Economics 101A
(Lecture 25)

Stefano DellaVigna

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Outline

1. Example of General Equilibrium

2. Existence and Welfare Theorems

3. Asymmetric Information: Introduction

4. Hidden Action (Moral Hazard)

1 Example

- Consumer 1 has Leontieff preferences:

$$u(x_{1,}x_2) = \min\left(x_1^1, x_2^1\right)$$

- Bundle demanded by consumer 1:

$$\begin{aligned} x_1^{1*} &= x_2^{1*} = x^{1*} = \frac{p_1\omega_1^1 + p_2\omega_2^1}{p_1 + p_2} = \\ &= \frac{\omega_1^1 + (p_2/p_1)\,\omega_2^1}{1 + (p_2/p_1)} \end{aligned}$$

- Graphically

- Comparative statics:
  - increase in ω
  - increase in p_2/p_1:

$$\frac{dx_1^{1*}}{dp_2/p_1} = \frac{\begin{array}{c}\omega_2^1\left(1+(p_2/p_1)\right)\\ -\left(\omega_1^1+(p_2/p_1)\,\omega_2^1\right)\end{array}}{\left(1+(p_2/p_1)\right)^2} = $$

$$= \frac{\omega_2^1-\omega_1^1}{\left(1+(p_2/p_1)\right)^2}$$

  - Effect depends on income effect through endowments:
    * A lot of good 2 –> increase in price of good 2 makes richer
    * Little good 2 –> increase in price of good 2 makes poorer

- Notice: Only ratio of prices matters (general feature)

- Consumer 2 has Cobb-Douglas preferences:

$$u(x_1, x_2) = \left(x_1^2\right)^{.5} \left(x_2^2\right)^{.5}$$

- Demands of consumer 2:

$$x_1^{2*} = \frac{.5 \left(p_1 \omega_1^1 + p_2 \omega_2^1\right)}{p_1} = .5 \left(\omega_1^1 + \frac{p_2}{p_1} \omega_2^1\right)$$

and

$$x_2^{2*} = \frac{.5 \left(p_1 \omega_1^1 + p_2 \omega_2^1\right)}{p_2} = .5 \left(\frac{p_1}{p_2} \omega_1^1 + \omega_2^1\right)$$

- Comparative statics:
  - increase in ω –> Increase in final consumption
  - increase in p_2/p_1 –> Unambiguous increase in x_1^{2*} and decrease in x_2^{2*}

- Impose Walrasian equilibrium in market 1:

$$x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2$$

This implies

$$\frac{\omega_1^1 + (p_2/p_1)\,\omega_2^1}{1 + (p_2/p_1)} + .5\left(\omega_1^1 + \frac{p_2}{p_1}\omega_2^1\right) = \omega_1^1 + \omega_1^2$$

or

$$\frac{.5 - .5\,(p_2/p_1)}{1 + (p_2/p_1)}\omega_1^1 + \frac{.5\,(p_2/p_1) + .5\,(p_2/p_1)^2 - 1}{1 + (p_2/p_1)}\omega_2^1 = 0$$

or

$$\left(\omega_1^1 - 2\omega_2^1\right) + \left(\omega_1^1 + \omega_2^1\right)(p_2/p_1) + \omega_2^1\,(p_2/p_1)^2 = 0$$

- Solution for p_2/p_1:

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\begin{array}{c}\left(\omega_1^1 + \omega_2^1\right)^2 \\ -4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1\end{array}}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

- Some complicated solution!

- Problem set has solution that is easier to compute (and interpret)

2 Existence and Welfare Theorems

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

- Is Walrasian Equilibrium always unique? Not necessarily

- Is Walrasian Equilibrium efficient? Yes.

- **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

- Figure

- **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation $\Big((x_1^1, x_1^1), (x_1^2, x_2^2)\Big)$ there exists some endowment (ω_1, ω_2) such that $\Big((x_1^1, x_1^1), (x_1^2, x_2^2)\Big)$ is a Walrasian Equilibrium for endowment (ω_1, ω_2).

- Figure

- Significance of these results:
  - First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
  - BUT: problems with externalities and public good
  - BUT: what about distribution?
  - Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.
  - But redistribution is hard to implement, and distortive.

3 Asymmetric Information: Introduction

- Nicholson, Ch. 18, pp. 627-632 [*NOT* in 9th Ed.]

