{ij})\frac{\partial \mathbf{q}_{ij}}{\partial p_{ij}}\frac{\partial p_{ij}}{\partial \zeta}\end{aligned} \tag{30}$$

which may be rewritten as:

$$\begin{aligned}\frac{\partial p_{ij}}{\partial \zeta} =& \rho_{ij}\frac{\partial((\Gamma_{ij}-\Gamma_{0j})-(\gamma_{ij}-\gamma_{i0}))}{\partial \zeta} + \rho_{ij}\frac{\partial((\mathbf{Q}_{ij}-\mathbf{Q}_{0j})-(\mathbf{q}_{ij}-\mathbf{q}_{i0}))}{\partial Z}\frac{\partial Z}{\partial \zeta} \\ &-\rho_{ij}\frac{\partial T_{ij}}{\partial \zeta} \\ \rho_{ij} \equiv& [(1+p_{ij})\frac{\partial \mathbf{q}_{ij}}{\partial p_{ij}}]^{-1} > 0\end{aligned} \tag{31}$$

A change in ζ induces three changes in the husband's power. The first is the effect of a change in relative spousal invariant gains on power. The second term is proportional to the change in the difference in expected spousal utilities (felicities) due to a change in the wage and non-labor income distribution in market r caused by a change in ζ. The third term is proportional to the change in marriage market tightness. Since $\rho_{ij} > 0$, when market tightness increases increases, the husband's power is predicted to fall. It is important to emphasize that T_{ij} and p_{ij} are both endogenous variables and simultaneously determined, thus (31) is not a statement about the causal effect of T_{ij} on p_{ij}.

We now use Proposition 2 and equation (31) to derive testable implications of our theory regarding the effect of marriage matching on spousal labor supplies. Let H^r_{ijG} be the hours of work of wife G in an $\{ij\}$ marriage in society r. Consider the following reduced form labor supply regression:

$$\ln H^r_{ijG} = z^r_{ij}{}'\beta_1 + \beta_{ij}T^r_{ij} + u^r_{ijG},\ G = 1,..,G^r;\ ij = 1,..,\Psi^r;\ r = 1,..,R \quad (32)$$

where u^r_{ijk} is the error term in the regression. The vector z^r_{ij} includes (1) proxies for the labor market and asset conditions of type i and type j individuals in society r, (2) society specific behavior which is independent of i and j (r fixed effects), and (3) labor supply effects that are common to ij marriages (i.e. ij fixed effects).

Recall from Section 2.1 that the labor supply of wife G married to husband g in an $\{i,j\}$ marriage in society r is $H_{ij}(p^r_{ij}, S^r_{ijgG})$. Using a log linear approximation, we obtain

$$\ln H^r_{ijgG} = \sigma_p p^r_{ij} + \sigma_S S^r_{ijgG} \quad (33)$$

A first order Taylor series approximation for p^r_{ij}, substituting into (33) yields:

$$\begin{aligned}\ln H^r_{ijgG} = \sigma_{ij} + \sigma_p\rho_{ij}((\Gamma^r_{ij} - \Gamma^r_{0j}) - (\gamma^r_{ij} - \gamma^r_{0i}) \\ + \frac{\partial((\mathbf{Q}_{ij} - \mathbf{Q}_{0j}) - (\mathbf{q}_{ij} - \mathbf{q}_{i0}))}{\partial Z}Z^r) - \sigma_p\rho_{ij}T^r_{ij} + \sigma_S S^r_{ijgG}\end{aligned} \quad (34)$$

where σ_{ij} contains all the zero order terms of the Taylor series expansion. In equation (32), z^r_{ij} includes ij fixed effects to capture differences in invariant gains to marriage across different matches, r fixed effects and measures of the wage and non-labor income distributions that are $\{i,j,r\}$ specific to capture differences in labor market conditions across marriage markets. We assume that variations in z^r_{ij} are sufficient to capture variations in $(\Gamma^r_{ij} - \Gamma^r_{0j}) - (\gamma^r_{ij} - \gamma^r_{0j})$, and variations in $F(S^r_{ijgG}|Z^r)$ across societies. That is:

$$(\Gamma^r_{ij} - \Gamma^r_{0j}) - (\gamma^r_{ij} - \gamma^r_{i0}) = z^r_{ij}{}'\psi_\Gamma \quad (35)$$

$$Z^r = z^r_{ij}{}'\psi_Z \quad (36)$$

$$S^r_{ijgG} = z^r_{ij}{}'\psi_S + \varepsilon^r_{ijgG} \quad (37)$$

where ε^r_{ijgG} is the idiosyncratic wage and non-labor income variations across $\{i,i,r\}$ families and is by definition uncorrelated with $\{i,j,r\}$ specific variables. Finally, substituting (35) through (37) into (34) yields

$$\begin{aligned} \ln H^r_{ijgG} = {} & \sigma_{ij} + \sigma_p \rho_{ij} (z^r_{ij}{}' \psi_\Gamma + \frac{\partial((\mathbf{Q}_{ij} - \mathbf{Q}_{0j}) - (\mathbf{q}_{ij} - \mathbf{q}_{i0}))}{\partial Z} z^r_{ij}{}' \psi_Z) \\ & - \sigma_p \rho_{ij} T^r_{ij} + \sigma_S z^r_{ij}{}' \psi_S + \varepsilon^r_{ijgG} \end{aligned} \tag{38}$$

which reduces to (32).

Comparing the reduced from labor supply equation (32) with (38), $\beta_{ij} = -\sigma_p \rho_{ij}$ estimates the elasticity of mean hours of work of the wives in ij marriages with respect to marriage market tightness, holding z^r_{ij} constant. In other words, β_{ij} measures the reduced form impact of market tightness on the labor supply of wives, holding the ij match production function, society wide differences, spousal invariant gains, and labor market and non-labor income conditions constant. The parameter β_{ij} is identified because there remains independent variation in T^r_{ij} due to differences in population supplies, M^r and F^r, across societies. *Identification thus relies upon variation in population supplies, within ij matches, across different marriage markets.* Since $\rho^r_{ij} > 0$ and Proposition 2 imply $\sigma_p > 0$, the model predicts β_{ij} should be negative.

In the above regression, we have $\Psi^r \leq I \times J$ types of marriages. Because (31) must hold for every ij marriage match and β_{ij} is match dependent, we do not need to include all marriage matches in our reduced form labor supply regression (32). The regression is valid for any subset of marital matches. For example, due to thin cell problems, we will focus only on own race marriages in the empirical analysis. The fact that there are cross race marriages, which we leave out in the empirical analysis, does not invalidate our statistical inference.

Following the same logic, we can derive the labor supply equation for husband g in an ij marriage in society r as

$$\ln h^r_{ijg} = \alpha_{ij} T^r_{ij} + z^r_{ij}{}' \alpha_1 + v^r_{ijg},\ g = 1, .., g^r;\ ij = 1, .., \Psi^r;\ r = 1, .., R \tag{39}$$

where h^r_{ijg} is the hours of work of the husband and v^r_{ijg} is the error term of the regression. Following the argument for the wife, we expect α_{ij} to be positive.

