Econ 101A – Midterm 1
Th 3 October.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 12.30 sharp. Show your work!

Problem 1. Labor supply with social comparison. (48 points) In this exercise, we consider a labor supply model with Cobb-Douglas preferences. The non-standard feature in this problem is that the preferences for consumption depend on a reference level C. Individuals in this society are only happy if they consumer more than a reference level C. You can think of this level as consumption of the neighbour. Consider the following utility function:

$$u(c, l; C) = (c - C)^{\alpha} l^{1-\alpha}$$

with $0 < \alpha < 1$ and $C > 0$. The utility from consumption of good c depends on the average consumption in society C. The price of the consumption good is p. Leisure is l.

1. How does the utility function change as C changes? In other words, compute $\partial u(c, l; C)/\partial C$. Why is this term negative? (4 points)
2. We are looking at labor supply in one day. We assume that, if the individual does not work, s/he takes leisure. In other words, the hours worked h equal $24 - l$. The hourly wage equals w. There are no sources of income other than income from hours worked. Write down the budget constraint as a function of c and l. [Hint: amount spent on the consumption good has to be smaller or equal than income earned] (6 points)
3. Write down the maximization problem of the worker with respect to c and l. Assume that the budget constraint is satisfied with equality. Why can we assume that the budget constraint is satisfied with equality? Provide as complete an explanation as you can. (10 points)
4. Write down the Lagrangean function. (2 points)
5. Write down the first order conditions for this problem with respect to c, l, and λ. (4 points)
6. Solve explicitely for c^* and l^* as a function of p, w, C, and α. (8 points)
7. Notice that the utility function $(c^* - C)^{\alpha} (l^*)^{1-\alpha}$ is defined only for $c^* > C, l^* > 0$. Show that these conditions are satisfied if $C \leq 24w/p$. From now on we assume these conditions satisfied.(2 points)
8. Use the expression for l^* that you obtained in point 6. Differentiate it with respect to C, that is, compute $\partial l^*/\partial C$. Why is this amount negative? [Hint: The individual needs to work more if...] (4 points)
9. Now calculate $\partial u(c^*(p, w), l^*(p, w))/\partial C$. Do it two ways. First, substitute into the utility function the expressions for $c^*(p, w)$ and $l^*(p, w)$ obtained at point 6, and differentiate the resulting expression with respect to C. Second, use the envelope theorem. The two results should coincide! What happens to utility at the optimum as the reference level increases? Is this result surprising? (8 points)

Problem 2. Short answers. (12 points) In this part, you are required to provide short answers to the following two questions:

1. Consider the set $\{a, b, c, d\}$ with the following preferences defined on it: $a \succeq b$, $b \succeq c$, $c \succeq d$, $d \succ a$. Are these preferences transitive? (6 points)
2. Consider the implicit function $y - \exp(x * y) = 0$. Use the implicit function theorem to write down $\partial y/\partial x$. (assume that all the assumption needed to apply the theorem are satisfied) (6 points)