- Common economic relationship

- Contract between two parties:

  - Principal

  - Agent

- Two parties have asymmetric information

  - Principal offers a contract to the agent

  - Agent chooses an action

  - Action of agent (or his type) is not observed by principle

- Example 1: *Manager and worker*
  - Manager employs worker and offers wage
  - Worker exerts effort (not observed)
  - Manager pays worker as function of output
- Example 2: *Car Insurance*
  - Car insurance company offers insurance contract
  - Driver chooses quality of driving (not observed)
  - Insurance company pays for accidents
- Example 3: *Shareholders and CEO*
  - Shareholders choose compensation for CEO
  - CEO puts effort
  - CEO paid as function of stock price

- In all of these cases (and many more!), common structure
  - Principal would like to observe effort (of worker, of CEO, of driver)
  - Unfortunately, this is not observable
  - Only a related, noisy proxy is observable: output, accident, success
  - Contract offered by principal is function of this proxy
- This means that occasionally an agent that put a lot of effort but has bad luck is 'punished'
- Also, agents that shirked may instead be compensated
- These principle-agent problems are called *hidden action* or *moral hazard*

- Second category (next lecture): *hidden type* or *adverse selection*

- Example 1: *Manager and worker*
  - Manager employs worker and offers wage
  - Worker can be hard-working or lazy

- Example 2: *Car Insurance*
  - Car insurance company offers insurance contract
  - Drivers ex ante can be careful or careless

- Example 3: *Shareholders and CEO*
  - Shareholders choose compensation for CEO
  - CEO is high-quality or thief

- Problem is similar (action is not observed), but with a twist
  - *Hidden action*: principal can convince agent to exert high effort with the appropriate incentives
  - *Hidden type*: agent's behavior is not affected by incentives, but by her type
- Different task for principal:
  - *Hidden action*: Principal wants to incentivize agent to work hard
  - *Hidden type*: Principal wants to make sure to recruit 'good' agent, not 'bad' one
- Two look similar, but analysis is different
- Start from *Hidden Action*

4 Hidden Action (Moral Hazard)

- Nicholson, Ch. 18, pp. 632-637 [*NOT* in 9th Ed.]

- Example 3: *Shareholders and CEO*

  - Division of ownership and control

- Shareholders (owners of firm):

  - Have capital, but do not have time to run company themselves

  - Want firm run so as to maximize profits

- CEO (manager)

  - Has time and managerial skill

  - Does not have capital to own the firm

- If CEO owns the company (private enterprises), problem is solved –> Infeasible in large companies

- Agent chooses effort e (unobserved)
  - Induces output $y = e + \varepsilon$, where ε is a noise term, with $E(\varepsilon) = 0$
  - Example: Despite putting effort, investment project did not succeed

- Principal pays a salary w to the agent
  - Salary is a function of output y: $w = w(y)$
  - Remember: Salary cannot be function of effort e

- Principal maximizes expected profits

$$E[\pi] = E[y - w(y)] = e - E[w(y)]$$

- Agent is risk averse and maximizes

$$E[U(w(e+\varepsilon))] - c(e)$$

  - $c(e)$ is cost of effort: assume $c'(e) > 0$ and $c''(e) > 0$ for all e
  - Utility function U satisfies $U' > 0$ and $U'' < 0$
  - Notice: Agent is risk-averse, Principal is risk-neutral

- Assume $U(w) = -e^{-\gamma w}$ and $\varepsilon \sim N\left(0, \sigma^2\right)$

- Can solve explicitly for $EU(w)$:

$$EU(w) = -\frac{1}{\sqrt{2\pi}} \int e^{-\gamma w} e^{-\frac{1}{2}\frac{w-\mu_w}{\sigma_w^2}} dw = \mu_w - \frac{\gamma}{2}\sigma_w^2$$

  [Take this for granted]

- Expected utility of agent is $EU(w) = \mu_w - \frac{\gamma}{2}\sigma_w^2$

- Note: μ_w is average salary and σ_w^2 is variance of salary

  - Agent likes high mean salary μ_w

  - Agent dislikes variance in salary σ_w^2

  - Dislike for variance increses in risk aversion γ

- Assume that contract is linear: $w = a + by = a + be + b\varepsilon$

  - Compute $\mu_w = E(w) = E[a + be + b\varepsilon] = a + be + bE[\varepsilon] = a + be$

  - Compute $\sigma_w^2 = Var[a + be + b\varepsilon] = b^2\sigma^2$

- Rewrite expected utility as

$$EU(w) = a + be - \frac{\gamma}{2}b^2\sigma^2$$

- Back to Principal-Agent problem

- Solve problem in three Steps, starting from last stage (backward induction)

  - **Step 1** *(Effort Decision)*. Given contract $w(y)$, what effort e^* is agent going to put in?

  - **Step 2.** *(Individual Rationality)* Given contract $w(y)$ and anticipating to put in effort e^*, does agent accept the contract?

  - **Step 3.** *(Profit Maximization)* Anticipating that the effort of the agent e^* (and the acceptance of the contract) will depend on the contract, what contract $w(y)$ does principal choose to maximize profits?

- **Step 1.** Solve effort maximization of agent:

$$Max_e a + be - \frac{\gamma}{2} b^2 \sigma^2 - c(e)$$

- Solution:

$$c'(e) = b$$

- If assume $c(e) = ce^2/2$ –> $e^* = b/c$

- Check comparative statics

  – With respect to b –> What happens with more pay-for-performance?