A large number of studies estimate the effect of marriage market conditions, typically measured by the sex ratio, on labor supply. The above discussion, highlights three important implications for empirical work in this area:

1. **Marital substitution effects.** Our theoretical model highlights the point that the relevant measure of marriage market conditions is not the aggregate sex ratio but a measure of an individual's option value, captured here by market tightness for two reasons. First, the within-region aggregate sex ratio does not capture the notion that matches to spouses of different types are not equally valued by the agents. Our measure of market tightness, a measure of the relative supply singles within a match type, directly captures this notion. Second, if the numbers of other types of men and women change, there is no way to predict their effect on labor supply. The problems with simply adding the sex ratio of substitutes are twofold. First, it is not clear to the researcher who are better substitute spouses. Second, many of the own and 'obvious' substitute sex ratios (such as those from adjacent age groups) are highly collinear and therefore it is difficult to estimate each effect separately. For empirical tractability, researchers have primarily restricted their empirical specifications to own sex ratios. However, since spousal substitutes have been shown to be quantitatively significant (Angrist, *; Brandt, Siow and Vogel, 2007) it is useful to find an empirical proxy for overall market conditions for each marital match. Market tightness provides such a proxy.

   Market tightness is an endogenous variable. To the extent that it is correlated with labor supplies shocks, u^r_{ijk} and $v^r_{ijk'}$, we will use sex ratios to instrument for market tightness in the empirical work.

2. **Labor market conditions and the sex ratio.** As pointed out in several studies in the past (Angrist, CFL), the sex ratio may be determined by labor market conditions and therefore endogenous in the labor supply equation. For example, regions with high relative demand for male labor may have high sex ratios and high male labor supply for reasons unrelated to the marriage market. Failing to control for local labor market conditions will incorrectly attribute the increase in male labor supply in high sex ratio regions to marriage market effects. We might therefore expect the coefficient on tightness to have an upward bias on the male labor supply equation and a downward bias in the female labor supply equation. We account for this issue by directly controlling for local labor market conditions in our regression. In particular, we control for both the state level wage and asset distributions, by gender and type, in all of our labor supply regressions.

3. **Heterogeneity in marital production technologies.** The standard empirical specification assumes that the marital technology is the same across different types of marriages. If there is heterogeneity in the marital production technology across different types of marriages, changes in market tightness will confound changes in supplies in the marriage market and changes in marital technologies. Suppose for instance that the gains to specialization are higher in certain types of marriages than others. Tightness will be higher in marriages where the gains to marriage are higher. Also, women's labor supply will likely be lower when the gains to specialization are greater. In this case, heterogeneity in marital production technologies will induce a positive correlation between women's time at home and market tightness (gains to specialization result in more marriages). Our empirical specification will consider the implications of ignoring this potential source of bias. Our identification strategy is equivalent to the difference in differences estimation of treatment effects using state and time panel data. Instead of the usual time variation, we use marital match (ij) variations. It is less restrictive than most existing empirical work on the effects of marriage market conditions on spousal labor supplies.[8]

Although we consider several new and important issues, other issues are ignored in our empirical analysis. Perhaps the main difficulty with our identification strategy is when there is variation in labor demand by state and individual types. Different types of individuals may migrate to high labor demand states and also work more in those states. This migration will lead to variations in the sex ratio and thus market tightness. If the increase in labor supplies, as a response to increased labor demand, is not captured by our variables characterizing the earnings distributions faced by these individuals, β_{ij} and α_{ij} will not be consistently estimated. Thus the reliability of our identification strategy depends on how well our labor market variables capture labor demand variation by state and individual types.[9] Classical labor supply theory, as assumed here, implies that wages and non-labor income are sufficient to characterize the labor market opportunities faced by

[8]For example, Angrist uses individual types variation alone and CFL use across state variation alone to identify their sex ratio effects.

[9]Substantial endogenous migration will also invalidate most panel estimates of treatment effects which make use of state and time variation. This is a well known caveat of such studies.

individuals. We include measures of these variables.

There is another selection issue. If our observed matches do not accord with the matches as perceived by market participants, then the marriages in each observed match may contain mixtures of different unobserved marital matches. As labor market conditions and other exogenous variables change, the mix of unobserved marital matches used to construct observed market tightness and other variables may change. How these unobserved resorting affects our results is unclear. This problem is not unique to our paper. To the extent that changes in exogenous variables change the composition of observed sex ratios, this problem affects all work this area.[10]

A secondary implication of our model is based on the observation that both β_{ij} and α_{ij} should depend on i and j, the marriage match. In other words, there should be interaction effects between spousal characteristics, $\{i, j\}$, and market tightness in the above reduced form spousal labor supplies regressions. For p^r_{ij} large, $(1 + p^r_{ij})$ is large but $\frac{\partial \mathbf{q}_{ij}}{\partial p^r_{ij}}$ is likely to be small. So the effect of p^r_{ij} on ρ^r_{ij} is unclear. Although β_{ij} and α_{ij} may be proportional to ρ^r_{ij}, without further restrictions, their magnitudes are not informative on the magnitude of the husband's power, p^r_{ij}. For ease of interpretation, we allow β_{ij} and α_{ij} to vary with race only to explore this issue.

Equations (32) and (39) do not include individual spousal wages or non-labor incomes as covariates. The theory implies that the labor supply responses to spousal wages should satisfy Slutsky symmetry. However, this restriction cannot be tested with Census data, which is used here, because wages and non-labor income are measured with error and we do not have instruments for the idiosyncratic component of individual wages and non-labor income. Systematic components cannot be used as instruments to test Slutsky symmetry because the systematic components are known at the time of marriage, and therefore affect husband's power p^r_{ij}, and are also collinear with z^r_{ij}.[11]

[10] For example, Angrist uses the sex ratio of immigrants as his measure of substitutes which he argues was driven by immigration policy. Differences in immigration policies will change the quality of mix of immigrants.

[11] CFL did not include Z^r_{ij} as covariates. They used systematic characteristics of couples to instrument their wages and rejected Slutsky symmetry. Their rejection is consistent with the theory developed here. See the literature review for more discussion.

7.2 Data

The data used in our analysis comes from two sources: the 5% sample from the 2000 US Census and the 2003 American Time Use Survey (ATUS). The Census data is used to construct measures of sex ratios, marriage market tightness and labor market conditions in each marriage market and to estimate our reduced form labor supply regressions. The reduced form home production regressions are estimated using time use data from the ATUS.

We define an individual's type as a combination of race, age and education. For each gender, there are four contiguous age categories of 5 years each. The ages are slightly staggered across gender to reflect the fact that men tend to marry slightly younger women. The youngest female and male age categories, are 25-29 and 27-31 respectively. For each gender, we consider two schooling categories: high school graduates (at least 12 and up to and including 15 years of education) and college graduates (16 years of education and higher). For each race and gender, there are 8 potential types of individuals. Since we are only considering same race marriages, there are potentially $64 \times 3 = 254$ types of marital matches for each society.

