Economics 101A
(Lecture 1)

Stefano DellaVigna

August 31, 2004

Outline

1. Who are we?
2. Prerequisites for the course
3. A test in maths
4. The economics of discrimination
5. Optimization with 1 variable
6. Multivariate optimization (Today or on Th)

1 Who are we?

Stefano DellaVigna

- Assistant Professor, Department of Economics

- Bocconi (Italy) undergraduate (Econ.), Harvard PhD (Econ.)

- Psychology and economics, applied microeconomics, behavioral finance, aging, media

- Evans 515

Ada Chen (2 Sections)

Adriana Espinosa (1 Section)

- Graduate Students, Department of Economics
- Rooms: To be announced

2 Prerequisites

- Mathematics
  - Good knowledge of multivariate calculus – Maths 1A or 1B
  - Basic knowledge of probability theory and matrix algebra

- Economics
  - Knowledge of fundamentals – Ec1 or 2 or 3
  - High interest!

3 A Test in Maths

1. Can you differentiate the following functions with respect to x?

   (a) $y = \exp(x)$

   (b) $y = a + bx + cx^2$

   (c) $y = \frac{\exp(x)}{b^x}$

2. Can you partially differentiate these functions with respect to x and w?

   (a) $y = axw + bx - c\frac{x}{w} + d\sqrt{xw}$

   (b) $y = \exp(x/w)$

   (c) $y = \int_0^1 (x + aw^2 + xs)ds$

3. Can you plot the following functions of one variable?

   (a) $y = \exp(x)$

   (b) $y = -x^2$

   (c) $y = \exp(-x^2)$

4. Are the following functions concave, convex or neither?

   (a) $y = x^3$

   (b) $y = -\exp(x)$

   (c) $y = x^{.5} y^{.5}$ for $x > 0$, $y > 0$

5. Consider an urn with 20 red and 40 black balls?

   (a) What is the probability of drawing a red ball?

   (b) What is the probability of drawing a black ball?

6. What is the determinant of the following matrices?

   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$

4 The economics of discrimination

- Ok, I got the maths. But where is the economics?

- Workers:

  - A and B. They produce 1 widget per hour

  - Both have reservation wage $\bar{u}$

- Firm:

  - sells widgets at price $p > \bar{u}$ (assume p given)

  - dislikes worker B

  - Maximizes profits ($p*$ no of widgets $-$ cost of labor) – disutility d if employs B

- Wages and employment in this industry?

- Employment
  - Net surplus from employing A: $p - \bar{u}$
  - Net surplus from employing B: $p - \bar{u} - d$
  - If $\bar{u} < p < \bar{u} + d$, Firm employs A but not B
  - If $\bar{u} + d < p$, Firm employs both

- What about wages?

- Case I. Firm monopolist and no worker union
  - Firm maximizes profits and gets all the net surplus
  - Wages of A and B equal $\bar{u}$
- Case II. Firm monopolist and worker union
  - Firm and worker get half of the net surplus each
  - Wage of A equals $\bar{u} + .5 * (p - \bar{u})$
  - Wage of B equals $\bar{u} + .5 * (p - \bar{u} - d)$
- Case III. Perfect competition among firms that discriminate ($d > 0$)
  - Prices are lowered to the cost of production
  - Wage of A equals p
  - B is not employed

- The magic of competition

- Case IIIb. Perfect competition + At least **one** firm does not discriminate ($d = 0$)
  - This firm offers wage p to both workers
  - What happens to worker B?
  - She goes to the firm with $d = 0$!
  - In equilibrium now:
    * Wage of A equals p
    * Wage of B equals p as well!

- Is this true? Any evidence?

- S. Black and P. Strahan, AER 2001.

  - Local monopolies in banking industry until mid 70s

  - Mid 70s: deregulation

  - From local monopolies to perfect competition.

  - Wages?

    * Wages fall by 6.1 percent

  - Discrimination?

    * Wages fall by 12.5 percent for men

    * Wages fall by 2.9 percent for women

    * Employment of women as managers increases by 10 percent

5 Optimization with 1 variable

- Nicholson, Ch.2, pp. 22-26

- Example. Function $y = -x^2$

- What is the maximum?

- Maximum is at 0

- General method?

- Sure! Use derivatives

- Derivative is slope of the function at a point:

$$\frac{\partial f(x)}{\partial x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Necessary condition for maximum** x^* is

$$\frac{\partial f(x^*)}{\partial x} = 0 \tag{1}$$

- Try with $y = -x^2$.

- $\frac{\partial f(x)}{\partial x} =$ $\qquad\qquad\qquad = 0 \Longrightarrow x^* =$

- Does this guarantee a maximum? No!

- Consider the function $y = x^3$

- $\frac{\partial f(x)}{\partial x} = \qquad\qquad\qquad\qquad = 0 \Longrightarrow x^* =$

- Plot $y = x^3$.

- **Sufficient condition for a (local) maximum**:

$$\frac{\partial f(x^*)}{\partial x} = 0 \text{ and } \left.\frac{\partial^2 f(x)}{\partial^2 x}\right|_{x^*} < 0 \qquad (2)$$

- At a maximum, $f(x^* + h) - f(x^*) < 0$ for all h.

- Taylor Rule: $f(x^*+h)-f(x^*) = \frac{\partial f(x^*)}{\partial x}h+\frac{1}{2}\frac{\partial^2 f(x^*)}{\partial^2 x}h^2+$ higher order terms.

- Notice: $\frac{\partial f(x^*)}{\partial x} = 0$.

- $f(x^* + h) - f(x^*) < 0$ for all $h \Longrightarrow \frac{\partial^2 f(x^*)}{\partial^2 x}h^2 < 0 \Longrightarrow \frac{\partial^2 f(x^*)}{\partial^2 x} < 0$

- Careful: Maximum may not exist: $y = \exp(x)$

6 Multivariate optimization

- Nicholson, Ch.2, pp. 26–32

- Function from R^n to R: $y = f(x_1, x_2, ..., x_n)$

- Partial derivative with respect to x_i:

$$\begin{aligned} & \frac{\partial f(x_1, ..., x_n)}{\partial x_i} \\ = \quad & \lim_{h \to 0} \frac{f(x_1, ..., x_i + h, ... x_n) - f(x_1, ..., x_i, ... x_n)}{h} \end{aligned}$$

- Slope along dimension i

- Total differential:

$$df = \frac{\partial f(x)}{\partial x_1} dx_1 + \frac{\partial f(x)}{\partial x_2} dx_2 + ... + \frac{\partial f(x)}{\partial x_n} dx_n$$

- One important economic example

- Example 1: Partial derivatives of $y = f(L, K) = L^{.5}K^{.5}$

- $f'_L =$
  (marginal productivity of labor)

- $f'_K =$
  (marginal productivity of capital)

- $f''_{L,K} =$

Maximization over an open set (like R)

- **Necessary condition for maximum** x^* is

$$\frac{\partial f(x^*)}{\partial x_i} = 0 \ \forall i \tag{3}$$

or in vectorial form

$$\nabla f(x) = 0$$

- These are commonly referred to as first order conditions (f.o.c.)

7 Next Class

- Multivariate Maximization (ctd.)

- Comparative Statics

- Implicit Function Theorem

- Envelope Theorem

- Going toward:

  - Preferences

  - Utility Maximization (where we get to apply maximization techniques the first time)