Economics 101A
(Lecture 20)

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Outline

1. Oligopoly?
2. Game Theory
3. Oligopoly: Cournot
4. Oligopoly: Bertrand

1 Oligopoly?

- Extremes:
  - Perfect competition
  - Monopoly
- Oligopoly if there are n (two, five...) firms
- Examples:
  - soft drinks: Coke, Pepsi;
  - cellular phones: Sprint, AT&T, Cingular,...
  - car dealers

- Firm i maximizes:

$$\max_{y_i} p\left(y_i + y_{-i}\right) y_i - c\left(y_i\right)$$

  where $y_{-i} = \sum_{j \neq i} y_j$.

- First order condition with respect to y_i:

$$p'_Y\left(y_i + y_{-i}\right) y_i + p - c'_y\left(y_i\right) = 0.$$

- Problem: what is the value of y_{-i}?
  - simultaneous determination?
  - can firms $-i$ observe y_i?

- Need to study strategic interaction

2 Game Theory

- Nicholson, Ch. 15, pp. 440–449 [OLD: Ch. 10, pp. 246–255].

- Unfortunate name

- Game theory: study of decisions when payoff of player i depends on actions of player j.

- Brief history:

  – von Neuman and Morgenstern, Theory of Games and Economic Behavior (1944)

  – Nash, Non-cooperative Games (1951)

  – ...

  – Nobel Prize to Nash, Harsanyi (Berkeley), Selten (1994)

- Definitions:

  - Players: $1, ..., I$

  - Strategy $s_i \in S_i$

  - Payoffs: $U_i\left(s_i, s_{-i}\right)$

- Example: Prisoner's Dilemma
  - $I = 2$
  - $s_i = \{D, ND\}$
  - Payoffs matrix:

$1 \setminus 2$	D	ND
D	$-4, -4$	$-1, -5$
ND	$-5, -1$	$-2, -2$

- What prediction?

- Maximize sum of payoffs

- Choose dominant strategies

- **Equilibrium in dominant stategies**

- Strategies $s^* = \left(s_i^*, s_{-i}^*\right)$ are an Equilibrium in dominant stategies if

  $$U_i\left(s_i^*, s_{-i}\right) \geq U_i\left(s_i, s_{-i}\right)$$

  for all $s_i \in S_i$, for all $s_{-i} \in S_{-i}$ and all $i = 1, ..., I$

- Battle of the Sexes game:

He \ She	Ballet	Football
Ballet	$2, 1$	$0, 0$
Football	$0, 0$	$1, 2$

- Choose dominant strategies? Not possible

- **Nash Equilibrium**.

- Strategies $s^* = \left(s_i^*, s_{-i}^*\right)$ are a Nash Equilibrium if

$$U_i\left(s_i^*, s_{-i}^*\right) \geq U_i\left(s_i, s_{-i}^*\right)$$

  for all $s_i \in S_i$ and $i = 1, ..., I$

- Is Nash Equilibrium unique?

- Does it always exist?

- Penalty kick in soccer (matching pennies)

Kicker \ Goalie	L	R
L	$0, 1$	$1, 0$
R	$1, 0$	$0, 1$

- Equilibrium always exists in mixed strategies σ

- Mixed strategy: allow for probability distibution.

- Back to penalty kick:
  - Kicker kicks left with probability k
  - Goalie kicks left with probability g

  - utility for kicker of playing L :

$$\begin{aligned} U_K(L,\sigma) &= gU_K(L,L) + (1-g)\,U_K(L,R) \\ &= (1-g) \end{aligned}$$

  - utility for kicker of playing R :

$$\begin{aligned} U_K(R,\sigma) &= gU_K(R,L) + (1-g)\,U_K(R,R) \\ &= g \end{aligned}$$

- Optimum?
  - $L \succ R$ if $1 - g > g$ or $g < 1/2$
  - $R \succ L$ if $1 - g < g$ or $g > 1/2$
  - $L \sim R$ if $1 - g = g$ or $g = 1/2$

- Plot best response for kicker

- Plot best response for goalie

- Nash Equilibrium is:
  - fixed point of best response correspondence
  - crossing of best response correspondences

3 Oligopoly: Cournot

- Nicholson, Ch. 14, pp. 418–419, 421–422 [OLD: p. 531, 534–535].

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity y_i

- Firm i maximizes:

$$\max_{y_i} p(y_i + y_{-i}) y_i - cy_i.$$

- First order condition with respect to y_i:

$$p'_Y \left(y_i^* + y_{-i}^*\right) y_i^* + p - c = 0, \ i = 1, 2.$$

- Nash equilibrium:
  - y_1 optimal given y_2;
  - y_2 optimal given y_1.

- Solve equations:

$$p'_Y\left(y_1^* + y_2^*\right) y_1^* + p - c = 0 \text{ and}$$

$$p'_Y\left(y_2^* + y_1^*\right) y_2^* + p - c = 0.$$

- Cournot -> Pricing above marginal cost

4 Oligopoly: Bertrand

- Previously, we assumed firms choose quantities

- Now, assume firms first choose prices, and then produce quantity demanded by market

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - c)\,Y(p_i) & \text{if } \; p_i < p_{-i} \\ (p_i - c)\,Y(p_i)/2 & \text{if } \; p_i = p_{-i} \\ 0 & \text{if } \; p_i > p_{-i} \end{cases}$$

- First show that $p_1 = c = p_2$ is Nash Equilibrium

- Does any firm have a (strict) incentive to deviate?

- Show that this equilibrium is unique

- Case 1. $p_1 > p_2 > c$

- Case 2. $p_1 = p_2 > c$

- Case 3. $p_1 > c \geq p_2$

- Case 4. $c > p_1 \geq p_2$

- Case 5. $p_1 = c > p_2$

- Case 6. $p_1 = c = p_2$

- It is unique!

- Marginal cost pricing

- Two firms are enough to guarantee perfect competition!

- Price wars

5 Next lecture

- Dynamic games
- Stackelberg duopoly
- Auctions