We define each state as a separate society. With 50 states, there are potentially $254 \times 50 = 12,700$ cells across all marriage markets. However, the majority of these potential cells (marital match $\times$ state) have few or no marriages. To avoid thin cell problems, we delete a cell if the number of marriages in that cell is less than 5.[12] For most regressions, we have 2995 different cells (marital match $\times$ state), with 189 distinct marital matches. Most of the missing cells are due to non-white marriages, with large spousal age differences, in states with small populations. There are 750,000 same race couples in our Census sample before dropping the thin cell couples. After dropping the thin cell couples, about 3,000 couples, our base Census sample has approximately 747,000 couples. In other words, most of the thin cells that we dropped were empty cells. We also exclude mixed race couples to mitigate thin cells and also because we would need to present separate coefficients on market tightness for each type of mixed race couple.[13]

There is one selection criteria that is commonly imposed in the empirical collective labor supply literature that we do not impose here, at least for the

[12] We have other minor selection rules.

[13] Market tightness for mixed race couples which include white spouses are very different from own race couples because there are so many more whites than other races in the data. So we would need to have separate coefficients on tightness for each mixed race couples.

labor supply regressions. Because we allow for public goods within marriage, we do not restrict our analysis of labor supply to childless individuals or couples. In contrast, we only consider childless couples in our home production model, as it is difficult to distinguish between home production and leisure for certain activities in households with children.

Market tightness for marital type ij in state r is defined as the log of the ratio of the number of unmarried type i males to the number of unmarried type j females in state r.[14] Across cells, mean market tightness (in levels) for whites, blacks and Hispanics is 0.9974, 0.6771, and 0.8869, respectively. On average, there are more single females than males for all racial groups, but marriage market tightness is greatest in black marriage markets and lowest for whites.

We use five measures of log sex ratios. The most refined measure is the sex ratio measured at the cell level (log of the ratio of the number of males of type i to the number of females of type j in state r). There are also sex ratios by education matches and state, age matches and state, and race and state. Finally, there is an aggregate sex ratio by state. For all measures, mean log sex ratios are slightly less than zero which implies that there are slightly more women than men. Again, the standard deviations are large.[15] As expected, more narrower definitions of marital type leads to larger standard deviations.

To control for aggregate labor market conditions in an individual's local marriage market, we define the following three variables to characterize the earnings and non-labor income distributions. First, conditional on positive annual labor earnings for a type of unmarried individual, we construct the mean and standard deviation of log annual labor earnings for the distribution of unmarried individuals (wage and salary income). The second measure is the fraction of individuals with zero labor earnings for each match type in each marriage market. Finally, we construct the analogous variables for non-labor earnings, defined from the Census as total personal income minus wage and salary income.[16]

Table 1 contains the summary statistics of our base sample from the Census and the ATUS. Within the sample of 747,000 married same race

[14] An individual is unmarried if he or she is currently not married in the Census form (not code 1 or 2).

[15] As in the case for market tightness, treating each marital type (cell) as one observation, the standard deviations are at least twice as large.

[16] To be precise, we measure the fraction of individuals with non-positive non-labor income rather than zero non-labor income.

couples, roughly 86% of respondents are white, 8% are black, and 6% are Hispanic.[17] Approximately two-thirds of individuals are college graduates. The ATUS contains 408 childless couples with a slightly more Hispanics and a lower proportion of college graduates.[18]

Table 1 also contains information on the labor supply behavior of married couples in the 2000 Census. The labor force participation rates for husbands and wives are 94% and 73%, respectively. We consider two measures of labor supply and one measure of home production in our empirical analysis. Conditional on participating in the labor force, our first measure of labor supply is the log of usual hours worked per week. Mean usual hours worked for men and women were 45 and 34 hours respectively. Conditional on being in the labor force, the second measure is log weeks worked per year. Mean weeks worked per year for men and women were 49 and 41 weeks respectively.

Our measure of home production, presented in Table 1 is the "total" non-market work definition of Aguiar and Hurst (2007), minus shopping activities (obtaining goods and services). In particular, an individual's hours supplied to home production is defined as the total time spent on meals (preparation, presentation, and cleanup), housework (interior cleaning, laundry, sewing, repairing and maintaining textiles, storing interior household items including food) and interior and exterior maintenance, repair, and decoration, vehicle repair and maintenance, and appliance and tool set-up, repair, and maintenance, household management (except mail and email), and lawn, garden, houseplant and pet care. Our measure of home production does not include time spent obtaining goods as services, as it is arguably more difficult to disentangle home production time and leisure time for both of these categories. For similar reasons, we limit our analysis to couples with no children in the household to abstract from decisions regarding time spent with children decisions. On average, husbands supply 10 hours per week to home production while wives supply 16 hours.

[17] If an individual chooses any Hispanic label in the US Census or reports being Hispanic in the ATUS, they are classified as Hispanic. Data on reported race in each survey is subsequently used to classify blacks and whites accordingly. We do this to maximize the number of hispanics and blacks.

[18] We are currently exploring the source of the difference in the demographic composition of the ATUS.

7.3 Determinants of market tightness

Table 2 presents estimates of market tightness. There are a total of 2995 cells (state $\times$ marital match) and each cell is one observation. This is our empirical estimate of $T(\Gamma^r, \gamma^r, Z^r, M^r, F^r)$. Column 1 regresses market tightness on sex ratios. The estimates show that all measures of sex ratios affect market tightness even though we include the sex ratio (by cell) as a covariate. Thus, substitution effects are central to marriage market behavior. Given the complex relationship between population supplies and marriage matching, we will not attempt to interpret the estimated relationship. The R^2 is 0.943 which says that sex ratios are major determinants of market tightness. Column 2 adds controls for race, age and education. As both the individual estimated coefficients and the F test indicate, in addition to population supplies, an individual's race, age and education also affect market tightness. Column 3 add state effects. Although the F test shows that state effects matter, their explanatory power is marginal.

We next consider the effect of labor market conditions on marriage market tightness. Column 4 includes the earnings distributions of the unmarried men and women in addition to state and race effects. Increasing (decreasing) unmarried female (male) mean log earnings increases market tightness. This is consistent with the interpretation that an increase in the earnings of a type of individual increases their desirability in marriage. [19] Similarly, increasing the fraction of unmarried individuals zero labor or non-labor income decreases the desirability of their type in marriage. These findings are important for our empirical strategy because we are using unmarried earnings of a type of individual as a proxy for labor market conditions for both married and unmarried individuals with the same type.[20]

Even after controlling for match-specific effects sex ratios and unmarried incomes continue to have explanatory power, as illustrated in Column 5. Column 5 is a standard difference in differences regression using state and

[19] It is not consistent with the interpretation that an increase in unmarried mean earnings leading to a relative increase of that type of single individuals alone. Note that we are not holding the earnings of the married individuals constant.

