Economics 101A
(Lecture 7)

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Outline

1. Utility maximization – Tricky Cases

2. Indirect Utility Function

3. Comparative Statics (Introduction)

4. Income Changes

1 Utility maximization – tricky cases

- First, re-solve CES utility function.

$$\max_{x_1,x_2} \left(\alpha x_1^\rho + \beta x_2^\rho\right)^{1/\rho}$$
$$s.t.\ p_1 x_1 + p_2 x_2 - M = 0$$

- Solution:

$$x_1^* = \frac{M}{p_1\left(1 + \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\rho-1}}\left(\frac{p_2}{p_1}\right)^{\frac{\rho}{\rho-1}}\right)}$$
$$x_2^* = \frac{M}{p_2\left(1 + \left(\frac{\beta}{\alpha}\right)^{\frac{1}{\rho-1}}\left(\frac{p_1}{p_2}\right)^{\frac{\rho}{\rho-1}}\right)}$$

- Special case 1: $\rho \to 1^-$ (Perfect Substitutes)

  - $lim_{\rho \to 1^-} \frac{1}{\rho - 1} = lim_{\rho \to 1^-} \frac{\rho}{\rho - 1} = -\infty$
    (here notice the convergence from the left)

  - If $\frac{\alpha}{\beta}\frac{p_2}{p_1} > 1$ (or $p_1/p_2 < \alpha/\beta$),

$$\begin{aligned} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\rho-1}} \left(\frac{p_2}{p_1}\right)^{\frac{\rho}{\rho-1}} &\to 0 \\ x_1^* &\to M/p_1 \end{aligned}$$

  - If $\frac{\alpha}{\beta}\frac{p_2}{p_1} < 1$ (or $p_1/p_2 > \alpha/\beta$),

$$\begin{aligned} \left(\frac{\alpha}{\beta}\right)^{\frac{1}{\rho-1}} \left(\frac{p_2}{p_1}\right)^{\frac{\rho}{\rho-1}} &\to \infty \\ x_1^* &\to 0 \end{aligned}$$

- Solution for Perfect Substitutes Case is

$$
\begin{aligned}
x_1^* &= \begin{cases} 0 & \text{if } p_1/p_2 > \alpha/\beta \\ M/p_1 & \text{if } p_1/p_2 < \alpha/\beta \\ \text{any } x_1 \in [0, M/p_1] & \text{if } p_1/p_2 = \alpha/\beta \end{cases} \\
x_2^* &= \begin{cases} M/p_2 & \text{if } p_1/p_2 > \alpha/\beta \\ 0 & \text{if } p_1/p_2 < \alpha/\beta \\ x_2 \text{ such that B.C. holds} & \text{if } p_1/p_2 = \alpha/\beta \end{cases}
\end{aligned}
$$

- Case $p_1/p_2 = \alpha/\beta$ has to be analyzed separately

- This is case in which budget line and indifference curves are parallel –> All points on budget line are tangent and hence optimal.

- Tricky Cases (ctd)

2. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\max x_1 * (x_2 + 5)$$
$$s.t.\ p_1 x_1 + p_2 x_2 = M$$

- In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?

3. Multiplicity of solutions.

   - Example 1: Perfect Substitutes with $p_1/p_2 = \alpha/\beta$

   - Example 2: Non-convex preferences with two optima

2 Indirect utility function

- Nicholson, Ch. 4, pp. 124-127 (106–108, 9th)

- Define the indirect utility $v(\mathbf{p}, M) \equiv u(\mathbf{x}^*(\mathbf{p}, M))$, with $\mathbf{p}$ vector of prices and $\mathbf{x}^*$ vector of optimal solutions.

- $v(\mathbf{p}, M)$ is the utility at the optimum for prices $\mathbf{p}$ and income M

- Some comparative statics: $\partial v(\mathbf{p}, M)/\partial M =$?

- Hint: Use Envelope Theorem on Lagrangean function

- What is the sign of λ?

- $\lambda = u'_{x_i}/p > 0$

- $\partial v(\mathbf{p}, M)/\partial p_i =$?

- Properties:
  - Indirect utility is always increasing in income M
  - Indirect utility is always decreasing in the price p_i

3 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 141-151 (121–131, 9th)

- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$

- Quantity consumed as a function of income and price

- What happens to quantity consumed x_i^* as prices or income varies?

- Simple case: Equal increase in prices and income.

- $M' = tM$, $p_1' = tp_1$, $p_2' = tp_2$.

- Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.

- What happens?

- Write budget line: $tp_1x_1 + tp_2x_2 = tM$

- Demand is homogeneous of degree 0 in $\mathbf{p}$ and M:

  $x^*(tM, tp_1, tp_2) = t^0 x^*(M, p_1, p_2) = x^*(M, p_1, p_2)$.

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, x_2^* = \frac{\beta}{\alpha + \beta} M/p_2$$

- What is $\partial x_1^*/\partial M$?

- What is $\partial x_1^*/\partial p_1$?

- What is $\partial x_1^*/\partial p_2$?

- General results?

4 Income changes

- Income increases from M to to $M' > M$.

- Budget line ($p_1x_1 + p_2x_2 = M$) shifts out:

$$x_2 = \frac{M'}{p_2} - x_1\frac{p_1}{p_2}$$

- New optimum?

- Engel curve: $x_i^*(M)$: demand for good i as function of income M holding fixed prices p_1, p_2

- Does x_i^* increase with M?
  - Yes. Good i is *normal*
  - No. Good i is *inferior*

5 Next Class

- More comparative statics:
  - Price Effects
  - Intuition
  - Slutzky Equation
- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism