Econ 101A – Final exam
Th 15 December.

Do not turn the page until instructed to.

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Please solve Problem 1, 2, and 3 in the first blue book and Problems 4 and 5 in the second Blue Book. Good luck!

Problem 1. Shorter problems. (50 points) Solve the following shorter problems.

1. Compute the pure-strategy and mixed strategy equilibria of the following coordination game. Call u the probability that player 1 plays Up, $1-u$ the probability that player 1 plays Down, l the probability that Player 2 plays Left, and $1-l$ the probability that Player 2 plays Right. (20 points)

1\2	Left	Right
Up	3, 2	1, 1
Down	1, 1	2, 3

2. For each of these cost functions, plot the marginal cost function and the supply function, *and* write out the supply function $S(p)$, with quantity as a function of price p (30 points):

   (a) $C(q) = 2q$ (8 points)

   (b) $C(q) = 2q^2 - q + 2$ (12 points)

   (c) $C(q) = q^3 + 10q$ (10 points)

Problem 2. Monopoly and Duopoly. (40 points). Initially there is one firm in a market for cars. The firm has a linear cost function: $C(q) = 2q$. The market inverse demand function is given by $P(Q) = 9 - Q$.

1. What price will the firm charge? What quantity of cars will the firm sell? (8 points)
2. How much profit will the firm make? (4 points)
3. Now, a second firm enters the market. The second firm has an identical cost function. What will the Cournot equilibrium output for each firm be? (8 points)
4. What is the Stackelberg equilibrium output for each firm if firm 2 enters second? (7 points)
5. How much profit will each firm make in the Cournot game? How much in Stackelberg? (5 points)
6. Which type of market do consumers prefer: monopoly, Cournot duopoly or Stackelberg duopoly? Why? (8 points)

Problem 3. Voting. (23 points) This paper proides a simple model of voting to illustrate the difficulties (and the strength) of an economic model of voting. Consider George, a committed Republican that is deciding whether to vote for Presidential elections. George's utility function is $U(P, v) = u(P) - cv$, where $u(P)$ equals U if Republicans win the election ($P = R$) and 0 if Democrats win the election ($P = D$). The variable $c \geq 0$ is the effort cost of going to vote, which George pays only if he votes ($v = 1$). If Goerge does not vote ($v = 0$), George pays no voting cost. Finally, George believes that there is a probability p that his vote will decide the election, and probability $1 - p$ that his vote will not affect the elections. In addition, George believes that the average share of Republican voters is .5.

1. Compute the expected utility of George from voting ($v = 1$) and from non-voting ($v = 0$). (6 points)
2. Under what condition does George vote? Provide intuition (4 points)
3. Assume that the cost of voting is \$10 (an hour's wage) and the value of voting is U is \$1,000. What would this imply about the cutoff level of p such that George votes? Is it plausible that George will vote? (5 points)
4. Two empirical facts about votings are that (i) voter turnout is higher in closer elections; (ii) voter turnout is higher for more educated voters; (iii) voter turnout is higher for individuals with higher ernings; (iv) voter turnout is lower for younger people. Interpret these results in light of the model (8 points)

[New Blue Book]

Problem 4. Driving risk and insurance (38 points). Robert is an expected-utility maximizer that likes to drive fast, so his probability of an accident is 2/3. Robert's preferences over wealth are $u(w) = w^{.5}$. Suppose that Robert's initial wealth is \$100. If Robert has an accident, he incurs a \$51 loss.

1. What is Robert's expected utility? (5 points)

2. Now, assume there is one insurance company in existence with one policy available: Full insurance. That is to say, the insurance company charges a fixed premium and then pays Robert \$51 if he gets in an accident. If the insurance company is risk neutral, what is the premium π they need to charge to break even? (5 points)

3. Compute the expected utility for Robert if he purchases the insurance at premium π. Will Robert purchase the insurance? (Note: $(66)^{.5}$ is approximately 8.1) (5 points)

4. Repeat the exercise for the case in which Robert has utility function $u(w) = w^2$. What is his utility if he does/does not purchase the insurance? Does he purchase the insurance? (Note: $(66)^2 = 4356$ and $(49)^2 = 2401$) (6 points)

5. Discuss the intuition for why Robert purchases the insurance in one case, but not in the other. (5 points)

6. (Harder) Consider now the general case with $u(w) = w^{\alpha}$ with $\alpha > 0$. Remember that Jensen's inequality says $Ef(x) \geq f(Ex)$ if and only if f concave. Use Jensen's inequality to show analytically that, in fact, Robert purchases the insurance at premium π if and only if $\alpha < 1$ (and is indifferent for $\alpha = 1$). If you cannot do the maths, provide intuition on this. (12 points)

Problem 5. Bertrand Competition in discrete increments (64 points) (Note: This problem resembles one on last year's exam, but it's not the same, read carefully) Consider a variant of the Bertrand model of competition with two firms that we covered in class. The difference from the model in class is that prices are not a continuous variable, but rather a discrete variable. Prices vary in multiples of 1 cent. Firms can charge prices of 0, .01, .02, .03,... etc. The profits of firm i are

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c_i) D(p_i) & \text{if} \quad p_i < p_j \\ (p_i - c_i) D(p_i)/2 & \text{if} \quad p_i = p_j. \\ 0 & \text{if} \quad p_i > p_j. \end{cases}$$

Demand is extremely inelastic, that is, $D(p) = Q$ for all p. Both firms have the same marginal cost $c_1 = c_2 = c$, and the marginal cost c is a multiple of 1 cent. (The firm can charge $c - .01$, c, $c + .01$, $c + .02$, etc.) Consider first the case in which the two firms move simultaneously (as we did in class), and apply the Nash Equilibrium concept.

1. Write down the definition of Nash Equilibrium as it applies to this game, that is, with p_i as the strategy of player i and $\pi_i(p_i, p_j)$ as the function that player 1 maximizes. Provide both the formal definition and the intuition. Do not substitute in the expression for π_i. (8 points)

2. Show that $p_1^* = p_2^* = c$ (that is, marginal cost pricing) is a first Nash Equilibrium of this game. (8 points)

3. Show that $p_1^* = p_2^* = c + .01$ is a second Nash Equilibrium of this game. (8 points)

4. (Harder) Can you find another Nash Equilibrium (you need to prove that it is a Nash Equilibrium) [Hint: The peculiar feature of this setup is that the firm can only charge prices that are multiples of 1 cent] Why does it matter that demand is inelastic? (10 points)

5. Now, we change the setup in just one way. The game is now played sequentially, that is, firm 1 moves first, and firm 2 follows after observing the price choice of firm 1. We apply Subgame Perfection to solve this game, and therefore start from the last period, from the choice of player 2. Player 2's strategy will be a function of Player 1's price p_1. Find the best response for player 2 as a function of p_1, that is, find $p_2^*(p_1)$. (10 points)

6. Now let's continue with the backward induction and go back to player 1. Player 1 anticipates the best response of player 2 and chooses the price p_1 that will yiled the highest profit. What is this price p_1^*? To simplify the solution, assume that player 2 responds to a price of $c + .02$ by also setting price $c + .02$ (8 points)

7. Write down the subgame perfect equilibrium. How does it differ from the set of Nash Equilibria of the symultaneous game? (8 points)

8. Can you conjecture how the solution of the dynamic Bertrand game will differ if firms can set price continuously? (4 points)