Economics 101A
(Lecture 24, Revised)

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Outline

1. Second-price Auction
2. Dynamic Games

1 Second-price Auction

- Sealed-bid auction
- Highest bidder wins object
- Price paid is second highest price

- Two individuals: $I = 2$
- Strategy s_i is bid b_i
- Each individual knows value v_i

- Payoff for individual i is

$$u_i(b_i, b_{-i}) = \begin{cases} v_i - b_{-i} & \text{if} \quad b_i > b_{-i} \\ (v_i - b_{-i})/2 & \text{if} \quad b_i = b_{-i} \\ 0 & \text{if} \quad b_i < b_{-i} \end{cases}$$

- Show: weakly dominant to set $b_i^* = v_i$

- To show:

$$u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$$

for all b_i, for all b_{-i}, and for $i = 1, 2$.

1. Assume $b_{-i} > v_i$

   - $u_i(v_i, b_{-i}) = 0 = u_i(b_i, b_{-i})$ for any $b_i < b_{-i}$ [REVISED]

   - $u_i(b_{-i}, b_{-i}) = (v_i - b_{-i})/2 < 0$ [REVISED]

   - $u_i(b_i, b_{-i}) = (b_i - b_{-i}) < 0$ for any $b_i > b_{-i}$ [REVISED]

2. Assume now $b_{-i} = v_i$

3. Assume now $b_{-i} < v_i$

2 Dynamic Games

- Nicholson, Ch. 10, pp. 256–259.

- Dynamic games: one player plays after the other

- Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node

- Example: battle of the sexes game

She \ He	Ballet	Football
Ballet	$2, 1$	$0, 0$
Football	$0, 0$	$1, 2$

- Dynamic version: she plays first

- **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players' strategy

- Backward induction. Find optimal action in last period and then work backward

- Solution

- Example 2: Entry Game

$1 \setminus 2$	Enter	Do not Enter
Enter	$-1, -1$	$10, 0$
Do not Enter	$0, 5$	$0, 0$

- Exercise. Dynamic version.

- Coordination games solved if one player plays first

- Can use this to study finitely repeated games

- Suppose we play the prisoner's dilemma game ten times.

$1 \setminus 2$	D	ND
D	$-4, -4$	$-1, -5$
ND	$-5, -1$	$-2, -2$

- What is the subgame perfect equilibrium?