[20] We use unmarried labor earnings rather than wages because the Census does not have data on individual wages. To construct wages, we would need to divide labor earnings by hours of work. Since the model implies that unmarried earnings are unaffected by marriage market considerations, we decide to use unmarried labor earnings as proxies for labor market conditions. This proxy ameliorates the problem of having a proxy for hours of work on the right hand side when we do labor supply regressions in the next section.

marital-match effects. Identification of the sex ratio and unmarried income effects are through state and marital match interactions.

7.4 Market tightness and labor supplies

Table 3 presents estimates of the effect of market tightness on log usual hours of work per week for wives.[21] Recall, our model and others in the literature (Becker, Grossbard-Schectman, Seitz) predict that an increase in the supply of men relative to women in the marriage market induces a reduction in female labor supply. As a benchmark specification, Column 1 includes only tightness measures by race. Column 2 introduces state fixed effects and Column 3 allows for race-specific effects. With the exception of Columns 1 and 2, market tightness has a small and insignificant effect on the labor supply of white wives and generally has a significant negative effect on the labor supply of black and Hispanic wives, as predicted by the theory.

Three model comparisons are of particular interest. First, to assess the importance of controls for labor market conditions, we compare the estimation results in Column 3 to those in Column 5. Although the effect of market tightness on labor supply remains roughly unchanged for whites and blacks, the coefficient on tightness for Hispanics moves closer to zero, suggesting the coefficient in Column 3 is biased downwards. This result suggests that positive labor demand shocks are negatively correlated with tightness (i.e. with an increase in the supply of unmarried females), at least for Hispanic women.

Second, to assess the importance of heterogeneity in marital production technologies, we include match-specific effects in Column 6. A comparison of Columns 4 (homogenous technology) and 6 (heterogeneous technology) suggests the marital production function differs across types of marriages for blacks and Hispanics. The estimated effect of tightness on labor supply becomes close to zero and insignificant for blacks, while for Hispanics the role of tightness becomes larger. Finally, the labor supply model is estimated by IV in Column 6 to correct for the endogeneity of market tightness. In this last specification, market tightness has a economically, but not statistically, effect on labor supply for Hispanic women and insignificant effects for white and black wives.

To gauge the quantitative importance of the results, we report the pooled

[21] The standard errors of all individual level regressions in this paper are clustered at the cell level.

estimates across Columns 3 to 6 for white tightness as approximately zero, black tightness as -0.02 and Hispanic tightness as -0.019. From the pooled estimates, a one standard deviation increase in tightness will decrease annual weeks worked by $0.02 \times 1.2/0.09 = 0.27$ standard deviations for blacks and , $0.019 \times 1.1/0.12 = 0.17$ standard deviations for hispanics.

Similar results can be observed upon examination of Table 4. In particular, the effect of tightness on annual weeks worked is opposite in sign that predicted by the theory in the benchmark specifications for both whites and Hispanics and there is substantial variation in the parameter estimates for across specifications that fail to control for labor market conditions and heterogeneity in production functions (e.g. Columns 3 and 4, respectively) and comparable specifications that do (Columns 5 and 6, respectively).

We next turn to the effect of marriage market tightness on labor supply for husbands. Table 5 presents estimates of the effect of tightness on husbands' log usual hours of work per week. In general, as predicted by the theory, an increase in market tightness results in an increase in labor supply for men. Pooling the point estimates from Columns 3 to 6, we obtain 0.002, 0.012 and 0.006 for white, black and hispanic tightness. Using the pooled estimates, a one standard deviation increase in tightness will increase white, black and hispanic usual hours per week by $0.002 \times 0.74/.07 = 0.02$, $0.012 \times 1.2/0.14 = 0.10$ and $0.006 \times 1.05/0.147 = 0.043$ standard deviations, respectively. Measured in terms of standard deviations, the response of black husbands response is largest and that of white husbands is smallest, as consistent with the results for wives. The white husbands' response is the smallest.

A comparison of Columns 3 and 5 suggest that estimation results for husbands seem quite robust to controls for labor market conditions, although there appears to be a downward bias in Column 3. The negative correlation between labor demand conditions and tightness for black males suggests high demand for black males in the labor market is consistent with high demand for black males in the marriage market. As is the case for women, the estimates in Column 6 (as compared to Column 4) indicate the presence of substantial heterogeneity in the marital production function for blacks and hispanics. Similar results are presented in Table 6. In Column 7, the labor supply model is estimated by IV to correct for the endogeneity of market tightness. In this last specification, market tightness has a economically, but not statistically, effect on labor supply for hispanic and black husbands and an insignificant effect for whites. There is no systematic evidence that the

IV estimates are larger or more precisely estimated. Thus the endogeneity of market tightness, as far as husbands' labor supplies are concerned, is not a serious concern.[22]

7.5 Home Production

Changes in marriage market conditions are also likely to affect other uses of time within the household.[23] In this section, we present estimation results on the effects of marriage market tightness on hours in home production. In general, the results are very consistent with the labor supply estimates. First, tightness has a large negative effect on hours in home production for white and black women, although the results are only significant for black women. For hispanics, the results are mixed and insignificant. A comparison of Columns 3 and 5 for black women suggests that ignoring labor market conditions results in a downward bias in the estimated effect of tightness on home production. The behavior of white and hispanic wives does not appear to be influenced by labor market conditions. Controlling for invariant gains results in large changes in the parameter estimates. For blacks and hispanics, the effect of tightness on hours now has a positive effect in contrast to the predictions of the theory. Due to the limited sample size, introducing match-specific effects results in a large increase in the standard errors as well.

For men, the effects of tightness are also consistent with the theory and the labor supply estimates. Increases in tightness increase male hours in home production. Failing to control for labor market conditions results in an upward bias in the tightness parameter estimates. Controlling for invariant gains to marriage tends to increase coefficients on tightness for blacks and whites, but the effects are imprecisely estimated to due the limited sample size.

[22] One explanation for the lack of responsiveness in the labor supply and weeks worked for husbands is that we consider prime working age males. If the primary role of these husbands within the marriage is to work, there may be little room for adjustment of labor supply in response to variation in tightness variation. The variation in labor supply among these husbands may be primarily due to involuntary layoffs or disability.

[23] For a related model of marriage and household time allocation decisions, see Del Boca and Flinn (2006).

7.6 Alternative specifications

For husbands, we also estimated one tightness coefficient for all races. The results were as expected, essentially an average of the three separate estimates by races. The results using a single coefficient reinforced the finding that husbands primarily adjust usual hours of work per week, then their labor force participation rate and least by weeks worked per year.

We deleted observations where usual hours of work exceeded 80 hours. Except for a tiny increase in precision, the estimates are unchanged. We also estimate the effects of tightness on annual hours of work (weekly hours multiplied by weeks worked per year). The estimated effects are consistent with that presented here for the two labor supply measures considered separately.

We were also worried about measurement error in constructing tightness and sex ratios due to thin cells. We deleted cells with less than 20 observations. This resulted in the deletion of about 100 cells. Using this smaller sample, the empirical results, both in terms of the point estimates and the estimated standard errors, are similar to that using the larger sample. Thus measurement error when constructing tightness or sex ratios due to thin cells is not a first order problem.

Finally, we investigated tightness effects interacted with other individual characteristics. There is some evidence that spouses' labor supply responses to market tightness also differ by their husbands' education.

8 Literature review (incomplete)

As discussed in the introduction, our collective model of marriage matching integrates the collective model with marriage matching. Our collective model of the household builds on BCM. The collective model has a long history beginning with Chiappori (1988, 1992). A large body of empirical work tested the restrictions of the unitary model versus the collective model and consistently finds the restrictions implied by the unitary model are rejected while those implied by the collective model are not (a partial list includes Lundberg, 1988; Thomas, 1990; Fortin and Lacroix, 1997; Chiappori, Fortin, and Lacroix, 2002; and Duflo, 2003).

We add to BCM efficient spousal risk sharing. Efficient spousal risk sharing models have been discussed by....

The marriage matching model builds on the transferable utilities models of the marriage market, and in particular CS. Dagsvik have a closely related non-transferable utilities model of the marriage market.

Starting with Grossbard-Schectman (1984), there is a large empirical literature which studies the impact of sex ratios on spousal labor supplies. Grossbard-Schectman (1984) constructs a model where more favorable conditions in the marriage market improve the bargaining position of individuals within marriage. One implication of Grossbard-Schectman and related models that has been tested extensively in the literature is that, for example, an improvement in marriage market conditions for women translates into a greater allocation of household resources towards women, which has a direct income effect on labor supply. Tests of this hypothesis have received support in the literature (see among others, Becker, 1981; Grossbard-Schectman, 1984, 1993, 2000; Grossbard-Schectman and Granger, 1998; Chiappori, Fortin, and Lacroix, 2002; Seitz, 2004; Grossbard and Amuedo-Dorantes, 2007). Our empirical work considers the link between the sex ratio and both marriage and labor supply decisions in a general version of the collective model with matching.

Seitz (2004) first proposed the measure of marriage market tightness used in this paper. She constructs and estimates a dynamic model in which the sex ratio, marriage, and employment decisions are jointly determined. She finds that variation in the ratio of single men to single women across race can explain much of the black-white differences in marriage and employment in the US.

It is also convenient at this point to discuss empirical tests of the static collective model using spousal labor supplies such as CFL. In their paper, they estimate restricted spousal labor supplies models where the restrictions are derived from a static collective model. They instrument spousal wages, children, and nonlabor income with education, age, father's education, city size and religion. Different values of these instruments define different types of individuals in different regions. There is no instrument which captures the transitory component of wages.[24] Our interpretation of their empirical results is that they provide evidence of (1) efficient bargaining between different types of spouses and (2), spousal bargaining power depends on the type of marriage matches as we assume in this paper. Their empirical results are not informative about whether there is efficient risk sharing with the household as we suppose, or whether there is not as they supposed. In order to empirically

[24] Although age changes for an individual over time, the changes are deterministic.

distinguish between whether there is efficient risk sharing or not, one would need an instrument for transitory wage shocks when one estimates spousal labor supplies equations. As mentioned in Section 6, the results in this paper also do not shed light on whether there is efficient risk sharing or not. Our rationale for using the risk sharing interpretation as we do here is primarily for empirical convenience.

Our static formulation of the collective model in this section is also close to Del Boca and Flinn's formulation Instead of competitive marriage market clearing as we use in this paper, they use two different household allocation models and the deferred acceptance algorithm to construct a marriage market equilibrium. The difference in equilibrium constructions may not be significant in large marriage markets.[25] The empirically significant difference between their paper and ours is that, like CFL, they impose the restriction that the invariant gains to marriage and utilities from consumption and labor supply are the same for all types of marriages.[26] This restriction imposes restrictions on marriage matching patterns and spousal labor supplies in a single marriage market. We use the exactly opposite assumption: we do not impose any structure on invariant gains and utilities from consumption and labor supply across different types of marriages. Thus we do not impose any marriage matching and spousal labor supplies pattern in a single marriage market.

9 Conclusion

[25]Dagsvik () has shown that when individuals' preferences over different spouses are characterized by McFadden's random utility model, using a non-transferable utility deferred acceptance algorithm to construct a large marriage market equilibrium results in a marriage matching function that is closely related to that discussed in this paper (See CS for further discussion).

[26]Their household production functions depend on the specific marital match which generates demand for different types of matches.

Table 1: Sample Statistics for the 2000 US Census
and the 2003 American Time Use Survey

	2000 Census		2003 ATUS	
	Racial Composition			
White	85.6		84.8	
Black	8.1		4.6	
Hispanic	6.2		10.6	
	Marriage Characteristics			
	Husbands	Wives	Husbands	Wives
College graduate	0.6634	0.6653	0.4398	0.4812
Participation rate	0.9412	0.7269		
Usual weekly hours	44.9972	34.1112		
Weeks per year	48.6386	41.4128		
Weekly housework			10.3	15.5
Observations	746,908		408	

Table 2: Determinants of Market Tightness

	1	2	3	4	5
			Regional Sex Ratios, by Type		
Match-specific	1.020*				1.039*
	(0.013)				(0.023)
Age-specific	-1.245*				-0.316*
	(0.040)				(0.049)
Race-specific	0.886*				0.328*
	(0.049)				(0.062)
Education-specific	0.165*				0.004
	(0.017)				(0.029)
Region Only	0.574*				0.000
	(0.126)				(0.000)
			Race		
Black		-0.416*	-0.419*	-1.084*	
		(0.019)	(0.021)	(0.040)	
Hispanic		-0.115*	-0128*	-0.440*	
		(0.020)	(0.023)	(0.040)	
Labor market controls	No	No	No	Yes	Yes
Age and education controls	No	Yes	Yes	No	No
Match-specific controls	No	No	No	No	Yes
State controls	No	No	Yes	Yes	Yes
Observations	2995	2995	2995	2976	2976

Table 3: Market tightness on log weekly hours for wives

	1	2	3	4	5	6	7
				Market tightness, by race			
				Race			
White	0.074*	0.073*	0.006	0.006	0.008	0.002	-0.003
	(0.004)	(0.003)	(0.005)	(0.004)	(0.005)	(0.006)	(0.006)
Black	-0.031*	-0.021*	-0.026*	-0.031*	-0.027*	0.009	0.003
	(0.006)	(0.004)	(0.003)	(0.003)	(0.004)	(0.009)	(0.010)
Hispanic	0.011	0.012*	-0.026*	-0.019*	-0.017*	-0.039*	0.023
	(0.006)	(0.004)	(0.004)	(0.004)	(0.004)	(0.015)	(0.017)
				Race indicators			
Black			0.085*		0.009		
			(0.004)		(0.007)		
Hispanic			0.045*		0.001		
			(0.005)		(0.006)		
State controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Labor market controls	No	No	No	Yes	Yes	Yes	Yes
Match-specific controls	No	No	No	No	No	Yes	Yes
Race controls	No	No	Yes	No	Yes	No	Yes
Age and education controls	No	No	Yes	No	No	No	No
Observations	589,374	589374,	589374	589,300	589,300	589,300	589,300

