Econ 101A – Final exam
F 13 December.

Do not turn the page until instructed to.

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Problem 1. Cost functions (17 points) Consider the cost functions in Figures 1a and 1b.

1. Take the total cost function in Figure 1a and draw the marginal cost function c'_y and the average cost function $c(y)/y$. What is the supply function, that is, the quantity $y^*(p)$ that a perfectly-competitive firm will produce as a function of the price p? What can we say about production if the price p is higher than the marginal cost $c'(y)$? (6 points)

2. Take the total cost function in Figure 1b and draw the marginal cost function c'_y and the average cost function $c(y)/y$. Draw the supply function, that is, the quantity $y^*(p)$ that a perfectly-competitive firm will produce as a function of the price p. Here you do not need to solve analytically. (6 points)

3. Draw the industry supply function for firms that have cost function as in Figure 1b if there are 3 firms in the market. (5 points)

Problem 2. Short answer problems. (13 points) In the following problems, you are required to give a short answer.

1. Annibal has homework to do. Instead he goes out with friends and gets a D on the homework. Does this imply that Annibal is time-inconsistent? (4 points)

2. Annabel does not buy icecream boxes from a shop because she knows that if she buys them, she will eat them. Today Annabel came home from school and discovered that her sister has bought icecream. Annable eats half of the 1gallon icrecream box. Does this imply that Annabel is time-inconsistent? (4 points)

3. Find the *pure-strategy* Nash Equilibria of the following simultaneous game [do NOT look for mixed-strategy equilibria]: (5 points)

1\2	Left	Middle	Right
Up	$1,1$	$3,0$	$2,1$
Middle	$1,0$	$0,-1$	$0,-2$
Down	$0,5$	$1,1$	$1,1$

Problem 3. Public good contribution (33 points) In this exercise, we consider the problem of contributions to public goods. Assume that in a community of n individuals each individual i decides a contribution g_i toward a public good. The total quantity of public good provided will equal $G = \sum_{i=1}^n g_i$. Each individual pays a cost of effort $\gamma (g_i)^2/2$ for the contribution. Therefore the utility function of individual i is

$$U_i(g_i, g_{-i}) = G - \gamma \frac{(g_i)^2}{2} = \sum_{i=1}^n g_i - \gamma \frac{(g_i)^2}{2}.$$

You can think as being public radio ('funded by our listeners'), the garbage collection, or contribution to a charity like Doctors without Borders.

1. Consider first the socially optimal solution. Assume that the social welfare measure V is the sum of the individual utility functions $V = \sum_{i=1}^n U_i(g_i, g_{-i})$. Maximize V with respect to g_i for $i = 1, ..., n$. Using the first order conditions, determine the socially optimal $g^*_{SO,i}$ for $i = 1, ..., n$ and $G^*_{SO} = \sum_{i=1}^n g^*_{SO,i}$ (5 points)

2. How do you know that the solutions for $g^*_{SO,i}$ are a maximum? Argue that, even though you may not know how to compute the determinant of an n-by-n Hessian, what you found is indeed an optimum. (6 points)

3. Consider now the problem of **simultaneous contribution** to public goods in the community. Find the Nash Equilibrium in the contribution level. As in Cournot duopoly, each individual maximizes holding the contribution of others g^*_{-i} constant. What is the quantity contributed g^*_i? (4 points)

4. Compare the contributions in the Nash equilibium to the social optimum quantity $g^*_{SO,i}$? In particular, compute $g^*_i/g^*_{SO,i}$. How does this vary with the number of individuals n? In which communities is the problem of underprovision of public goods more serious? (3 points)

5. Assume now altruistic individuals that maximize $U_i(g_i, g_{-i}) + \alpha \sum_{j \neq i} U_i(g_i, g_{-i})$. That is, individuals put weight $\alpha > 0$ on the utility of other people in the community. Recompute the Nash equilibrium g^*_i for the case of altruism. What is the comparative static with respect to α? (6 points)

6. Suppose now that the government wishes to attain the socially optimal level of contribution $g^*_{SO,i}$. Some marketing groups just discovered that with appropriate advertisement campaigns it is possible to change the level of altruism α of citizens. Suppose that the campaign is costless for the government. Compute the level of α the the government with induce. (5 points)

7. What does economics suggest in this case? Is it better if people are nice to each other or selfish? Would you choose to live in a society with high or low α? 4 points)

Problem 4. Hotelling model of spatial competition (36 points) In this exercise, we consider the problem of political parties that seeks to determine the optimal placement on the Left-right spectrum. Denote the placement of party i as $t_i \in [0,1]$, where $t = 0$ indicates left wing and $t = 1$ indicates right wing. The parties seek to maximize the number of votes received. The voter political views are uniformly distributed between 0 and 1. Voters vote for the political party that is closer to them and, if two parties are equally close, they randomize.

