# Economics 101A (Lecture 9) 

Stefano DellaVigna

September 28, 2004

## Outline

## 1. Labor Supply

2. Intertemporal choice

## 1 Labor Supply

- Nicholson Ch. 16, pp. 477-484 [OLD: Ch. 22, pp. 606-613.]
- Labor supply decision: how much to work in a day.
- Goods: consumption good $c$, hours worked $h$
- Price of good $p$, hourly wage $w$
- Consumer spends $24-h=l$ hours in units of leisure
- Utilify function: $u(c, l)$
- Budget constraint?
- Income of consumer: $M+w h=M+w(24-l)$
- Budget constraint: $p c \leq M+w(24-l)$ or

$$
p c+w l \leq M+24 w
$$

- Notice: leisure $l$ is a consumption good with price w. Why?
- General category: opportunity cost
- Instead of enjoying one hour of TV, I could have worked one hour and gained wage $w$.
- You should value the marginal hour of TV $w$ !
- Opportunity costs are very important!
- Example 2. CostCo has a warehouse in SoMa
- SoMa used to have low cost land, adequate for warehouses
- Price of land in SoMa triples in 10 years.
- Should firm relocate the warehouse?
- Did costs of staying in SoMa go up?
- No.
- Did the opportunity cost of staying in SoMa go up?
- Yes!
- Firm can sell at high price and purchase land in cheaper area.
- Let's go back to labor supply
- Maximization problem is

$$
\begin{aligned}
& \max u(c, l) \\
& \text { s.t. } p c+w l \leq M+24 w
\end{aligned}
$$

- Standard problem (except for $24 w$ )
- First order conditions
- Assume utility function Cobb-Douglas:

$$
u(c, l)=c^{\alpha} l^{1-\alpha}
$$

- Solution is

$$
\begin{aligned}
c^{*} & =\alpha \frac{M+24 w}{p} \\
l^{*} & =(1-\alpha)\left(24+\frac{M}{w}\right)
\end{aligned}
$$

- Both $c$ and $l$ are normal goods
- Unlike in standard Cobb-Douglas problems, $c^{*}$ depends on price of other good $w$
- Why? Agents are endowed with $M$ AND 24 hours of $l$ in this economy
- Normally, agents are only endowed with $M$


## 2 Intertemporal choice

- Nicholson Ch. 17, pp. 502-506 [OLD: Ch. 23, pp. 628-632.]
- So far, we assumed people live for one period only
- Now assume that people live for two periods:
- $t=0$ - people are young
- $t=1$ - people are old
- $t=0$ : income $M_{0}$, consumption $c_{0}$ at price $p_{0}=1$
- $t=1$ : income $M_{1}>M_{0}$, consumption $c_{1}$ at price $p_{1}=1$
- Credit market available: can lend or borrow at interest rate $r$
- Budget constraint in period 1 ?
- Sources of income:
- $M_{1}$
$-\left(M_{0}-c_{0}\right) *(1+r)$ (this can be negative)
- Budget constraint:

$$
c_{1} \leq M_{1}+\left(M_{0}-c_{0}\right) *(1+r)
$$

or

$$
c_{0}+\frac{1}{1+r} c_{1} \leq M_{0}+\frac{1}{1+r} M_{1}
$$

- Utility function?
- Assume

$$
u\left(c_{0}, c_{1}\right)=U\left(c_{0}\right)+\frac{1}{1+\delta} U\left(c_{1}\right)
$$

- $U^{\prime}>0, U^{\prime \prime}<0$
- $\delta$ is the discount rate
- Higher $\delta$ means higher impatience
- Elicitation of $\delta$ through hypothetical questions
- Person is indifferent between 1 hour of TV today and $1+\delta$ hours of TV next period
- Maximization problem:

$$
\begin{aligned}
& \max U\left(c_{0}\right)+\frac{1}{1+\delta} U\left(c_{1}\right) \\
& \text { s.t. } c_{0}+\frac{1}{1+r} c_{1} \leq M_{0}+\frac{1}{1+r} M_{1}
\end{aligned}
$$

- Lagrangean
- First order conditions:
- Ratio of f.o.c.s:

$$
\frac{U^{\prime}\left(c_{0}\right)}{U^{\prime}\left(c_{1}\right)}=\frac{1+r}{1+\delta}
$$

- Case $r=\delta$
$-c_{0}^{*} \quad c_{1}^{*}$ ?
- Substitute into budget constraint using $c_{0}^{*}=$ $c_{1}^{*}=c^{*}$ :

$$
\frac{2+r}{1+r} c^{*}=\left[M_{0}+\frac{1}{1+r} M_{1}\right]
$$

or

$$
c^{*}=\frac{1+r}{2+r} M_{0}+\frac{1}{2+r} M_{1}
$$

- We solved problem virtually without any assumption on $U$ !
- Notice: $M_{0}<c^{*}<M_{1}$
- Case $r>\delta$

$$
-c_{0}^{*} \quad c_{1}^{*} ?
$$

- Comparative statics with respect to income $M_{0}$
- Rewrite ratio of f.o.c.s as

$$
U^{\prime}\left(c_{0}\right)-\frac{1+r}{1+\delta} U^{\prime}\left(c_{1}\right)=0
$$

- Substitute $c_{1}$ in using $c_{1}=M_{1}+\left(M_{0}-c_{0}\right)(1+r)$ to get

$$
U^{\prime}\left(c_{0}\right)-\frac{1+r}{1+\delta} U^{\prime}\left(M_{1}+\left(M_{0}-c_{0}\right)(1+r)\right)=0
$$

- Apply implicit function theorem:

$$
\frac{\partial c_{0}^{*}(r, \mathbf{M})}{\partial M_{0}}=-\frac{-\frac{1+r}{1+\delta} U^{\prime \prime}\left(c_{1}\right)(1+r)}{U^{\prime \prime}\left(c_{0}\right)-\frac{1+r}{1+\delta} U^{\prime \prime}\left(c_{1}\right) *(-(1+r))}
$$

# - Denominator is always negative 

- Numerator is positive
- $\partial c_{0}^{*}(r, \mathbf{M}) / \partial M_{0}>0$ - consumption at time 0 is a normal good.
- Can also show $\partial c_{0}^{*}(r, \mathbf{M}) / \partial M_{1}>0$
- Comparative statics with respect to interest rate $r$
- Apply implicit function theorem:

$$
\begin{aligned}
\frac{\partial c_{0}^{*}(r, \mathbf{M})}{\partial r}= & -\frac{-\frac{1}{1+\delta} U^{\prime}\left(c_{1}\right)}{U^{\prime \prime}\left(c_{0}\right)-\frac{1+r}{1+\delta} U^{\prime \prime}\left(c_{1}\right) *(-(1+r))} \\
& -\frac{-\frac{1+r}{1+\delta} U^{\prime \prime}\left(c_{1}\right) *\left(M_{0}-c_{0}\right)}{U^{\prime \prime}\left(c_{0}\right)-\frac{1+r}{1+\delta} U^{\prime \prime}\left(c_{1}\right) *(-(1+r))}
\end{aligned}
$$

- Denominator is always negative
- Numerator: First term negative (substitution eff.)
- Numerator: Second term (income effect:)
- positive if $M_{0}>c_{0}$
- negative if $M_{0}<c_{0}$.