Table 4: Market tightness on log annual weeks for wives

	1	2	3	4	5	6	7
				Market tightness, by race			
				Race			
White	0.036*	0.037*	-0.003	-0.006	-0.020*	0.012*	0.006
	(0.002)	(0.002)	(0.005)	(0.004)	(0.004)	(0.005)	(0.006)
Black	0.002	0.004	-0.010*	-0.014*	-0.026*	-0.014	-0.004
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.012)	(0.013)
Hispanic	0.012*	0.012*	-0.017*	-0.008	-0.020*	-0.020	-0.011
	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)	(0.017)	(0.021)
				Race indicators			
Black			0.018*		-0.039*		
			(0.003)		(0.007)		
Hispanic			-0.025*		-0.050*		
			(0.005)		(0.007)		
State controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Labor market controls	No	No	No	Yes	Yes	Yes	Yes
Match-specific controls	No	No	No	No	No	Yes	Yes
Race controls	No	No	Yes	No	Yes	No	No
Age and education controls	No	No	Yes	No	No	No	No
Observations	589,374	589,374	589,374	589,300	589,300	589,300	589,300

Table 5: Market tightness on log weekly hours for husbands

	1	2	3	4	5	6	7
				Market tightness, by race			
White	-0.017*	-0.017*	0.000	0.012*	-0.001	-0.002	-0.002
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Black	0.024*	0.025*	0.007*	0.022*	0.009*	0.010	0.013
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.007)	(0.008)
Hispanic	0.004	0.004	0.004*	0.013*	0.004*	0.005	0.007
	(0.004)	(0.004)	(0.002)	(0.003)	(0.002)	(0.008)	(0.010)
				Race indicators			
Black			-0.064*		-0.037*		
			(0.002)		(0.003)		
Hispanic			-0.052*		-0.033*		
			(0.002)		(0.002)		
State controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Labor market controls	No	No	No	Yes	Yes	Yes	Yes
Match-specific controls	No	No	No	No	No	Yes	Yes
Race controls	No	No	Yes	No	Yes	No	No
Age and education controls	No	No	Yes	No	No	No	No
Observations	725,315	725,315	725,315	725,177	725,177	725,177	725,177

Table 6: Market tightness on log annual weeks for husbands

	1	2	3	4	5	6	7
			Market tightness, by race				
			Race				
White	-0.009*	-0.009*	-0.002	0.007*	-0.007*	0.001*	-0.000
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Black	0.020*	0.021*	0.004	0.015*	0.001	0.004	0.009
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.010)	(0.011)
Hispanic	0.004	0.003	-0.001	0.007*	-0.003	-0.013	0.007
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.011)	(0.012)
			Race indicators				
Black			-0.054*		-0.041*		
			(0.002)		(0.003)		
Hispanic			-0.049*		-0.040*		
			(0.002)		(0.003)		
State controls	No	Yes	Yes	Yes	Yes	Yes	Yes
Labor market controls	No	No	No	Yes	Yes	Yes	Yes
Match-specific controls	No	No	No	No	No	Yes	Yes
Race controls	No	No	Yes	No	Yes	No	No
Age and education controls	No	No	Yes	No	No	No	No
Observations	725,315	725,315	725,315	725,177	725,177	725,177	725,177

Table 7: Market tightness on log weekly hours of home production for wives, couples with no children

	1	2	3	4	5	6
			Market tightness, by race			
			Race			
White	-0.030	-0.158	-0.668	-0.657	-0.740	-1.061
	(0.031)	(0.300)	(0.545)	(0.458)	(0.504)	(0.858)
Black	-1.226*	-1.485*	-1.528*	-1.940*	-1.990*	3.556
	(0.404)	(0.506)	(0.534)	(0.607)	(0.605)	(4.402)
Hispanic	-0.011	0.005	-0.018	0.140	0.076	2.084
	(0.329)	(0.373)	(0.532)	(0.446)	(0.510)	(1.865)
			Race indicators			
Black			-0.133		-0.627	
			(0.402)		(0.724)	
Hispanic			-0.196		-0.387	
			(0.503)		(0.703)	
State controls	No	Yes	Yes	Yes	Yes	Yes
Labor market controls	No	No	No	Yes	Yes	Yes
Match-specific controls	No	No	No	No	No	Yes
Race controls	No	No	Yes	No	Yes	No
Age and education controls	No	No	Yes	No	No	No
Observations	169	169	169	169	169	169

Table 8: Market tightness on log weekly hours of home production for husbands, couples with no children

	1	2	3	4	5	6
			Market tightness, by race			
			Race			
White	0.472*	0.432	1.728*	0.763	0.378	0.931
	(0.218)	(0.297)	(0.739)	(0.799)	(0.955)	(2.406)
Black	-1.283	2.664*	2.425	5.129	0.462	20.458
	(1.839)	(3.351)	(5.420)	(4.459)	(6.432)	(15.134)
Hispanic	0.205	7.016*	7.161	11.271*	5.493	3.757
	(0.307)	(3.278)	(5.530)	(5.359)	(8.020)	(3.99)
			Race indicators			
Black			0.709		-1.812	
			(0.981)		(1.576)	
Hispanic			0.104		0.014	
			(1.183)		(1.288)	
State controls	No	Yes	Yes	Yes	Yes	Yes
Labor market controls	No	No	No	Yes	Yes	Yes
Match-specific controls	No	No	No	No	No	Yes
Race controls	No	No	Yes	No	Yes	No
Age and education controls	No	No	Yes	No	No	No
Observations	125	125	125	125	125	125

A Proof of Proposition 1

Abstracting from i, j, g, G, the social planner solves:

$$\max_{\{C,c,H,h,K\}} \mathbf{E}(\widehat{Q}(C, 1-H, K|Z) + p\mathbf{E}(\widehat{q}(c, 1-h, K)|Z)$$

subject to, for each state S,

$$c + C + K \leq A + WH + wh \tag{40}$$

Let Z^* be the value of Z evaluated at the optimum. The first order conditions with respect to c, C, H, h, K and the multiplier λ for each state S are:

$$\widehat{Q}^*_C = \lambda \tag{41}$$

$$p\widehat{q}^*_c = \lambda \tag{42}$$

$$\widehat{Q}^*_{1-H} = \lambda W \tag{43}$$

$$p\widehat{q}^*_{1-h} = \lambda w \tag{44}$$

$$\widehat{Q}^*_K + p\widehat{q}^*_K = \lambda \tag{45}$$

Using the first order conditions, as p changes, for each state S,

$$\frac{\partial \widehat{Q}^*}{\partial p} = \lambda(C^*_p - WH^*_p + K^*_p) - p\widehat{q}^*_K K^*_p \tag{46}$$

$$\frac{\partial \widehat{q}^*}{\partial p} = \frac{\lambda}{p}(c^*_p - wh^*_p) + \widehat{q}^*_K K^*_p \tag{47}$$

which imply:

$$\frac{1}{p}\frac{\partial \widehat{Q}^*}{\partial p} + \frac{\partial \widehat{q}^*}{\partial p} = \frac{\lambda}{p}(c^*_p - wh^*_p + K^*_p + C^*_p - WH^*_p) \tag{48}$$

Since the budget constraint has to hold for every S,

$$c^*_p + C^*_p + K^*_p - wh^*_p - WH^*_p = 0$$

$$\Rightarrow \frac{\partial \widehat{Q}^*}{\partial p} = -p\frac{\partial \widehat{q}^*}{\partial p} \tag{49}$$

Since (49) holds for every state S, (6) obtains.

