Economics 101A
(Lecture 24)

Stefano DellaVigna

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Outline

1. Game Theory II
2. Dynamic Games
3. Oligopoly: Stackelberg

1 Game Theory II

- Penalty kick in soccer (matching pennies)

Kicker \ Goalie	L	R
L	$0, 1$	$1, 0$
R	$1, 0$	$0, 1$

- Equilibrium always exists in mixed strategies σ

- Mixed strategy: allow for probability distibution.

- Back to penalty kick:
  - Kicker kicks left with probability k
  - Goalie kicks left with probability g

  - utility for kicker of playing L :

$$
\begin{aligned}
U_K(L,\sigma) &= gU_K(L,L) + (1-g)\,U_K(L,R) \\
&= (1-g)
\end{aligned}
$$

  - utility for kicker of playing R :

$$
\begin{aligned}
U_K(R,\sigma) &= gU_K(R,L) + (1-g)\,U_K(R,R) \\
&= g
\end{aligned}
$$

- Optimum?
  - $L \succ R$ if $1 - g > g$ or $g < 1/2$
  - $R \succ L$ if $1 - g < g$ or $g > 1/2$
  - $L \sim R$ if $1 - g = g$ or $g = 1/2$

- Plot best response for kicker

- Plot best response for goalie

- Nash Equilibrium is:
  - fixed point of best response correspondence
  - crossing of best response correspondences

2 Dynamic Games

- Nicholson, Ch. 15, pp. 449–454.[OLD: Ch. 10, pp. 256–259]

- Dynamic games: one player plays after the other

- Decision trees
  - Decision nodes
  - Strategy is a plan of action at each decision node

- Example: battle of the sexes game

She \ He	Ballet	Football
Ballet	$2, 1$	$0, 0$
Football	$0, 0$	$1, 2$

- Dynamic version: she plays first

- **Subgame-perfect equilibrium.** At each node of the tree, the player chooses the strategy with the highest payoff, given the other players' strategy

- Backward induction. Find optimal action in last period and then work backward

- Solution

- Example 2: Entry Game

$1 \setminus 2$	Enter	Do not Enter
Enter	$-1, -1$	$10, 0$
Do not Enter	$0, 5$	$0, 0$

- Exercise. Dynamic version.

- Coordination games solved if one player plays first

- Can use this to study finitely repeated games

- Suppose we play the prisoner's dilemma game ten times.

$1 \setminus 2$	D	ND
D	$-4, -4$	$-1, -5$
ND	$-5, -1$	$-2, -2$

- What is the subgame perfect equilibrium?

3 Oligopoly: Stackelberg

- Setting as in problem set.

- 2 Firms

- Cost: $c(y) = cy$, with $c > 0$

- Demand: $p(Y) = a - bY$, with $a > c > 0$ and $b > 0$

- Difference: Firm 1 makes the quantity decision first

- Use subgame perfect equilibrium

- Solution:

- Solve first for Firm 2 decision as function of Firm 1 decision:

$$\max_{y_2} \left(a - by_2 - by_1^*\right) y_2 - cy_2$$

- F.o.c.:

$$a - 2by_2^* - by_1^* - c = 0$$

or

$$y_2^* = \frac{a-c}{2b} - \frac{y_1^*}{2}.$$

$$p_D^* = a - bY_D^* = a - b\left(2\frac{a-c}{3b}\right) = \frac{1}{3}a + \frac{2}{3}c.$$

- Firm 1 takes this response into account in the maximization:

$$\max_{y_1} (a - by_1 - by_2^* (y_1)) y_1 - cy_1$$

or

$$\max_{y_1} \left(a - by_1 - b\left(\frac{a-c}{2b} - \frac{y_1}{2}\right)\right) y_1 - cy_1$$

- F.o.c.:

$$a - 2by_1 - \frac{(a-c)}{2} + by_1 - c = 0$$

or

$$y_1^* = \frac{a-c}{2b}$$

and

$$y_2^* = \frac{a-c}{2b} - \frac{y_1^*}{2} = \frac{a-c}{2b} - \frac{a-c}{4b} = \frac{a-c}{4b}.$$

- Total production:

$$Y_D^* = y_1^* + y_2^* = 3\frac{a-c}{4b}$$

- Price equals

$$p^* = a - b\left(\frac{3}{4}\frac{a-c}{b}\right) = \frac{1}{4}a + \frac{3}{4}c$$

- Compare to monopoly:

$$y_M^* = \frac{a-c}{2b}$$

and

$$p_M^* = \frac{a+c}{2}.$$

- Compare to Cournot:

$$Y_D^* = y_1^* + y_2^* = 2\frac{a-c}{3b}$$

and

$$p_D^* = \frac{1}{3}a + \frac{2}{3}c.$$

- Figure

- Compare with Cournot outcome

4 Next lecture

- General equilibrium

- Edgeworth Box