Economics 101A
(Lecture 4, Revised)

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Outline

1. Constrained Maximization (from last lecture)

2. Envelope Theorem II

3. Preferences

4. Properties of Preferences

1 Constrained Maximization (ctnd)

- **Constrained Maximization, Sufficient condition for the case $n = 2, m = 1$.**

- If $\mathbf{x}^*$ satisfies the Lagrangean condition, and the determinant of the bordered Hessian

$$H = \begin{pmatrix} 0 & -\frac{\partial h}{\partial x_1}(\mathbf{x}^*) & -\frac{\partial h}{\partial x_2}(\mathbf{x}^*) \\ -\frac{\partial h}{\partial x_1}(\mathbf{x}^*) & \frac{\partial^2 L}{\partial^2 x_1}(\mathbf{x}^*) & \frac{\partial^2 L}{\partial x_2 \partial x_1}(\mathbf{x}^*) \\ -\frac{\partial h}{\partial x_2}(\mathbf{x}^*) & \frac{\partial^2 L}{\partial x_1 \partial x_2}(\mathbf{x}^*) & \frac{\partial^2 L}{\partial x_2 \partial x_2}(\mathbf{x}^*) \end{pmatrix}$$

  is positive, then $\mathbf{x}^*$ is a constrained maximum.

- If it is negative, then $\mathbf{x}^*$ is a constrained minimum.

- Why? This is just the Hessian of the Lagrangean L with respect to λ, x_1, and x_2

- Example 4: $\max_{x,y} x^2 - xy + y^2$ s.t. $x^2 + y^2 - p = 0$

- $\max_{x,y,\lambda} x^2 - xy + y^2 - \lambda(x^2 + y^2 - p)$

- F.o.c. with respect to x:

- F.o.c. with respect to y:

- F.o.c. with respect to λ:

- Candidates to solution?

- Maxima and minima?

2 Envelope Theorem II

- Nicholson, Ch. 2, pp. 46-47.

- **Envelope Theorem for Constrained Maximization.** In problem above consider $F(p) \equiv f(\mathbf{x}^*(\mathbf{p}); \mathbf{p})$. We are interested in $dF(p)/dp$. We can neglect indirect effects:

$$\frac{dF}{dp_i} = \frac{\partial f(\mathbf{x}^*(\mathbf{p}); \mathbf{p})}{\partial p_i} - \sum_{j=0}^{m} \lambda_j \frac{\partial h_j(\mathbf{x}^*(\mathbf{p}); \mathbf{p})}{\partial p_i}$$

- Example 4 (continued). $\max_{x,y} x^2 - xy + y^2$ s.t. $x^2 + y^2 - p = 0$

- $df(x^*(p), y^*(p))/dp$?

- Envelope Theorem.

3 Preferences

- Part 1 of our journey in microeoconomics: *Consumer Theory*

- Choice of consumption bundle:

  1. vegetables in Berkeley Bowl
  2. work, study, and leisure
  3. spend today or spend tomorrow

- Starting point: preferences.

  1. 5 Roma tomatoes $\succ$ 3 zucchini
  2. 1 hour out with friends $\succ$ 1 hour in class $\succ$ 1 hour doing problem set
  3. 1 egg today $\succ$ 1 chicken tomorrow

4 Properties of Preferences

- Nicholson, Ch.3, p. 66.

- Commodity set X (apples vs. strawberries, work vs. leisure, consume today vs. tomorrow)

- Preference relation $\succeq$ over X

- A preference relation $\succeq$ is *rational* if

  1. It is *complete*: For all x and y in X, either $x \succeq y$, or $y \succeq x$ or both

  2. It is *transitive*: For all x, y, and z, $x \succeq y$ and $y \succeq z$ implies $x \succeq z$

- Preference relation $\succeq$ is *continuous* if for all y in X, the sets $\{x : x \succeq y\}$ and $\{x : y \succeq x\}$ are closed sets.

- Example: $X = R^2$ with map of indifference curves

- Counterexamples:

  1. Incomplete preferences. Dominance rule.

  2. Intransitive preferences. Quasi-discernible differences.

  3. Discontinuous preferences. Lexicographic order

- Indifference relation $\sim$: $x \sim y$ if $x \succeq y$ and $y \succeq x$
- Strict preference: $x \succ y$ if $x \succeq y$ and not $y \succeq x$
- Exercise. If $\succeq$ is rational,
  - $\succ$ is transitive
  - $\sim$ is transitive
  - Reflexive property of $\succeq$. For all x, $x \succeq x$.

- Other features of preferences

- Preference relation $\succeq$ is:

  - *monotonic* if $x \geq y$ implies $x \succeq y$.

  - *strictly monotonic* if $x \geq y$ and $x_j > y_j$ for some j implies $x \succ y$.

  - *convex* if for all x, y, and z in X such that $x \succeq z$ and $y \succeq z$, then $tx + (1 - t)y \succeq z$ for all t in $[0, 1]$