Economics 101A
(Lecture 15)

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Outline

1. Two-step Cost Minimization II

2. Cost Minimization: Example

3. Cost Curves and Supply Function

1 Two-step Cost minimization II

- *First Step.* Minimize input costs for given production

- Firm objective function:

$$\min_{L,K} wL + rK$$
$$s.t. f(L,K) \geq y$$

- Derived demand for inputs:
  - $L = L^* (w, r, y)$
  - $K = K^* (w, r, y)$

- Value function at optimum is **cost function:**

$$c(w, r, y) = wL^* (r, w, y) + rK^* (r, w, y)$$

- *Second step.* Given cost function, choose optimal quantity of y as well

- Price of output is p.

- Firm's objective:

$$\max py - c(w, r, y)$$

- First order condition:

$$p - c'_y(w, r, y) = 0$$

- Price equals marginal cost – very important!

- Second order condition:

$$-c''_{y,y}(w, r, y^*) < 0$$

- For maximum, need increasing marginal cost curve.

2 Cost Minimization: Example

- Continue example above: $y = f(L, K) = AK^{\alpha}L^{\beta}$

- Cost minimization:

$$\begin{aligned} &\min wL + rK \\ &s.t. AK^{\alpha}L^{\beta} = y \end{aligned}$$

- What is the return to scale for this example?

- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$

- How much does input increase?

  – Decreasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

- Constant returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

- Increasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

• Returns to scale depend on $\alpha+\beta \lesseqgtr 1$: $f(tK, tL) = A(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}f(K, L)$

- Solutions:

  - Optimal amount of labor:

$$L^*(r, w, y) = \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}}$$

  - Optimal amount of capital:

$$\begin{aligned} K^*(r, w, y) &= \frac{w}{r}\frac{\alpha}{\beta}\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} = \\ &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{aligned}$$

- Check various comparative statics:

  - $\partial L^*/\partial A < 0$ (technological progress and unemployment)

  - $\partial L^*/\partial y > 0$ (more workers needed to produce more output)

- $\partial L^*/\partial w < 0$, $\partial L^*/\partial r > 0$ (substitute away from more expensive inputs)

- Parallel comparative statics for K^*

- Cost function

$$\begin{aligned} c(w,r,y) &= wL^*(r,w,y) + rK^*(r,w,y) = \\ &= \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}} \left[\begin{array}{c} w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + \\ +r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \end{array} \right] \end{aligned}$$

- Define $B := w\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{-\frac{\alpha}{\alpha+\beta}} + r\left(\frac{w}{r}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$

- Cost-minimizing output choice:

$$\max py - B\left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta}}$$

- First order condition:

$$p - \frac{1}{\alpha+\beta}\frac{B}{A}\left(\frac{y}{A}\right)^{\frac{1-(\alpha+\beta)}{\alpha+\beta}} = 0$$

- Second order condition:

$$-\frac{1}{\alpha+\beta}\frac{1-(\alpha+\beta)}{\alpha+\beta}\frac{B}{A^2}\left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$

- When is the second order condition satisfied?

- Solution:
  - $\alpha + \beta = 1$ (CRS):
    * S.o.c. equal to 0
    * Solution depends on p
    * For $p > \frac{1}{\alpha+\beta}\frac{B}{A}$, produce $y^* \to \infty$
    * For $p = \frac{1}{\alpha+\beta}\frac{B}{A}$, produce any $y^* \in [0, \infty)$
    * For $p < \frac{1}{\alpha+\beta}\frac{B}{A}$, produce $y^* = 0$

- $\alpha + \beta > 1$ (IRS):
  - S.o.c. positive
  - Solution of f.o.c. is a minimum!
  - Solution is $y^* \to f\infty$.
  - Keep increasing production since higher production is associated with higher returns

– $\alpha + \beta < 1$ (DRS):

  * s.o.c. negative. OK!

  * Solution of f.o.c. is an interior optimum

  * This is the only "well-behaved" case under perfect competition

  * Here can define a supply function

3 Cost Curves

- Nicholson, Ch. 10, pp. 330-338; Ch. 11, pp. 365-369 (Ch. 8, pp. 220–228; Ch. 9, pp. 256–259, 9th)

- Marginal costs $MC = \partial c/\partial y \rightarrow$ Cost minimization
$$p = MC = \partial c\left(w, r, y\right)/\partial y$$

- Average costs $AC = c/y \rightarrow$ Does firm break even?
$$\begin{aligned} \pi &= py - c\left(w, r, y\right) > 0 \text{ iff} \\ \pi/y &= p - c\left(w, r, y\right)/y > 0 \text{ iff} \\ c\left(w, r, y\right)/y &= AC < p \end{aligned}$$

- **Supply function.** Portion of marginal cost MC above average costs.(price equals marginal cost)

- Assume only 1 input (expenditure minimization is trivial)

- **Case 1.** Production function. $y = L^{\alpha}$

  – Cost function? (cost of input is w):

$$c(w, y) = wL^{*}(w, y) = wy^{1/\alpha}$$

  – Marginal cost?

$$\frac{\partial c(w, y)}{\partial y} = \frac{1}{\alpha} wy^{(1-\alpha)/\alpha}$$

  – Average cost $c(w, y)/y$?

$$\frac{c(w, y)}{y} = \frac{wy^{1/\alpha}}{y} = wy^{(1-\alpha)/\alpha}$$

- **Case 1a.** $\alpha > 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1b.** $\alpha = 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 1c.** $\alpha < 1$. Plot production function, total cost, average and marginal. Supply function?

- **Case 2.** *Non-convex technology.* Plot production function, total cost, average and marginal. Supply function?

- **Case 3.** *Technology with setup cost.* Plot production function, total cost, average and marginal. Supply function?

3.1 Supply Function

- Supply function: $y^* = y^*(w, r, p)$

- What happens to y^* as p increases?

- Is the supply function upward sloping?

- Remember f.o.c:

$$p - c'_y(w, r, y) = 0$$

- Implicit function:

$$\frac{\partial y^*}{\partial p} = -\frac{1}{-c''_{y,y}(w, r, y)} > 0$$

as long as s.o.c. is satisfied.

- Yes! Supply function is upward sloping.

4 Next Lectures

- Profit Maximization
- Aggregation
- Market Equilibrium