Economics 101A
(Lecture 18, Revised)

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Outline

1. One-Step Profit Maximization

2. Introduction to Market Equilibrium

3. Aggregation

4. Maket Equilibrium in Short-Run

5. Comparative Statics of Equilibrium

- Pages covered on Nicholson so far include (not necessarily exhaustive):
  - Nicholson, Ch. 8, pp. 98–110. (Expected Utility)
  - Nicholson, Ch. 11, pp. 268–278, 280–285
  - Nicholson, Ch. 12, pp. 298–302, 304–312, 318–324.
  - Nicholson, Ch. 13, pp. 342–350.

1 One-Step Profit Maximization

- Firm optimization:

$$\max_{L,K} pf(L,K) - wL - rK$$

- First order conditions:

$$pf'_L(L,K) - w = 0$$

and

$$pf'_K(L,K) - r = 0$$

- Second order conditions? $pf''_{L,L}(L,K) < 0$, and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix} = \\ &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0 \end{aligned}$$

- Comparative statics with respect to to p, w, and r.

- What happens if w increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial p} =$$

- $\partial L^*/\partial p > 0$ if $f''_{L,K} > 0$.

2 Introduction to Market Equilibrium

- Nicholson, Ch. 14, pp. 368–382.

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs L^*, K^* (see above)
  - Optimal quantity produced y^*

- **Supply function**. $y = y^* (p, w, r)$
  - From profit maximization:

    $$y = f (L^* (p, w, r), K^* (p, w, r))$$

  - From cost minimization:

    MC curve above AC [REVISED]

  - Supply function is increasing in p

- Market Equilibrium. Equate demand and supply.

- Aggregation?

- Industry supply function!

3 Aggregation

3.1 Producers aggregation

- J companies, $j = 1, ..., J$, producing good i

- Company j has supply function

$$y_i^j = y_i^{j*}(p_i, w, r)$$

- Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r)$$

- Graphically,

3.2 Consumer aggregation

- Nicholson, Ch. 7, pp. 172–176

- *One-consumer economy*

- Utility function $u(x_1, ..., x_n)$

- prices $p_1, ..., p_n$

- Maximization $\Longrightarrow$

$$\begin{aligned} x_1^* &= x_1^*(p_1, ..., p_n, M), \\ &\vdots \\ x_n^* &= x_n^*(p_1, ..., p_n, M). \end{aligned}$$

- Focus on good i. Fix prices $p_1, ..., p_{i-1}, p_{i+1}, ..., p_n$ and M

- **Single-consumer demand function**:

$$x_i^* = x_i^* \left(p_i | p_1, ..., p_{i-1}, p_{i+1}, ..., p_n, M\right)$$

- What is sign of $\partial x_i^* / \partial p_i$?

- Negative if good i is normal

- Negative or positive if good i is inferior

- *Aggregation:* J consumers, $j = 1, ..., J$

- Demand for good i by consumer j :

$$x_i^{j*} = x_i^{j*}\left(p_1, ..., p_n, M^j\right)$$

- Market demand X_i:

$$\begin{aligned} & X_i\left(p_1, ..., p_n, M^1, ..., M^J\right) \\ = & \sum_{j=1}^{J} x_i^{j*}\left(p_1, ..., p_n, M^j\right) \end{aligned}$$

- Graphically,

- Notice: market demand function depends on distribution of income M^J

- Market demand function X_i:
  - Consumption of good i as function of prices $\mathbf{p}$
  - Consumption of good i as function of income distribution M^j

4 Market Equilibrium in the Short-Run

- Nicholson, Ch. 14, pp. 368–382.

- What is equilibrium price p_i?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $\mathbf{p}^*$ equates demand and supply of good i:

  $Y^* = Y_i^S(p_i^*, w, r) = X_i^D\left(p_1^*, ..., p_n^*, M^1, ..., M^J\right)$

- Graphically,

- Notice: in short-run firms can make positive profits

- Comparative statics exercises with endogenous price p_i :
  - increase in wage w or interest rate r:
  - change in income distribution

5 Comparative statics of equilibrium

- Supply and Demand function of parameter α :
  - $Y_i^S(p_i, w, r, \alpha)$
  - $X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$

- How does α affect p^* and Y^*?

- Comparative statics with respect to α

- Equilibrium:

$$Y_i^S(p_i, w, r, \alpha) = X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$$

- Can write equilibrium as implicit function:

$$Y_i^S(p_i, w, r, \alpha) - X_i^D(\mathbf{p}, \mathbf{M}, \alpha) = 0$$

- What is $dp^*/d\alpha$?

- Implicit function theorem:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- What is sign of denominator?

- Sign of $\partial p^*/\partial \alpha$ is negative of sign of numerator

- How do we interpret magnitudes of $\partial p^*/\partial\alpha$?

- Result depends on unit of measures

- Can we write $\partial p^*/\partial\alpha$ in a unit-free way?

- Yes! Use **elasticities.**

- Elasticity of x with respect to parameter p is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p}\frac{p}{x}$$

- Interpretation: Percent response in x to percent change in p :

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p}\frac{p}{x} = \frac{dx/x}{dp/p}$$

- Exercise:

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$

- Use elasticities to rewrite response of p to change in α :

$$\frac{\partial p^*}{\partial \alpha}\frac{\alpha}{p} = -\frac{\left(\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}\right)\frac{\alpha}{Y}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right)\frac{p}{Y}}$$

or

$$\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- We are likely to know elasticities from empirical studies.