  – With respect to c –> What happens with higher cost of effort?

- **Step 2.** Agent needs to be willing to work for principal

- *Individual rationality* condition:

$$EU\left(w\left(e^*\right)\right) - c\left(e^*\right) \geq 0$$

- Substitute in the solution for e^* and obtain

$$a + be^* - \frac{\gamma}{2}b^2\sigma^2 - c\left(e^*\right) \geq 0$$

- Will be satisfied with equality: $a^* = -be^* + \frac{\gamma}{2}b^2\sigma^2 + c\left(e^*\right)$

- **Step 3**: Owner maximizes expected profits

$$\max_{a,b} E[\pi] = e - E[w(y)] = e - a - be$$

- Substitute in the two constraints: $c'(e) = b$ (Step 1) and $a^* = -be^* + \frac{\gamma}{2}b^2\sigma^2 + c(e^*)$ (Step 2)

- Obtain

$$\begin{aligned} E[\pi] &= e - \left(-be + \frac{\gamma}{2}b^2\sigma^2 + c(e)\right) - c'(e)\,e \\ &= e + be - \frac{\gamma}{2}b^2\sigma^2 - c(e^*) - c'(e)\,e \\ &= e + c'(e)\,e - \frac{\gamma}{2}\left(c'(e)\right)^2\sigma^2 - c(e^*) - c'(e)\,e \\ &= e - \frac{\gamma}{2}\left(c'(e)\right)^2\sigma^2 - c(e^*) \end{aligned}$$

- Profit maximization yields f.o.c.

$$1 - \gamma c'(e)\,\sigma^2 c''(e) - c'(e) = 0$$

and hence

$$c'(e^*) = \frac{1}{1 + \gamma\sigma^2 c''(e^*)}$$

- Notice: This implies $c'(e^*) < 1$

- Substitute $c(e) = ce^2/2$ to get

$$e^* = \frac{1}{c}\frac{1}{1 + \gamma\sigma^2 c}$$

- Comparative Statics:

  – Higher risk aversion γ –>...

  – Higher variance of output σ –>...

  – Higher effort cost c –>...

- Also, remember $b^* = c'(e^*) = ce^*$ and hence

$$b^* = ce^* = c\frac{1}{c}\frac{1}{1+\gamma\sigma^2 c} = \frac{1}{1+\gamma\sigma^2 c}$$

- Notice $0 < b^* < 1$:
  - Agent gets paid increasing function of output to incentivize
  - Does not get paid one-on-one ($b = 1$) because that would pass on too much risk to agent
  - (Remember $w^* = a^* + b^* y = a^* + b^* e + b^* \varepsilon$)
  - Comparative Statics: what happens to b^* if $\gamma = 0$ or $\sigma = 0$? Interpret

- Compare this solution to solution when effort is observable

- This is so-called **first best** since it eliminates the uncertainty involved in connecting pay to performance (as opposed to effort)

  – Principal offers a flat wage $w = a$ as long as agent works e^*

  – Agent accepts job if

  $$a - c(e^*) \geq 0$$

  – Principal wants to pay minimal necessary and hence sets $a^* = c(e^*)$

  – Substitute into profit of principal

  $$\max_{a,b} E[\pi] = e - E[w(y)] = e - a^* = e - c(e)$$

  - Solution for e^*: $c'(e^*) = 1$ or

$$e^*_{FB} = 1/c$$

- Compare e^* above and e^*_{FB} in first best

- –> With observable effort (first best) agent works harder

- Summary of hidden-action solution with risk-averse agent:

- **Risk-incentive trade-off:**
  - Agent needs to be incentivized ($b^* > 0$) or will not put in effort e
  - Cannot give too much incentive (b^* too high) because of risk-aversion
  - Trade-off solved if
    * Action e observable OR
    * No risk aversion ($\gamma = 0$) OR
    * No noise in outcome ($\sigma^2 = 0$)
  - Otherwise, effort e^* in equilibrium is sub-optimal

- Same trade-off applies to other cases

- Example 2: *Insurance* (Not fully solved)
  - Two states of the world: Loss and No Loss
  - Probability of Loss is $\pi(e)$, with $\pi'(e) < 0$
    * Example: Careful driving (Car Insurance)
    * Example: Maintaining your house better (House insurance)
    * Agent chooses quantity of insurance α purchased
  - Agent risk averse: $U(c)$ with $U' > 0$ and $U'' < 0$

- Qualitative solution:
  - No hidden action –> Full insurance: $\alpha^* = L$
  - Hidden action –>
    * Trade-off risk-incentives –> Only Partial insurance $0 < \alpha^* < L$
    * Need to make agent partially responsible for accident to incentivize
    * Do not want to make too responsible because of risk-aversion

5 Next lecture

- Asymmetric Information

- Moral Hazard