Econ 101A – Solution to Midterm 2
Th 7 November.

Problem 1. Cost functions, demand functions, market equilibrium. (41 points) In this exercise, we consider the market equilibrium of an economy of which we know the cost function of firms, as well as the demand function of consumers. We consider first the short-run equilibrium, and then the long-run equilibrium. Each firm in the industry has the same technology with cost function

$$c(y) = k^2 + y^2$$

if $y > 0$ and $c(0) = 0$, where y is the quantity produced and k is some fixed cost that the firm pays only if it produces anything. You should think of this cost function as coming out of a standard cost minimization problem with respect to the inputs.

1. Derive the average cost $c(y)/y$ and the marginal cost $c'_y(y)$ for $y > 0$. Graph the average cost and marginal cost [remember, p is on the vertical axis]. (3 points)
2. Draw the supply function in the graph, and write down the equation that represents the supply function $y(p)$. [If possible, write y as a function of p and not viceversa] (5 points)
3. Derive the aggregate supply function $Y^S(p)$ by summing the supply $y(p)$ over the J firms that are in the market. Write down the expression for $y^S(p)$ (2 points)
4. Consider now the demand side of the market. For simplicity, assume a linear demand function: $Y^D(p) = a - bp$ where Y^D is the total quantity demanded in the industry. Assume that this comes from aggregation of the individual demand functions derived from maximization. Find the short-run equilibrium price p^* by equating Y^D and Y^S. Assume that a and b are such that we are on the increasing part of the supply function (i.e., the firm produces a positive quantity.) Find the short-run equilibrium industry production $Y^* = Y^S(p^*) = Y^S(p^*)$. (4 points)
5. Under what condition for a, J, b, k the firms will indeed produce a positive quantity of output? We maintain this assumption for points 6 and 7. (3 points)
6. Assuming positive production, we consider several comparative statics predictions. What happens to Y^* and p^* as the number of firms J increases? What is the intuition? [You do not even have to take derivatives, as long as you can infer the sign of the effect from the equations you derived at point 5]. If this was Ec10, and you could not do any algebra but only shift curves, how would you prove the same result graphically? What happens to Y^* and p^* as the demand coefficient a increase? What is the intuition? (4 points)
7. What happens to Y^* and p^* as the fixed costs k increase? What is the intuition? (3 points)
8. Consider now the long-run equilibrium, in which firms are allowed to enter the market. Solve for the number of firms J^* that will enter into the market [You can assume that the number that you find is integer] (5 points)
9. How does the number of firms J^* depend on k and a? [Give the sign only] (2 points)
10. What is the equation for the long-run supply curve? Is it horizontal, increasing or decreasing? (4 points)
11. What does the answer to the previous question tell you about incidence of a tax t? You can argue intuitively, or using the expression we derived in class on $\partial p/\partial t$. In the long-run, who bears the burden of the tax, the consumers or the producers, or both? [you may find a graph helpful] (6 points)

Solution to Problem 1.

1. The average cost is $c(y)/y = \left(k^2 + y^2\right)/y = k^2/y + y$. The marginal cost is $c'_y(y) = 2y$. For the graph, see the Appendix.

2. The supply function is given by the equation $p = c_y'(y)$ for marginal costs higher than average costs, and zero otherwise. Therefore, the supply function is defined by $p = 2y$ for $2y > k^2/y + y$, or $k^2/y < y$ or $k^2 < y^2$ or $y > k$. Invert to find $y(p) = p/2$ for $p/2 > k$. We can write down the supply function as

$$y(p) = \begin{cases} \frac{p}{2} & \text{if } \ p/2 \geq k \\ 0 & \text{if } \ p/2 < k \end{cases}$$

For the graph, see the Appendix.

