

Economics 101A

(Lecture 14)

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Outline

1. Production: Introduction
2. Production Function
3. Returns to Scale
4. Two-step Cost Minimization

1 Production: Introduction

- Second half of the economy. **Production**
- Example. Ford and the Minivan (Petrin, 2002):
 - Ford had idea: "Mini/Max" (early '70s)
 - Did Ford produce it?
 - No!
 - Ford was worried of cannibalizing station wagon sector
 - Chrysler introduces Dodge Caravan (1984)
 - Chrysler: \$1.5bn profits (by 1987)!

- Why need separate treatment?
- Perhaps firms maximize utility...
- ...we can be more precise:
 - Competition
 - Institutional structure

2 Production Function

- Nicholson, Ch. 9, pp. 295-301; 306-311 (Ch. 7, pp. 183-190; 195-200, 9th)
- Production function: $y = f(\mathbf{z})$. Function $f : R_+^n \rightarrow R_+$
- Inputs $\mathbf{z} = (z_1, z_2, \dots, z_n)$: labor, capital, land, human capital
- Output y : Minivan, Intel CPU, mangoes (Philippines)
- Properties of f :
 - no free lunches: $f(0) = 0$
 - positive marginal productivity: $f'_i(\mathbf{z}) > 0$
 - decreasing marginal productivity: $f''_{i,i}(\mathbf{z}) < 0$

- Isoquants $Q(y) = \{\mathbf{x} | f(\mathbf{x}) = y\}$
- Set of inputs \mathbf{z} required to produce quantity y
- Special case. Two inputs:
 - $z_1 = L$ (labor)
 - $z_2 = K$ (capital)
- Isoquant: $f(L, K) - y = 0$
- Slope of isoquant $dK/dL = MRTS$

- Convex production function if convex isoquants
- Reasonable: combine two technologies and do better!
- Mathematically, convex isoquants if $d^2K/d^2L > 0$

- Solution:

$$\frac{d^2K}{d^2L} = -\frac{f''_{L,L}f'_K - 2f''_{L,K}f'_L + f''_{K,K}(f'_L)^2}{(f'_K)^2} / f'_K$$

- Hence, $d^2K/d^2L > 0$ if $f''_{L,K} > 0$ (inputs are complements in production)

3 Returns to Scale

- Nicholson, Ch. 9, pp. 302-305 (Ch. 7, pp. 190–193, 9th)
- Effect of increase in labor: f'_L
- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$
- How much does input increase?
 - Decreasing returns to scale: for all \mathbf{z} and $t > 1$,
$$f(t\mathbf{z}) < tf(\mathbf{z})$$

- Constant returns to scale: for all \mathbf{z} and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

- Increasing returns to scale: for all \mathbf{z} and $t > 1$,

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

- Example: $y = f(K, L) = AK^\alpha L^\beta$
- Marginal product of labor: $f'_L =$
- Decreasing marginal product of labor: $f''_{L,L} =$
- $MRTS =$
- Convex isoquant?
- Returns to scale: $f(tK, tL) = A(tK)^\alpha (tL)^\beta = t^{\alpha+\beta} AK^\alpha L^\beta = t^{\alpha+\beta} f(K, L)$

4 Two-step Cost minimization

- Nicholson, Ch. 10, pp. 323-330 (Ch. 12 , pp. 212–220, 9th)
- Objective of firm: Produce output that generates maximal profit.
- Decompose problem in two:
 - Given production level y , choose cost-minimizing combinations of inputs
 - Choose optimal level of y .
- *First step.* Cost-Minimizing choice of inputs

- Two-input case: Labor, Capital
- Input prices:
 - Wage w is price of L
 - Interest rate r is rental price of capital K
- Expenditure on inputs: $wL + rK$
- Firm objective function:

$$\begin{aligned} \min_{L, K} & wL + rK \\ \text{s.t.} & f(L, K) \geq y \end{aligned}$$

- Equality in constraint holds if:
 1. $w > 0, r > 0$;
 2. f strictly increasing in at least L or K .
- Counterexample if ass. 1 is not satisfied
- Counterexample if ass. 2 is not satisfied

- Compare with expenditure minimization for consumers

- First order conditions:

$$w - \lambda f'_L = 0$$

and

$$r - \lambda f'_K = 0$$

- Rewrite as

$$\frac{f'_L(L^*, K^*)}{f'_K(L^*, K^*)} = \frac{w}{r}$$

- MRTS (slope of isoquant) equals ratio of input prices

- Graphical interpretation

- Derived demand for inputs:

- $L = L^*(w, r, y)$

- $K = K^*(w, r, y)$

- Value function at optimum is **cost function**:

$$c(w, r, y) = wL^*(r, w, y) + rK^*(r, w, y)$$

- *Second step.* Given cost function, choose optimal quantity of y as well
- Price of output is p .
- Firm's objective:

$$\max py - c(w, r, y)$$

- First order condition:

$$p - c'_y(w, r, y) = 0$$

- Price equals marginal cost – very important!

- Second order condition:

$$-c''_{y,y}(w, r, y^*) < 0$$

- For maximum, need increasing marginal cost curve.

5 Next Lecture

- Solve an Example
- Cases in which s.o.c. are not satisfied
- Start Profit Maximization