1. Consider first the case of two parties. Assume that party 1 chooses to place at $t_1 < t_2$, the placement of party 2. Show that the share of votes that party 1 receives is $t_1 + (t_2 - t_1)/2$. [Consequently, the share of votes received by party 2 is $1 - (t_1 + (t_2 - t_1)/2)$] Similarly, show that if $t_1 = t_2$, the share of votes of party 1 is 1/2. Finally, for $t_1 > t_2$, show that the share of votes for party 1 is $1 - t_1 + (t_1 - t_2)/2$ [and the share of votes of party 2 is $t_1 - (t_1 - t_2)/2$] (5 points)

2. Consider first the case of **sequential decision**. Assume that in period 1 party 1 has chosen a placement at t_1. What placement t^*_2 will party 2 choose (as a function of t_1) in period 2 in order to maximize its share of votes? (5 points)

3. Now that you have determined the decision of party 2 in period 2, determine the optimal decision t^*_1 of party 1 in period 1.(5 points)

4. What features does this subgame-perfect equilibrium have? Do you think that it reflects some feature of the American two-party system? (4 points)

5. Consider now the case of **simultaneous decision**. Now parties choose simultaneously where to locate on the policy space. Show that $t^*_1 = t^*_2 = 1/2$ is a Nash Equilibrium. You do not need to find all the Nash Equilibria. However, I can tell you that it is unique. (5 points)

6. We now investigate whether the solution $t^*_1 = t^*_2 = 1/2$ is also socially optimal. Suppose that a voter with policy preference t has a disutility $(t - t^*)^2$ from voting a politician with policy stand t^*. That is, if I am conservative, I do not mind too much voting for a middle-of-the-road politician, but I really hate voting for Ralph Nader. Call t^*_1 and t^*_2 the two positions of the politicians and assume $t^*_1 \leq t^*_2$.

What is the average disutility U associated with the politician choices? I answer this question for you: it is

$$U = \int_0^{(t_1^*+t_2^*)/2} (t-t_1^*)^2\,dt + \int_{(t_1^*+t_2^*)/2}^1 (t-t_2^*)^2\,dt = \left[\frac{1}{3}(t-t_1^*)^3\right]_0^{(t_1^*+t_2^*)/2} + \left[\frac{1}{3}(t-t_2^*)^3\right]_{(t_1^*+t_2^*)/2}^1 =$$

$$= \frac{1}{3}\left\{\frac{1}{8}(t_2^*-t_1^*)^3 + (t_1^*)^3 + (1-t_2^*)^3 - \frac{1}{8}(t_1^*-t_2^*)^3\right\} = \frac{1}{3}\left\{(t_1^*)^3 + (1-t_2^*)^3 + \frac{1}{4}(t_2^*-t_1^*)^3\right\}.$$

Now take the final expression for the disutility U (the one after the last equality sign) and minimize it with respect to t_1^* and t_2^*, that is, write down the first order conditions. Solve for t_1^* and t_2^*. (8 points)

7. Check that the second order conditions for a minimum of U are met at t_1^* and t_2^*. [minimum, not maximum! If you could not solve for t_1^* and t_2^* in the previous point, state in general the second order conditions for a minimum] (4 points)

Problem 5. Wonderport economics. (28 points) You are in Wonderland and you just landed at the local airport Wonderport. In Wonderport shops sell exclusively toys. Each toy is produced at a constant marginal cost c. In the population there are kids and adults, with K being the number of kids and A the number of adults. People go to Wonderport once a year. That is, K kids and A adults visit Wonderport per year. In Wonderland both kids and adults earn money and can afford to shop. The value that a kid assings to a toy is v_k. That is, a kid will buy one toy if the price is lower than v_k, and not buy otherwise. Similarly, each adult values one toy v_a, with $v_k > v_a > c$. The value of a second toy purchase in a year is zero, both for kids and adults.

Unfortunately, we cannot offer you a free trip to Wonderland. However, as an apprentice economist, you get to guess the pricing at Wonderport. Have a safe journey of Wonderlanomics!

1. Once upon a time in Wonderport there used to be many independently-owned perfectly-competing shops selling toys. What was the price of a toy back then? Did both kids and adults purchase toys? What were the profits of the firms? What about the surplus of the consumers (measured as willingness to pay minus price paid)? (5 points)

2. In 5,670 W.T. (Wonder Time) the government decided to introduce a per-unit tax t on addictive goods like toys. How much did the price charged to consumers change between years 5,669 and 5,670? Who bore the burden of a tax? Did both kids and adults puchase toys? What were the profits of the firms? What about the surplus of the consumers? (4 points)

3. In 6,200 W.T. a large company consolidated the shop industry. Since then, a monopolist owns the toy shops in Wonderport. (assume no tax) The monopolist can price discriminate by designing fully separate kid-shops and adult-shops. Resale of toys carries the death penalty. What will the price be in kid-shops? And in adult-shops? Do both groups buy? What are the profits of the firm? What about the surplus of the consumers? (6 points)

4. In 6,500 W.T. the government imposed once again a tax t on toys. What is the price in kid-shops? And in adult-shops? Do both groups buy? What are the profits of the firm? What about the surplus of the consumers? Who bears the burden of the tax? (5 points)

5. Finally, in 7,000 W.T. the government decides to remove the tax under the condition that the monopolist stops unfairly discriminating against kids. Now that the monopolist charges one price for toys, what is it as a function of v_k, v_a, K, and A? Do both groups buy? What are the profits of the firm? How do they compare to the case of price discrimination (and no tax)? What about the surplus of the consumers? (8 points)