Economics 137
Aggregate Economics Seminar
Fall 2003
Professor Craine

CAPM

The CAPM explains the equilibrium relationship between expected return and risk. Two equations summarize the CAPM results: the securities market line (SML) shows the relationship among individual securities and the capital market line (CML) shows the expected return-risk tradeoff for the market portfolio.

The class project is to test the CAPM by estimating the SML.

Model

Environment

Tastes: U(ER, ω_R^2) for individual i

Technology: r ~ N(Er,O); r is an nx1 vector of returns

Trading Mechanism: Perfect and complete markets

Definitions:

rf is the return on the risk free asset

$$R = \sum_{j=1}^{n} w_j r_j + w_0 rf; \; w_0 = 1 - \sum_{j=1}^{n} w_j ,$$
$$= rf + w'(r - rf); \; w' \equiv \{ w_1, w_2, \ldots w_n \}$$

R is the return on a portfolio of risky assets,
w_j is the share of agent i's wealth invested in asset j.

Part I: Individual Behavior

Each investor chooses weights, w_j, to maximize utility subject to their "budget constraint",

$$ER \mid \; rf\ 2 \underset{j \mid 1}{—} w_j (Er_j\ 4\ rf)$$

$$\omega_R^{\ 2} \mid \underset{j \mid 1 \;\; i \mid 1}{——} w_j w_i \operatorname{cov}(r_i, r_j)$$

See Bodie-Kane-Marcus Chapters 7-8 for the solution.

Part II: Market Equilibrium

The Capital Market Line and the Mutual Fund Separation Theorem

Part I is an individual choice problem. If agents have different tastes, then they will hold different portfolios. An amazing implication (and empirically easily rejected) of the CAPM is that all agents hold the "market portfolio" and the risk free asset. This is the mutual fund separation theorem. The only thing that distinguishes agents is the share of their wealth they invest in the market portfolio. (If you think this is unrealistic--it is--recall that in deterministic models all assets are perfect substitutes so agents don't care what asset they hold. The goals of the CAPM are to explain why assets aren't perfect substitutes and explain the risk premia. The CAPM does this pretty well.)

The first market equilibrium condition is the CML which shows the expected return—risk trade-off for the market portfolio.

Definitions:

RM is the market portfolio,

$$RM = \underset{j=1}{\overset{n}{—}} q_j r_j \ \Sigma q \Re\!\!\!\;\text{—} q_j = 1.$$

where q_j is the fraction of the market portfolio invested in asset j. The market portfolio dominates all other efficient mean-variance portfolios when there exists a risk free asset.

Combining the market portfolio (the mutual fund) with the risk free asset gives a new portfolio,

$$\begin{aligned} R &\mid (1\,4\,v)rf\ 2\ vRM \mid rf\ 2\ v(RM\ 4\ rf) \\ ER &\mid rf\ 2\ v(ERM\ 4\ rf) \\ \omega_R &\mid v\omega_{RM} \end{aligned} \tag{1.1}$$

In equilibrium each investor, *i,* chooses the share of his wealth, v^i , to invest in the market

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portfolio.

Eliminating v in the expected return equation gives the CAPITAL MARKET LINE

$$ER - rf = \left(\frac{ERM - rf}{\omega_{RM}}\right)\omega_R$$

The slope of the capital market line is the expected risk-return tradeoff,

$$\frac{\mathrm{dER}}{\mathrm{d}\omega_R} = \frac{ERM - rf}{\omega_{RM}}$$

for the market portfolio.

Each investor’s indifference curve is tangent to the CML—investor risk aversion determines the fraction of their wealth that they invest in the market portfolio.

The Securities Market Line

The second equilibrium condition is the security market line: the expected return—risk relationship among risky securities. In equilibrium, the *risk adjusted* expected excess return on all assets is equal. The key is to use the correct measure of risk which is the covariance of the asset’s return with the return on the market portfolio, $cov(r_j,RM)$. The covariance measures asset *j’s* contribution to the risk of the market portfolio—the risk investors care about. The equilibrium condition is:

$$\frac{Er_j - rf}{\operatorname{cov}(r_j, RM)} = \frac{Er_i - rf}{\operatorname{cov}(r_i, RM)}; \ \forall \ i, j \qquad (1.2)$$

Notice that the equilibrium condition (1.2), of course, holds for any portfolio including the market portfolio,

$$\frac{Er_j \, 4 \, rf}{\mathrm{cov}(r_j, RM)} \mid \frac{ERM \, 4 \, rf}{\mathrm{cov}(RM, RM)} \mid \frac{ERM \, 4 \, rf}{\omega_{RM}^{\;2}}, or$$

$$Er_j \, 4 \, rf \mid \eta_j [ERM \, 4 \, rf] \qquad (1.3)$$

$$\eta_j \, \Sigma \frac{\mathrm{cov}(r_j, RM)}{\omega_{RM}^{\;2}}$$

Equation 1.3 gives the traditional η representation of the SML.

Testing the CAPM
The Model

Replacing the expectations with realizations in equation (1.3) gives a testable model of the SML,

$$(r_j \, 4 \, rf)_t \mid a_j \, 2 \, b_j (RM \, 4 \, rf)_t \, 2 \, e_{jt} \qquad (1.4)$$

Under null hypothesis the intercept, a—which represents an excess expected return—in the linear regression is zero. The b_j coefficient is the security's η, see Campbell, Lo, and MacKinlay, Chapter 5 for a detailed (probably more detailed than you want) discussion of the econometrics.

The Test

Test the security market line restriction that $a = 0$. Use an individual stock, decile, and an international return for the return on security (or portfolio) j. Choose a proxy for the "market" return and the risk free rate. Justify your choice.

You can get individual stock, decile, the market portfolio, and the risk free return from CRSP. Use GFD, or DataScream for the international return. (Be sure to get returns, not prices.) Use at least 10 years of monthly data.

Each team should write a one or two page summary of the project and attach results. Be prepared to discuss the project in class.

Due in class Thursday 10/16