Econ 101A – Final exam
Th 14 December.

Do not turn the page until instructed to.

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Please solve Problem 1, 2, and 3 in the first blue book and Problem 4 in the second Blue Book. Good luck!

Problem 1. Shorter problems. (35 points) Solve the following shorter problems.

1. Consider the following (simultaneous) game of chicken. This is a game in which two players drive cars at each other. The first to swerve away and slow down loses and is humiliated as the "chicken"; if neither player swerves, the result is a potentially fatal head-on collision. The principle of the game is to create pressure until one person backs down. Call s the probability that player 1 swerves, $1-s$ the probability that player 1 drives straight, S the probability that Player 2 swerves, and $1-S$ the probability that Player 2 drives straight. Compute all the pure-strategy and mixed strategy equilibria. (20 points)

1\2	Swerving	Driving Straight
Swerving	$0, 0$	$-1, 1$
Driving Straight	$1, -1$	$-10, -10$

2. Now assume that the game is played sequentially, with player 1 moving first and deciding whether to Swerve or Drive Straight, and player 2 moving second after observing what player 1 did, and deciding also whether to Swerve or Drive Straight. (I know, this makes for a boring game of chicken) Write the decision tree for this game, and find all the pure-strategy Subgame-Perfect Equilibria. In particular, write down the Subgame-Perfect Equilibrium **strategies** of this dynamic game. How do these equilibria compare to the equilibria of the static game? (15 points)

Problem 2. Consumption-Savings Problem with Habit Formation (35 points) In this exercise, we consider the choice of consumption over time. We assume two periods, $t = 1$ and $t = 2$. Kim receives no income in period 1 and earns income M in period 2. She can borrow at per-period interest r on each dollar. The peculiarity of this exercise is that Kim's preferences are characterized by habit formation: the more she consumes in the first period, the less she gets utility from consumption in the second period. (This is similar to the addiction problem that you solved in the problem sets). More precisely, Kim has utility function

$$u(c_1, c_2) = u(c_1) + u(c_2 - \gamma c_1)$$

with $0 \leq \gamma \leq 1$, and $u(x)$ with the usual concavity assumptions, $u' > 0$ and $u'' < 0$. (We assume that $\delta = 0$)

1. Derive the intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = \frac{M}{1+r}.$$

   (5 points)

2. Write down the utility maximization problem by substituting in the expression for c_2 from the budget constraint (4 points)

3. Derive the first-order condition for c_1^*. (5 point)

4. Check the second-order condition for c_1^*. (5 point)

5. Use the implicit function theorem to compute $\partial c_1^* / \partial \gamma$. Interpret the sign. (8 points)

6. Under the assumption $u(x) = \log(x)$, solve for c_1^* and c_2^*. In particular, assume $r = 0$ for simplicity and interpret the solution for the cases $\gamma = 0$ and $\gamma = 1$. (8 points)

Problem 3. Price Discrimination. (48 points) Consider a monopolistic firm that sells drugs in two markets, Europe and US. In Europe, inverse demand is given by $p(x_E) = 8 - 2x_E$, whereas in the US it is given by $p(x_{US}) = 10 - x_{US}$. The cost function of the monopolist is $c(x_E + x_{US}) = x_E + x_{US}$, that is, the marginal cost of production is constant and equal to 1.

1. Which market has a higher willingness to pay (that is, consumers are willing to pay more for a given quantity)? (5 points)

2. First, assume that the monopolist can price discriminate between the two markets, that is, can charge different prices in the two markets. Set up the maximization problem of the monopolist. (5 points)

3. Solve the problem. Compute the profit-maximizing choice of x_E^D and x_{US}^D, as well as the equilibrium prices p_E^D and p_{US}^D. (8 points)

4. Compare the quantities and prices across the two markets. Interpret the results in light of what you discussed in point 1. (5 points)

5. Now, assume that new legislation makes it illegal to price-discriminate between the two markets. That is, the monopolist must charge the same price $p_{US}^M = p_E^M = p^M$. Set up the new maximization problem for the firm. (Be careful how you set this problem up, this is not a trivial step, it may be useful to write the maximization as a function of p and X, the total production) (10 points)

6. Compute the profit-maximizing choice of the total quantity produced X^M, the price p^M, and the quantities sold in each market x_E^M and x_{US}^M. [Help: You should get $X^M = 25/4$] (10 points)

7. Compare the price p^M with the prices p_E^D and p_{US}^D with discrimination. Do a similar comparison for quantities produces with discrimination. Discuss. (5 points)

[New Blue Book!]

