Economics 101A
(Lecture 14)

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Outline

1. Production: Introduction

2. Production Function

3. Returns to Scale

4. Two-step Cost Minimization

1 Production: Introduction

- Second half of the economy. **Production**

- Example. Ford and the Minivan (Petrin, 2002):
  - Ford had idea: "Mini/Max" (early '70s)
  - Did Ford produce it?
  - No!
  - Ford was worried of cannibalizing station wagon sector
  - Chrysler introduces Dodge Caravan (1984)
  - Chrysler: $1.5bn profits (by 1987)!

- Why need separate treatment?

- Perhaps firms maximize utility...

- ...we can be more precise:
  - Competition
  - Institutional structure

2 Production Function

- Nicholson, Ch. 9, pp. 295-301; 306-311 (Ch. 7, pp. 183–190; 195–200, 9th)

- Production function: $y = f(\mathbf{z})$. Function $f : R_+^n \rightarrow R_+$

- Inputs $\mathbf{z} = (z_1, z_2, ..., z_n)$: labor, capital, land, human capital

- Output y: Minivan, Intel CPU, mangoes (Philippines)

- Properties of f:

  - no free lunches: $f(0) = 0$

  - positive marginal productivity: $f_i'(\mathbf{z}) > 0$

  - decreasing marginal productivity: $f_{i,i}''(\mathbf{z}) < 0$

- Isoquants $Q(y) = \{\mathbf{x} | f(\mathbf{x}) = y\}$

- Set of inputs $\mathbf{z}$ required to produce quantity y

- Special case. Two inputs:

  - $z_1 = L$ (labor)

  - $z_2 = K$ (capital)

- Isoquant: $f(L, K) - y = 0$

- Slope of isoquant $dK/dL = MRTS$

- Convex production function if convex isoquants

- Reasonable: combine two technologies and do better!

- Mathematically, convex isoquants if $d^2K/d^2L > 0$

- Solution:

$$\frac{d^2K}{d^2L} = -\frac{f''_{L,L}f'_K - 2f''_{L,K}f'_L + f''_{K,K}\left(f'_L\right)^2/f'_K}{\left(f'_K\right)^2}$$

- Hence, $d^2K/d^2L > 0$ if $f''_{L,K} > 0$ (inputs are complements in production)

3 Returns to Scale

- Nicholson, Ch. 9, pp. 302-305 (Ch. 7, pp. 190–193, 9th)

- Effect of increase in labor: f'_L

- Increase of all inputs: $f(t\mathbf{z})$ with t scalar, $t > 1$

- How much does input increase?

  – Decreasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) < tf(\mathbf{z})$$

– Constant returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) = tf(\mathbf{z})$$

– Increasing returns to scale: for all $\mathbf{z}$ and $t > 1$,

$$f(t\mathbf{z}) > tf(\mathbf{z})$$

- Example: $y = f(K, L) = AK^{\alpha}L^{\beta}$

- Marginal product of labor: $f'_L =$

- Decreasing marginal product of labor: $f''_{L,L} =$

- $MRTS =$

- Convex isoquant?

- Returns to scale: $f(tK, tL) = A(tK)^{\alpha}(tL)^{\beta} = t^{\alpha+\beta}AK^{\alpha}L^{\beta} = t^{\alpha+\beta}f(K, L)$

4 Two-step Cost minimization

- Nicholson, Ch. 10, pp. 323-330 (Ch. 12 , pp. 212–220, 9th)

- Objective of firm: Produce output that generates maximal profit.

- Decompose problem in two:
  - Given production level y, choose cost-minimizing combinations of inputs
  - Choose optimal level of y.

- *First step.* Cost-Minimizing choice of inputs

- Two-input case: Labor, Capital

- Input prices:
  - Wage w is price of L
  - Interest rate r is rental price of capital K

- Expenditure on inputs: $wL + rK$

- Firm objective function:

$$\min_{L,K} wL + rK$$
$$s.t. f(L,K) \geq y$$

- Equality in constraint holds if:

  1. $w > 0$, $r > 0$;

  2. f strictly increasing in at least L or K.

- Counterexample if ass. 1 is not satisfied

- Counterexample if ass. 2 is not satisfied

- Compare with expenditure minimization for consumers

- First order conditions:

$$w - \lambda f'_L = 0$$

and

$$r - \lambda f'_K = 0$$

- Rewrite as

$$\frac{f'_L(L^*, K^*)}{f'_K(L^*, K^*)} = \frac{w}{r}$$

- MRTS (slope of isoquant) equals ratio of input prices

- Graphical interpretation

- Derived demand for inputs:
  - $L = L^* (w, r, y)$
  - $K = K^* (w, r, y)$
- Value function at optimum is **cost function:**

$$c(w, r, y) = wL^* (r, w, y) + rK^* (r, w, y)$$

- *Second step.* Given cost function, choose optimal quantity of y as well

- Price of output is p.

- Firm's objective:

$$\max py - c(w, r, y)$$

- First order condition:

$$p - c'_y(w, r, y) = 0$$

- Price equals marginal cost – very important!

- Second order condition:

$$-c''_{y,y}(w, r, y^*) < 0$$

- For maximum, need increasing marginal cost curve.

5 Next Lecture

- Solve an Example
- Cases in which s.o.c. are not satisfied
- Start Profit Maximization