B Proof of $\frac{\partial \mathbf{E} H^*}{\partial p_{ij}} > 0$ and $\frac{\partial \mathbf{E} h^*}{\partial p_{ij}} < 0$

For an $\{i, j, G, g\}$ family, given realizations of wages and asset income, and taking p_{ij} as given, the planner solves a one period household maximization problem, P2. The objective of this appendix is to show that for any admissible realization of wages and asset income, and taking p_{ij} as fixed, labor supply of the wife will increase and labor supply of the husband will decrease as p_{ij} increases.

Ignoring the i, j, G, g subscripts, and assuming that realized wages and asset income are W, w and A, the planner's problem is:

$$\max_{C,L,c,l,K} \widehat{Q}(\Omega(C, L), K) + p\widehat{q}(\omega(c, l), K) \tag{50}$$

$$s.t.\ c + C + K + WL + wl \leq A + W + w = I \tag{51}$$

Given the weak separability between private goods and the public good in each spouse's utility function, let Y and y be the expenditure on the wife's and husband's private goods respectively. Then the wife will solve:

$$\max_{C,L} \widehat{Q}(\Omega(C, L), K) \tag{52}$$

$$s.t.\ C + WL \leq Y \tag{53}$$

Due to the weak separability, the optimal levels of private goods, C and L, only depend on W and Y, and are independent of K. We will assume that the optimal level of L is increasing in Y. The standard restriction on $\Omega(C, L)$, i.e. concavity and $\Omega_{LL} - \Omega_{CL} < 0$, that is leisure increases as Y increases, is sufficient. Solving (52) will result in an indirect utility:

$$\widetilde{Q}(Y, K) \tag{54}$$

The husband will solve:

$$\max_{c,l} \widehat{q}(\omega(c, l), K) \tag{55}$$

$$s.t.\ c + wl \leq y \tag{56}$$

Again, the optimal levels of private goods, c and l, only depend on w and y, and are independent of K. We will assume that the optimal level of

l is increasing in y. The standard restriction on $\omega(c, l)$, i.e. concavity and $\omega_{ll} - \omega_{cl} < 0$, that is leisure increases as y increases, is sufficient. Solving (55) will result in an indirect utility:

$$\widetilde{q}(y, K) \tag{57}$$

All the above implications of (50) and (51) are known from BCM.
Assume that $\widetilde{q}(y, K)$ is increasing and quasi-concave, and $\widetilde{q}_{yK} > 0$.
So we can rewrite the planner's problem as:

$$\max_{Y,y,K} \widetilde{Q}(Y, K) + p\widetilde{q}(y, K) \tag{58}$$

$$s.t.\ Y + y + K \leq I \tag{59}$$

Let $\mathbf{Y} = -Y$. Then the planner's problem, (58) and (59), can be rewritten as:

$$\max_{\mathbf{Y},y} R(\mathbf{Y}, y, p) = \widetilde{Q}(-\mathbf{Y}, I - y + \mathbf{Y}) + p\widetilde{q}(y, I - y + \mathbf{Y}) \tag{60}$$

$R(\mathbf{Y}, y, p)$ is supermodular in $\mathbf{Y}, y, K$ and p if:

$$R_{\mathbf{Y}y} = \widetilde{Q}_{YK} - \widetilde{Q}_{KK} + p(\widetilde{q}_{Ky} - \widetilde{q}_{KK}) > 0 \tag{61}$$

$$R_{\mathbf{Y}p} = \widetilde{q}_K > 0 \tag{62}$$

$$R_{yp} = \widetilde{q}_y - \widetilde{q}_K > 0 \tag{63}$$

The first order condition to the planner's problem is:

$$-\widetilde{Q}_Y + \widetilde{Q}_K + p\widetilde{q}_K = 0 \tag{64}$$

$$-\widetilde{Q}_K + p(\widetilde{q}_y - \widetilde{q}_K) = 0 \tag{65}$$

(65) implies (63).

(61) and (62) are implied by the assumption that $\widetilde{Q}(Y, K)$ is increasing in both arguments and quasi-concave in K, and $\widetilde{Q}_{YK} > 0$. An economically meaningful interpretation is that K is a normal good. In terms of the planner's primitive objective function (50), a sufficient condition is $\widehat{Q}(\Omega(C, L), K) + p\widehat{q}(\omega(c, l), K) = \Omega(C, L)\widehat{\Omega}(K) + p\omega(c, l)\widehat{\omega}(K)$ for increasing concave functions $\widehat{\Omega}$ and $\widehat{\omega}$.

Since $R(\mathbf{Y}, y, p)$ is supermodular, using the monotone theorem of Milgrom and Shannon (1994), $\mathbf{Y}$ and y are both increasing in p, and thus Y is decreasing in p. Since L and l are increasing in Y and y respectively, L will decrease and l will increase as p increases. Thus H and h are increasing and decreasing in p respectively.

See BCM for other implications of the weakly separable collective model of spousal labor supplies with public goods.

B.1 Cobb-Douglas preferences

Let the preferences of the husband and the wife be:

$$\widehat{q}(c, l, K) = l^{\alpha_h} c^{1-\alpha_h} K^{\delta_h} \tag{66}$$

$$\widehat{Q}(C, L, K) = L^{\alpha_f} C^{1-\alpha_f} K^{\delta_f} \tag{67}$$

Then:

$$\omega(c, l) = l^{\alpha_h} c^{1-\alpha_h} \tag{68}$$

$$\widehat{\omega}(K) = K^{\delta_h} \tag{69}$$

$$\Omega(C, L) = L^{\alpha_f} C^{1-\alpha_f} \tag{70}$$

$$\widehat{\Omega}(K) = K^{\delta_f} \tag{71}$$

Given y and Y, optimal leisure will satisfy:

$$l^* = \frac{\alpha_h y}{w} \tag{72}$$

$$L^* = \frac{\alpha_f Y}{W} \tag{73}$$

l^* and L^* are increasing in y and Y respectively as required.

