Economics 101A
(Lecture 13)

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Outline

1. Mid-Term Suggestions
2. Nobel Prize winners
3. Risk Aversion and Lottery
4. Insurance
5. Investment in Risky Asset
6. Measures of Risk Aversion

1 Mid-Term Suggestions

- Suggestions from you...

2 Nobel Prize winners

- Robert Aumann (Hebrew University)
- Thomas Schelling (University of Maryland)
- Game Theory
- (Coming in third part of course...)
- Repeated Games and Applications to Conflict

3 Risk Aversion and Lottery

- Are you risk-averse?

- Let's see...

4 Insurance

- Nicholson, Ch. 18, pp. 545–551 [OLD: Ch. 8, pp. 211-216] Notice: different treatment than in class

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$, $u'' < 0$

- Probability p of accident with loss L

- Insurance offers coverage:
  - premium \$$q$ for each \$1 paid in case of accident
  - units of coverage purchased α

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\,(w - q\alpha) + pu\,(w - q\alpha - L + \alpha)$$
$$s.t. \alpha \geq 0$$

- Assume $\alpha^* \geq 0$, check later

- First order conditions:

$$\begin{aligned} 0 &= -q\,(1-p)\, u'\,(w - q\alpha) \\ &\quad + (1-q)\, pu'\,(w - q\alpha - L + \alpha) \end{aligned}$$

or

$$\frac{u'\,(w - q\alpha)}{u'\,(w - q\alpha - L + \alpha)} = \frac{1-q}{q} \frac{p}{1-p}.$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* =$?

- $\alpha^* > 0$, so we are ok!

- What if $q > p$ (insurance needs to cover operating costs)?

- Insurance will be only partial (if at all)

- Exercise: Check second order conditions!

5 Investment in Risk Asset

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    - $r = r_+ > 0$ with probability p
    - $r = r_- < 0$ with probability $1 - p$
    - $Er = pr_+ + (1 - p)\, r_- > 0$

- Share of wealth invested in stock S $= \alpha$

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\left(w\left[(1-\alpha)+\alpha\left(1+r_{-}\right)\right]\right)+$$
$$+pu\left(w\left[(1-\alpha)+\alpha\left(1+r_{+}\right)\right]\right)$$
$$s.t. 0 \leq \alpha \leq 1$$

- Case of risk neutrality: $u(x)=a+bx$, $b>0$

- Assume $a=0$ (no loss of generality)

- Maximization becomes

$$\max_{\alpha} b\,(1-p)\left(w\left[1+\alpha r_{-}\right]\right)+bp\left(w\left[1+\alpha r_{+}\right]\right)$$

or

$$\max_{\alpha} bw+\alpha bw\left[(1-p)\, r_{-}+pr_{+}\right]$$

- Sign of term in square brackets? Positive!

- Set $\alpha^{*}=1$

- Case of risk aversion: $u'' < 0$

- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

$$\begin{aligned} 0 &= (1-p)(wr_-)\,u'(w[1+\alpha r_-]) + \\ &\quad + p(wr_+)\,u'(w[1+\alpha r_+]) \end{aligned}$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.

6 Measures of Risk Aversion

- Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

- How risk averse is an individual?

- Two measures:

  – Absolute Risk Aversion r_A:

$$r_A = -\frac{u''(x)}{u'(x)}$$

  – Relative Risk Aversion r_R:

$$r_R = -\frac{u''(x)}{u'(x)}x$$

- Examples in the Problem Set

7 Next lecture and beyond

- Tu:
  - Time consistency
  - Time Inconsistency
  - Application to health clubs

- Then:
  - Begin Production
  - Returns to scale
  - Cost minimization