Problem 4. Profit Maximization with Discrete Increments (118 points). In class, we studied all sorts of variants of profit maximization with continuous demand and supply functions. In this problem we study the case where the demand and supply function are discrete. You will not take any derivatives in this problem. Consumers in 101World like kiwis, but in different ways: 100 consumers value kiwis at \$5 a piece, 10 consumers value them at \$3 a piece, 90 consumers value them at \$2 a piece, and 100 consumers value them at \$1 a piece. No consumer wants more than 1 kiwi, that is, they value the second kiwi at \$0. *Assume that consumers purchase kiwis even if their valuation is exactly equal to the price of a kiwi (in which case they are indifferent).* For example, if the price is \$2, all the 90 consumers of the third type still purchase kiwis.

1. **Demand.** Plot the market demand for kiwis in the space (quantity of kiwis, price of a piece of kiwi). Put the price in the y axis. (4 points)
2. **Perfect Competition in the Short-Run.** Consider now farms producing kiwis in a perfectly competitive market. Each farm can produce 10 kiwis at the cost of \$1 each and additional kiwis at \$2 each. (To produce additional kiwis one needs to grow them on less productive land, which is more costly). Compute and plot the total cost, the average cost, the marginal cost for each firm. (4 points)
3. Derive the supply function for each firm. Remember, the supply function is a correspondence $y^*(p)$ from prices to quantity produced. (6 points)
4. Still under perfect competition: Assume that there are 10 firms in the market. (That is, we are in the short-run with a fixed number of firms) Compute and plot the industry supply function. (4 points)
5. Find the market equilibrium for price p^*_{PC} and total quantity produced under perfect competition Q^*_{PC} with 10 firms producing as the levels that equate demand and supply. (As I wrote above, assume that all consumers that are indifferent between purchasing and not purchasing actually purchase) (4 points)
6. Compute the firm surplus (that is, the profit) for each of the firms and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by all firms and consumers. (6 points)
7. Do firms make zero profits? If so, is it surprising? If not, is it surprising? (6 points)
8. **Perfect Competition in the Long-Run.** Now, still assume perfect competition, but allow for free entry. (That is, we are in the long-run) That is, more firms with cost function of the type above will enter the market as long as there are positive profits. How many firms will enter at a minimum ? (additional firms may enter beyond this number) Plot the industry supply function in this case and determine the equilibrium price and quantity produced. (8 points)
9. Compute the firm surplus (that is, the profit) for each of the firms and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by all firms and consumers. How do these differ from the case of perfect competition with fixed number of firms? Discuss. (6 points)
10. **Monopoly.** In the next year, expectations are that one big company will buy out all ten farms and act as a monopolist. Hence, this firm will be able to produce up to 100 kiwis for \$1 each and any additional kiwis for \$2 each. (That is, the costs for this firm are the sum of the costs for the 10 individual firms) Determine the profit-maximizing quantity q^*_M and price p^*_M produced by the monopoly. (The firm charges the same price to all consumers) (8 points)
11. Compute the firm surplus (that is, the profit) and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by the firm and all the consumers. How do these differ from the previous cases you considered? Discuss. (6 points)
12. **Monopoly with Perfect Price Discrimination.** Assume now that a single monopolist can charge different prices to different consumers. What will the prices and quantities in equilibrium be now? (6 points)

13. Compute the firm surplus (that is, the profit) and the consumer surplus for each type of consumer. Compute the total surplus, defined as the sum of the surpluses by the firm and all the consumers. (4 points)

14. Focus on the total surplus and compare the case of perfect competition in the short-run and the two monopoly cases. Discuss and relate to deadweight loss. (6 points)

15. **Bertrand Duopoly.** Based on the analysis above, the anti-trust authority outlaws the buy-out plans that would lead to monopoly. The authority instead will allow two firms to take over half the market each. That is, each firm will be able to produce up to 50 kiwis for \$1 each and any additional kiwis for \$2 each. Assume now that there are two firms are competing a la Bertrand, that is, on prices. To be more precise, each firm maximizes

$$\pi_i(p_i, p_j) = \begin{cases} p_i D(p_i) - C(D(p_i)) & \text{if} \quad p_i < p_j \\ p_i \frac{D(p_i)}{2} - C\left(\frac{D(p_i)}{2}\right) & \text{if} \quad p_i = p_j. \\ 0 & \text{if} \quad p_i > p_j. \end{cases}$$

That is, if the two firms charge the same price, they each sell half the output. We are also assuming that firms have to produce all the quantity that is demanded at a given price, that is, $D(p_i)$, unless they tie, in which case they have to produce $D(p_i)/2$. Show that $p_1^* = p_2^* = 1.5$ is a Nash Equilibrium of this game. (10 points)

16. Compare the quantity and price produced in this equilibrium to the case of perfect competition in the short-run. Comment on the difference between the two cases. Are you surprised? (8 points)

17. (Harder) Find another Nash Equilibrium of this game. (10 points)

18. (Hard) Find all the Nash Equilibria of this game (12 points)