Economics 101A
(Lecture 13)

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Outline

1. Expected Utility

2. Insurance

3. Investment in Risky Asset

1 Expected Utility

- Nicholson, Ch. 7, pp. 202-209 (Ch. 18, pp. 533–541, 9th)

- Consumer at time 0 asks: what is utility in time 1?

- At $t = 1$ consumer maximizes

$$\begin{aligned}&\max U(c^1)\\ &s.t.\ c_i^1 \leq M_i^1 + (1+r)\left(M^0 - c^0\right)\end{aligned}$$

  with $i = 1, 2, 3$.

- What is utility at optimum at $t = 1$ if $U' > 0$?

- Assume for now $M^0 - c^0 = 0$

- Utility $U\left(M_i^1\right)$

- This is uncertain, depends on which i is realized!

- How do we evaluate future uncertain utility?

- **Expected utility**

$$EU = \sum_{i=1}^{3} p_i U\left(M_i^1\right)$$

- In example:

$$EU = 1/3U(20) + 1/3U(25) + 1/3U(30)$$

- Compare with $U(EC) = U(25)$.

- Agents prefer riskless outcome EM to uncertain outcome M if

$$\begin{aligned} 1/3U(20) + 1/3U(25) + 1/3U(30) &< U(25) \text{ or} \\ 1/3U(20) + 1/3U(30) &< 2/3U(25) \text{ or} \\ 1/2U(20) + 1/2U(30) &< U(25) \end{aligned}$$

- Picture

- Depends on sign of U'', on concavity/convepwxity
- Three cases:
  - $U''(x) = 0$ for all x. (linearity of U)
    - $U(x) = a + bx$
    - $1/2U(20) + 1/2U(30) = U(25)$
  - $U''(x) < 0$ for all x. (concavity of U)
    - $1/2U(20) + 1/2U(30) < U(25)$
  - $U''(x) > 0$ for all x. (convexity of U)
    - $1/2U(20) + 1/2U(30) > U(25)$

- If $U''(x) = 0$ (linearity), consumer is indifferent to uncertainty

- If $U''(x) < 0$ (concavity), consumer dislikes uncertainty

- If $U''(x) > 0$ (convexity), consumer likes uncertainty

- Do consumers like uncertainty?

- **Theorem. (Jensen's inequality)** If a function $f(x)$ is concave, the following inequality holds:

$$f(Ex) \geq Ef(x)$$

where E indicates expectation. If f is strictly concave, we obtain

$$f(Ex) > Ef(x)$$

- Apply to utility function U.

- Individuals dislike uncertainty:

$$U(Ex) \geq EU(x)$$

- Jensen's inequality then implies U concave ($U'' \leq 0$)

- Relate to diminishing marginal utility of income

2 Insurance

- Nicholson, Ch. 7, pp. 216–221 (18, pp. 545–551, 9th) Notice: different treatment than in class

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$, $u'' < 0$

- Probability p of accident with loss L

- Insurance offers coverage:
  - premium \$$q$ for each \$1 paid in case of accident
  - units of coverage purchased α

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\,(w - q\alpha) + pu\,(w - q\alpha - L + \alpha)$$
$$s.t. \alpha \geq 0$$

- Assume $\alpha^* \geq 0$, check later

- First order conditions:

$$\begin{aligned} 0 &= -q\,(1-p)\, u'\,(w - q\alpha) \\ &\quad + (1-q)\, pu'\,(w - q\alpha - L + \alpha) \end{aligned}$$

or

$$\frac{u'\,(w - q\alpha)}{u'\,(w - q\alpha - L + \alpha)} = \frac{1-q}{q}\frac{p}{1-p}.$$

- Assume first $q = p$ (insurance is fair)

- Solution for $\alpha^* =?$

- $\alpha^* > 0$, so we are ok!

- What if $q > p$ (insurance needs to cover operating costs)?

- Insurance will be only partial (if at all): $\alpha^* < L$

- Exercise: Check second order conditions!

3 Investment in Risk Asset

- Individual has:
  - wealth w
  - utility function u, with $u' > 0$

- Two possible investments:
  - Asset B (bond) yields return 1 for each dollar
  - Asset S (stock) yields uncertain return $(1 + r)$:
    * $r = r_+ > 0$ with probability p
    * $r = r_- < 0$ with probability $1 - p$
    * $Er = pr_+ + (1 - p)\, r_- > 0$

- Share of wealth invested in stock S $= \alpha$

- Individual maximization:

$$\max_{\alpha} (1-p)\, u\left(w\left[(1-\alpha)+\alpha\left(1+r_{-}\right)\right]\right)+$$
$$+pu\left(w\left[(1-\alpha)+\alpha\left(1+r_{+}\right)\right]\right)$$
$$s.t. 0 \leq \alpha \leq 1$$

- Case of risk neutrality: $u(x)=a+bx$, $b>0$

- Assume $a=0$ (no loss of generality)

- Maximization becomes

$$\max_{\alpha} b\,(1-p)\left(w\left[1+\alpha r_{-}\right]\right)+bp\left(w\left[1+\alpha r_{+}\right]\right)$$

or

$$\max_{\alpha} bw+\alpha bw\left[(1-p)\, r_{-}+pr_{+}\right]$$

- Sign of term in square brackets? Positive!

- Set $\alpha^{*}=1$

- Case of risk aversion: $u'' < 0$

- Assume $0 \leq \alpha^* \leq 1$, check later

- First order conditions:

$$\begin{aligned} 0 &= (1-p)(wr_-)\,u'(w[1+\alpha r_-]) + \\ &\quad +p(wr_+)\,u'(w[1+\alpha r_+]) \end{aligned}$$

- Can $\alpha^* = 0$ be solution?

- Solution is $\alpha^* > 0$ (positive investment in stock)

- Exercise: Check s.o.c.

4 Next lecture and beyond

- Measures of Risk Aversion
- Production Function