3. The aggregate supply function $Y^S(p)$ is the sum $\sum_{j=1}^{J} y(p)$:

$$Y^S(p) = \begin{cases} J\frac{p}{2} & \text{if } \ p/2 \geq k \\ 0 & \text{if } \ p/2 < k \end{cases}$$

4. We equate the supply function and the demand function. We have to be careful since the supply function is defined in two pieces. Assume a positive production. We will check in point 5 the necessary conditions:

$$\begin{aligned} Y^S(p) &= J\frac{p^*}{2} = a - bp^* = Y^D(p) \text{ or} \\ & p^*\left(\frac{J}{2} + b\right) = a \text{ or} \\ p^* &= a/\left(\frac{J}{2} + b\right). \end{aligned} \tag{1}$$

The quantity produced is

$$Y^S = Jp^*/2 = \frac{a}{1 + 2(b/J)}. \tag{2}$$

We would have gotten the same expression for p^* if we had used the demand function:

$$Y^D = a - bp^* = a - ba/\left(\frac{J}{2} + b\right) = \left(\frac{a(J/2 + b) - ba}{J/2 + b}\right) = \left(\frac{aJ/2}{J/2 + b}\right) = \frac{a}{1 + 2(b/J)}.$$

5. Production is positive if $p^*/2 \geq k$ or

$$a/(J + 2b) \geq k \tag{3}$$

6. Using equation (1), it is easy to see that an increase in the number of firms in the market J leads to a reduction in the equilibrium price. Similarly, from (2), we see that it leads to an increase in output. More firms corresponds to a positive supply shock, that is a shift to the right of the supply curve in Ec10. Once again, using equations (1)and (2), we see that an increase in the demand coefficient a leads to an increase in price and in output. Higher exogenous demand corresponds to a positive demand shock, that is a shift to the right of the demand curve in Ec10.

7. A change in the fixed costs k does not affect the equilibrium price and output, as long as production remains positive. Fixed costs do not affect marginal decisions, or the intersection of demand and supply. However, an increase in fixed costs is likely to push firms to exit the market.

8. In the long-run, firm enter until profits are zero, or equivalently until firms are indifferent between staying in and staying out. This condition is equivalent to condition (3) with equality, or

$$\begin{aligned} \frac{a}{k} &= J + 2b \text{ or} \\ J^* &= \frac{a}{k} - 2b \end{aligned}$$

9. As the exogenous demand a increases, there is room for more firms in the market. As the fixed costs k increase, there is room for fewer firms.

10. In this setting, since the cost function of each firm is not affected by the entry of other firms, the long-run supply function is horizontal. The supply function is

$$Y^S(p) = \begin{cases} \infty & \text{if} \quad p/2 > k \\ \text{anything} & \text{if} \quad p/2 = k \\ 0 & \text{if} \quad p/2 < k \end{cases}$$

11. The fact that the supply curve is horizontal implies that the burden of a tax t falls fully on the consumers. Intuitively, an horizontal supply function is a very, very elastic supply function, and we know that the burden of taxation tends to fall on the side with least elastic function. Formally, we saw in class that

$$\frac{\partial p^*}{\partial t} = \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

and for $\varepsilon_{S,p} \to \infty$, $\frac{\partial p^*}{\partial t} = 1$ – the incidence is all on the consumer.

Problem 2. Uncertainty. (20 points) In the world, we observe many individuals that purchase both insurance and that gamble, a puzzling behavior. Define the problem as follows. An agent has utility function $u(w)$ defined over wealth w, with $u' > 0$. The agent has wealth w.

1. Consider the following stylized Las Vegas gamble: the agent wins \$10 with probability 1/10 and loses \$2 with probability 9/10. Write the expected value and the expected utility associated with this gamble. (5 points)

2. Try to show that a risk-averse agent (concave utility, $u'' < 0$) will prefer not to take this gamble using Jensen's inequality. Risk-averse people do not go to Las Vegas. If you do not remember Jensen's inequality, it's ok! Try a graphical or verbal argument. (5 points)

3. In class, we also showed that risk-averse agents purchase insurance. To sum up, risk-averse agents purchase insurance, but do not gamble. To reconcile the theory with the evidence that people do both, Friedman and Savage in 1950 proposed that the utility function over wealth is as in Figure 1: concave for low levels of wealth, and convex for high levels of wealth. Explain verbally why this theory predicts that we should observe in the world both insurance and gambling. (4 points)

4. Do you find this explanation convincing? What kind of evidence would imply that this theory is the wrong explanation? (6 points)

Solution to Problem 2.