The indirect utilities are:

$$\widetilde{Q}(Y, K) = \alpha_f Y K^{\delta_f} \tag{74}$$

$$\widetilde{q}(y, K) = \alpha_h y K^{\delta_h} \tag{75}$$

for positive constants α_f and α_h. $R(\mathbf{Y}, y, p)$ is supermodular as required. Thus l^* will increase and L^* will decrease as p increases.

C Proof of Existence of Equilibrium

In the proof, we need:

$$E_{ij}(\underline{p}) > 0 \text{ as } \underline{p} \to \infty \tag{Condition A1}$$

$$E_{ij}(\underline{p}) < 0 \text{ as } \underline{p} \to 0 \tag{Condition A2}$$

That is, the utility functions q and Q must be such that as $\underline{p}$ approaches 0, men will not want to marry. And as $\underline{p}$ approaches ∞, women will not want to marry.

Let $\beta_{ij} = (1 + p_{ij})^{-1}$ where $\beta_{ij} \in [0, 1]$ is the utility weight of the wife in an $\{i, j\}$ marriage and $(1 - \beta_{ij})$ is the utility weight of the husband.

We know:

$$\frac{\partial \underline{\mu}_{ij}}{\partial p_{ij}} > 0 \tag{76}$$

$$\frac{\partial \underline{\mu}_{ij}}{\partial p_{ik}} < 0,\ k \neq j \tag{77}$$

$$\frac{\partial \underline{\mu}_{kl}(\beta)}{\partial p_{ij}} = 0;\ k \neq i, l \neq j \tag{78}$$

$$\frac{\partial \overline{\mu}_{ij}}{\partial p_{ij}} < 0 \tag{79}$$

$$\frac{\partial \overline{\mu}_{ij}}{\partial p_{kj}} > 0,\ k \neq i \tag{80}$$

$$\frac{\partial \overline{\mu}_{kl}(\beta)}{\partial p_{ij}} = 0;\ k \neq i, l \neq j \tag{81}$$

Let β be a matrix with typical element β_{ij} and the IxJ matrix function $E(\beta)$ be:

$$E(\beta) = \underline{\mu}(\beta) - \overline{\mu}(\beta) \tag{82}$$

An element of $E(\beta)$, $E_{ij}(\beta)$, is the excess demand for j type wives by i type men given β.

An equilibrium exists if there is a β^* such that $E(\beta^*) = 0$.

Assume that there exists a function $f(\beta) = \alpha E(\beta) + \beta$, $\alpha > 0$ which maps $[0, 1]^{I*J} \to [0, 1]^{I*J}$ and is non-decreasing in β. Tarsky's fixed point theorem says if a function $f(\beta)$ maps $[0, k]^N \to [0, k]^N$, $k > 0$, and is non-decreasing in β, there exists $\beta^* \in [0, k]^N$ such that $\beta^* = f(\beta^*)$. Let $f(\beta) = \alpha E(\beta) + \beta$,

$k = 1$ and $N = I * J$, and apply Tarsky's theorem to get $\beta^* = \alpha E(\beta^*) + \beta^* \Rightarrow E(\beta^*) = 0$.

Thus the proof of existence reduces to showing $f(\beta)$ which has the required properties.

We know from (76) to (81) that:

$$\frac{\partial E_{ij}(\beta)}{\partial \beta_{ij}} < 0 \tag{83}$$

$$\frac{\partial E_{ik}(\beta)}{\partial \beta_{ij}} > 0 \tag{84}$$

$$\frac{\partial E_{kj}(\beta)}{\partial \beta_{ij}} > 0 \tag{85}$$

$$\frac{\partial E_{kl}(\beta)}{\partial \beta_{ij}} = 0; \; k \neq i, l \neq j \tag{86}$$

(83) to (86) imply that $E(\beta)$ satisfies the Weak Gross Substitutability (WGS) assumption.

We now show that the WGS property of $E(\beta)$ implies that we can construct $f(\beta)$, such that $f(\beta)$ maps $[0,1]^{I*J} \rightarrow [0,1]^{I*J}$ and is non-decreasing in β. The proof follows the solution to exercise 17.F.16^C of Mas-Colell, Whinston and Green given in their solution manual (Hara, Segal and Tadelis, 1996). N.B. Unlike them, we do not start with Gross Substitution, we begin from WGS, but it turns out to be sufficient for Tarsky's conditions.

For notational convenience, now onwards we'll treat the matrix function $E(\beta)$, as a vector function.

Let $N = I * J$ and 1_N be a $N \times 1$ vector of ones. $E(\beta) : [0,1]^N \rightarrow R^N$ is continuously differentiable and satisfies $E(0_N) >> 0_N$ and $E(1_N) << 0_N$ (Conditions A1 and A2).

For every $\beta \in [0,1]^N$ and any n, if $\beta_n = 0$, then $E_n(\beta) > 0$.

For every $\beta \in [0,1]^N$ and any n, if $\beta_n = 1$, then $E_n(\beta) < 0$.

If $\beta = \{0_N, 1_N\}$, the facts follow from Conditions A1 and A2. Otherwise, they are due to Conditions A1 and A2, and (83) to (86), i.e. WGS.

For each n, define $C_n = \{\beta \in [0,1]^N : E_n(\beta) \geq 0\}$ and $D_n = \{\beta \in [0,1]^N : E_n(\beta) \leq 0\}$.

Then $C_n \subset \{\beta \in [0,1]^N : \beta_n < 1\}$ and $D_n \subset \{\beta \in [0,1]^N : \beta_n > 0\}$.

Then by continuity, the following two minima, ${}_{ij}((1-\beta_n)/E_n(\beta) : \beta \in C_n)$ and ${}_{ij}(-\beta_n/E_n(\beta) : \beta \in D_n)$, exist and are positive. Let $\underline{\beta}_n > 0$ be smaller than those two minima. Then, for all $\alpha \in (0, \underline{\beta}_n)$ and any $\beta \in [0,1]^N$, we have $0 \leq \alpha E_n(\beta) + \beta_n \leq 1$.

For each n, define $L_n =_{ij} \{|\partial E_n(\beta)/\partial \beta_n| : \beta \in [0,1]^N\}$. Then, for all $\alpha \in (0, 1/L_n)$,

$$\frac{\partial(\alpha E_n(\beta) + \beta_n)}{\partial \beta_n} = \alpha \frac{\partial E_n(\beta)}{\partial \beta_n} + 1 \geq -\alpha L_n + 1 > 0$$
$$\frac{\partial(\alpha E_n(\beta) + \beta_n)}{\partial \beta_m} = \alpha \frac{\partial E_n(\beta)}{\partial \beta_m} \geq 0; n \neq m, \text{ follows from (83) to (86).}$$

Now let $K = _{ij}\{\underline{\beta}_1, .., \underline{\beta}_N, 1/L_1, .., 1/L_N\}$, choose $\alpha \in (0, K)$, then $f(\beta) = \alpha E(\beta) + \beta \in [0,1]^N$ and $\partial f(\beta)/\partial \beta_n \geq 0$ for every $\beta \in [0,1]^N$, and any n. Hence Tarsky's conditions are satisfied.