Econ 101A
Midterm 2

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Do not turn page unless instructed to.

Econ 101A – Midterm 2
Th 8 April 2009.

You have approximately 1 hour and 20 minutes to answer the questions in the midterm. I will collect the exams at 11.00 sharp. Show your work, and good luck!

Problem 1. Production (38 points)**.** Consider a farmer that produces corn using labor. The labor cost in dollar to produce y bushels of corn is $c(y) = Cy^2$, with $C > 0$. There are 100 identical farms which all behave competitively. Corn sells at a price p per bushel.

1. Derive the marginal cost $c'(y)$ and the average cost $c(y)/y$. Plot them. Derive graphically the supply curve. (Have the quantity y on the horizontal axis). (5 points)
2. Write the expression for the supply of corn $y(p)$ for each firm, and derive the expression for the aggregate supply function $Y^S(p)$ (5 points)
3. From now on, suppose that the demand curve of corn is $D(p) = 200 - 50p$ and assume $C = 1$. Derive the equilibrium price p^* and total quantity sold Y^{S*}. (5 points)
4. Derive the profits of each firm. (5 points)
5. Now assume that land is not free, and that the farmers are renting the land from a monopolist (that is, there is only one land owner, and it is impossible to rent land somewhere else). How much will the land-owner charge each of the farms? Explain. (8 points)
6. Taking the rent of the land into account, how much are the profits of each firm? (4 points)
7. In what sense there is a parallel between the presence of rents in the land and the entry of firms in the long-run equilibrium? (6 points)

Problem 2. Consumer Surplus (35 points) We evaluate here the change in consumer surplus associated with a change in the price of good 1 from p_1 to p_1' for a consumer with Cobb-Douglas utility $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, with $0 < \alpha < 1$.

1. Explain the intuition of why the change in consumer surplus is defined as $\Delta CS = e(p_1, p_2, u) - e(p_1', p_2, u)$, where e is the expenditure function. (6 points)
2. Define the expenditure function. (4 points)
3. Derive an expression for the expenditure function $e(p_1, p_2, u)$ for this Cobb-Douglas case given that the Hicksian demands in this case are

$$
\begin{aligned}
h_1^*(p_1, p_2, u) &= u \cdot \left(\frac{p_2}{p_1} \cdot \frac{\alpha}{1-\alpha}\right)^{1-\alpha} \\
h_2^*(p_1, p_2, u) &= u \cdot \left(\frac{p_1}{p_2} \cdot \frac{1-\alpha}{\alpha}\right)^{\alpha}
\end{aligned}
$$

   (5 points)
4. Solve for the change in consumer surplus $\Delta CS = e(p_1, p_2, u) - e(p_1', p_2, u)$. (If you get stuck here, just skip to the next point) (5 points)
5. If you also substituted for u the expression for the indirect utility $v(p_1, p_2, M)$, you would get that ΔCS is proportional to

$$\left[1 - \left(\frac{p_1'}{p_1}\right)^\alpha\right] \cdot M \tag{1}$$

(Do not attempt to do this, it involves a fair amount of algebra, take it as given) Using expression (1), show that the following statements are true, and provide intuitive explanations for each of them: (i) $\Delta CS > 0$ if and only if $p_1' < p_1$; (ii) holding constant p_1' and p_1 with $p_1' < p_1$, ΔCS is increasing in α; (iii) holding constant p_1' and p_1 with $p_1' < p_1$, ΔCS is increasing in M. (15 points)

Problem 3. Uncertainty (40 points)

1. A first consumer has an expected utility function of the form $u(w) = \sqrt{w}$. She initially has a wealth of \$4. She has a lottery ticket that will be worth \$14 with probability 1/2 and will be worth \$0 with probability 1/2. What is her expected utility? What is the lowest price p she is willing to accept to sell her ticket? (10 points)

2. A second consumer has an expected utility function of the form $u(w) = \ln(w)$. A friend that is fond of gambling offers him the opportunity to bet on the flip of a coin that has probability π of coming up heads. If he bets \$$x$, he will have $w + x$ if head comes up and $w - x$ is tails comes up. Notice that x has to be non-negative ($x \geq 0$).

   - Provide a definition of risk-aversion and show that the agent is risk-averse (or not). (6 points)
   - What is his expected utility? (4 points)
   - Solve for the optimal x^* as a function of π. Discuss the qualitative features of the solution (6 points)
   - In particular, discuss the optimal x^* for $\pi < 1/2$ and for $\pi > 1/2$. Draw a parallel with the case of investment in risky asset that we saw in class. (6 points)
   - True or not true. Explain as precisely as you can. 'Given that the agent is risk-averse, he will not bet in the lottery unless the lottery has a substantially positive expected value' (8 points)