1. The expected value of the gamble is $1/10 * 10 - 9/10 * 2 = 1 - 1.8 = -.8$. The expected utility is

$$EU = 1/10 * u(w + 10) + 9/10 * u(w - 2).$$

Notice that the expression for the expected utility intergrates the wealth w into the calculation.

2. For a risk-averse agent the utility function u is concave. Using Jensen's inequality for concave functions we have

$$EU = 1/10 * u(w + 10) + 9/10 * u(w - 2) \le u[1/10 * (w + 10) + 9/10 * (w - 2)] = u[w - .8] < u(w).$$

Therefore the agent prefers not to take the lottery.

3. This utility function implies that the agent is risk-averse for low levels of wealth, and risk-seeking for high levels of wealth. Therefore, a poor agent would puchase insurance, while a rich agent would gamble in Las Vegas. Gambling and insurance coexist.

4. This explanation implies that, while we can observe gambling and insurance occurring contemporaneously, the same individual should not do both. Moreover, only poor people would insure, and only rich people would gamble. Clearly, the former assersion is false. (a theory called prospect theory can explain this paradox better...)

Problem 3. Economics of crime (Becker). (18 points) Consider a risk-neutral agent that files taxes. She puts effort e, $e \in [0,1]$, to file taxes correctly. Effort e has cost $e^2/2$. The benefit of effort is that it reduces the probability of errors: the agent makes an error with probability $(1-e)$. If the agent makes an error, she is discovered with probability p, at which point she has to pay a fine f, so wealth w goes down to $w-f$. The maximization problem of the individual is

$$\max_e p(1-e)(w-f) + [1-p(1-e)]w - \frac{e^2}{2}.$$

1. Write down the first order conditions and solve for the optimal level of effort e^* (3 points)

2. Why is effort e^* increasing in the probability of being caught p? Why is it increasing in the fine f? (3 points)

3. (Hard) Here is now the interesting part: what is the optimal choice of p and f for the goverment? Suppose that the government has to pay wages x to agents that audit the taxes. Moreover, the government cares about deviations from the optimal level of effort $e = 1$. The government then maximizes

$$\begin{aligned}
&\max_{f,p} -px - (1-e^*(p,f))^2 \\
&s.t.\ 0 \le e^*(p,f) \le 1, \\
&s.t.\ 0 \le p \le 1
\end{aligned}$$

   You should substitute the expression for $e^*(p,f)$ that you found in point 1. Now compute the optimal levels of fine f^* and auditing probability p^*. What are the solutions? [Hint: Do not use the Lagrangeans, there are corner solutions. Use your intuition. Technically, there is no optimum, but there are sups] (8 points)

4. Why does the government adopt this enforcement strategy? (4 points)

Solution to Problem 3.

1. The first order condition is

$$-p(w-f) + pw - e = 0$$

   or $e^* = pf$.

2. The higher the probability of being caught, the more important it is to be careful about the tax filing. Similarly, a higher fine makes it worthwhile to spend more effort checking the taxes.

3. In the maximization problem we substitute $e^* = pf$ and get

$$\begin{aligned}
&\max_{f,p} -px - (1-pf)^2 \\
&s.t.\ 0 \le pf \le 1, \\
&s.t.\ 0 \le p \le 1
\end{aligned}$$

Notice now that the government can achieve the fully optimal level of e^* by setting $p^*f^* = 1$. this maximizes $-(1-pf)^2$. Then, in order to maximize $-px$ the government wants to set p as low as possible, compatibly with having $p^*f^* = 1$. The government can achieve this objective by letting $p^* \to 0$ and $f^* \to \infty$ so that $p^*f^* = 1$.

4. The government can adopt two strategies to enforce the law: high fines or high auditing probabilities. The latter option is much more costly than the former, since it requires the work of auditors. Therefore the government finds it optimal to set very, very high fines and reduce the probability of monitoring to almost zero.