Econ 101A – Midterm 1
Th 26 February 2009.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. Dan and Mariana will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Consumption and Leisure Decision. (58 points) In class, we considered separately a consumption decision between goods x_1 and x_2 and a leisure decision between consumption good x and leisure l. Now we consider those together. Yingyi likes three goods: consumption goods x_1, x_2, and leisure l. He maximizes the utility function

$$u(x_1, x_2, x_3) = x_1^{\alpha_1} x_2^{\alpha_2} l^{\gamma},$$

with $0 < \alpha_i < 1$ for $i = 1, 2$ and $0 < \gamma < 1$. The consumption good x_i has price p_i (for $i = 1, 2$), the hourly wage is w.

1. Write down the budget constraint. Consider that Yingyi has H hours to work and, if he does not work, he takes leisure. For example, H could be 24 hours if the time period is a day. Hence, the hours worked h equal $H - l$. There are no sources of income other than income from hours worked. Write down the budget constraint as a function of x_1, x_2, and l. [Hint: Money spent on goods has to be smaller than or equal to money earned] (5 points)

2. Write down the maximization problem of the worker with respect to x_1, x_2, and l with *all* the relevant constraints Assume that the budget constraint is satisfied with equality. Why can we assume that the budget constraint is satisfied with equality? Provide as complete an explanation as you can. (5 points)

3. Write down the Lagrangean and derive the first order conditions with respect to x_1, x_2, l, and λ. (4 points)

4. Solve for x_1^* as a function of the prices p_1, p_2, w, the total number of hours H, and the parameters α_1, α_2, and γ. [Hint: combine the first and second first-order condition, then combine the first and third first-order condition, and finally plug in budget constraint] Similarly solve for x_2^* and l^*. (6 points)

5. Plot the Engel function relating the demand for good 1 $x_1^*(H)$ to the number of hours available H. (Plot x_1 in the x axis and H in the y axis) In what sense H plays the role of income? Explain. (5 points)

6. Plot the demand function for good 1 $x_1^*(p_1)$ as a function of p_1. (Put x_1 in the x axis and price p_1 in the y axis) Is the demand function downward sloping? Interpret. (5 points)

7. Are goods x_1 and x_2 gross complements, gross substitutes, or neither? Define and answer. (5 points)

8. Plot the demand function for leisure $l^*(w)$ as a function of its (shadow) price w. (Put l in the x axis and price w in the y axis) Is the demand function downward sloping? Interpret. (5 points)

9. Relate the response to question 8 (leisure l and price w) to substitution and income effects. Be careful here, this is not exactly the case we saw in class. (6 points)

10. Using the envelope theorem, compute how the indirect utility $v(p_1, p_2, w, \alpha_1, \alpha_2, \gamma, H)$ changes as H changes: $\partial v/\partial H$. Remember that the indirect utility is the utility of Yingyi at the optimum level of the parameters: $v(p_1, p_2, w, \alpha_1, \alpha_2, \gamma, H) = u(x_1^*, x_2^*, l^*)$. What is the sign of $\partial v/\partial H$? Interpret (6 points)

11. (Extra credit) Solve for the Lagrangean multiplier $\lambda^*(p_1, p_2, w, \alpha_1, \alpha_2, \gamma)$ using the first order conditions above. Comment on what this implies for $\partial v/\partial H$. (6 points)

Problem 2. (28 points)

1. **Complements and Substitutes.**

   (a) Define when two goods x_1 and x_2 are gross substitutes/complements and net substitutes/complements. (4 points)

   (b) Is it possible for two goods to be gross substitutes and net complements if both goods are normal goods? And if both goods are inferior goods? Use the general form of the Slutsky equation and provide an explanation for the result. (10 points)

2. Kim has utility function $u(x_1, x_2) = \exp(x_1 + x_2)$ for $x_i \geq 0$, $i = 1, 2$.

   (a) Plot the indifference curves of Kim. What kind of goods do they represent? (4 points)

   (b) Are the preferences represented by this utility function monotonic? Define. (4 points)

   (c) Are they rational? Define. (6 points)