Economics 101A
(Lecture 25)

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Outline

1. Walrasian Equilibrium

2. Example

3. Existence and Welfare Theorems

1 Walrasian Equilibrium

- Prices p_1, p_2

- Consumer 1 faces a budget set:

$$p_1 x_1^1 + p_2 x_2^1 \leq p_1 \omega_1^1 + p_2 \omega_2^1$$

- How about consumer 2?

- Budget set of consumer 2:

$$p_1 x_1^2 + p_2 x_2^2 \leq p_1 \omega_1^2 + p_2 \omega_2^2$$

or (assuming $x_i^1 + x_i^2 = \omega_i$)

$$p_1(\omega_1 - x_1^1) + p_2 \left(\omega_1 - x_2^1\right) \leq p_1 \left(\omega_1 - \omega_1^1\right) + p_2 \left(\omega_2 - \omega_2^1\right)$$

or

$$p_1 x_1^1 + p_2 x_2^1 \geq p_1 \omega_1^1 + p_2 \omega_2^1$$

- **Walrasian Equilibrium.** $\Big((x_1^{1*}, x_2^{1*}), (x_1^{2*}, x_2^{2*}), p_1^*, p_2^*\Big)$ is a Walrasian Equilibrium if:

  – Each consumer maximizes utility subject to budget constraint:

$$\begin{aligned}(x_1^{i*}, x_2^{i*}) &= \arg\max_{x_1^i, x_2^i} u_i\Big((x_1^i, x_2^i\Big) \\ s.t.\ p_1^* x_1^i + p_2^* x_2^i &\leq p_1^* \omega_1^i + p_2^* \omega_2^i\end{aligned}$$

  – All markets clear:

$$x_j^{1*} + x_j^{2*} \leq \omega_j^1 + \omega_j^2 \text{ for all } j.$$

- Compare with partial (Marshallian) equilibrium:
  - each consumer maximizes utility
  - market for good i clears.
  - (no requirement that all markets clear)

- How do we find the Walrasian Equilibria?

- **Graphical method.**

  1. Compute first for each consumer set of utility-maximizing points as function of prices

  2. Check that market-clearing condition holds

- *Step 1.* Compute optimal points as prices p_1 and p_2 vary

- Start with Consumer 1. Find points of tangency between budget sets and indifference curves

- Figure

- **Offer curve** for consumer 1:

$$\left(x_1^{1*}\left(p_1, p_2, (\omega_1, \omega_2)\right), x_2^{1*}\left(p_1, p_2, (\omega_1, \omega_2)\right)\right)$$

- Offer curve is set of points that maximize utility as function of prices p_1 and p_2.

- Then find offer curve for consumer 2:

$$\left(x_1^{2*}\left(p_1, p_2, (\omega_1, \omega_2)\right), x_2^{2*}\left(p_1, p_2, (\omega_1, \omega_2)\right)\right)$$

- Figure

- *Step 2.* Find intersection(s) of two offer curves

- Walrasian Equilibrium is intersection of the two offer curves!
  - Both individuals maximize utility given prices
  - Total quantity demanded equals total endowment

- Relate Walrasian Equilibrium to barter equilbrium.

- Walrasian Equilibrium is a subset of barter equilibrium:
  - Does WE satisfy Individual Rationality condition?
  - Does WE satisfy the Pareto Efficiency condition?

- Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.

2 Example

- Consumer 1 has Leontieff preferences:

$$u(x_{1,}x_2) = \min\left(x_1^1, x_2^1\right)$$

- Bundle demanded by consumer 1:

$$\begin{aligned} x_1^{1*} &= x_2^{1*} = x^{1*} = \frac{p_1\omega_1^1 + p_2\omega_2^1}{p_1 + p_2} = \\ &= \frac{\omega_1^1 + (p_2/p_1)\,\omega_2^1}{1 + (p_2/p_1)} \end{aligned}$$

- Graphically

- Comparative statics:
  - increase in ω
  - increase in p_2/p_1:

$$\frac{dx_1^{1*}}{dp_2/p_1} = \frac{\begin{array}{c}\omega_2^1 \left(1 + (p_2/p_1)\right) \\ -\left(\omega_1^1 + (p_2/p_1)\, \omega_2^1\right)\end{array}}{\left(1 + (p_2/p_1)\right)^2} = $$

$$= \frac{\omega_2^1 - \omega_1^1}{\left(1 + (p_2/p_1)\right)^2}$$

  - Effect depends on income effect through endowments:
    * A lot of good 2 –> increase in price of good 2 makes richer
    * Little good 2 –> increase in price of good 2 makes poorer

- Notice: Only ratio of prices matters (general feature)

- Consumer 2 has Cobb-Douglas preferences:

$$u(x_{1,}x_2) = \left(x_1^2\right)^{.5} \left(x_2^2\right)^{.5}$$

- Demands of consumer 2:

$$x_1^{2*} = \frac{.5\left(p_1\omega_1^1 + p_2\omega_2^1\right)}{p_1} = .5\left(\omega_1^1 + \frac{p_2}{p_1}\omega_2^1\right)$$

and

$$x_2^{2*} = \frac{.5\left(p_1\omega_1^1 + p_2\omega_2^1\right)}{p_2} = .5\left(\frac{p_1}{p_2}\omega_1^1 + \omega_2^1\right)$$

- Impose Walrasian equilibrium in market 1:

$$x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2$$

This implies

$$\frac{\omega_1^1 + (p_2/p_1)\,\omega_2^1}{1 + (p_2/p_1)} + .5\left(\omega_1^1 + \frac{p_2}{p_1}\omega_2^1\right) = \omega_1^1 + \omega_1^2$$

or

$$\frac{.5 - .5\,(p_2/p_1)}{1 + (p_2/p_1)}\omega_1^1 + \frac{.5\,(p_2/p_1) + .5\,(p_2/p_1)^2 - 1}{1 + (p_2/p_1)}\omega_2^1 = 0$$

or

$$\left(\omega_1^1 - 2\omega_2^1\right) + \left(\omega_1^1 + \omega_2^1\right)(p_2/p_1) + \omega_2^1\,(p_2/p_1)^2 = 0$$

- Solution for p_2/p_1:

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\begin{array}{c}\left(\omega_1^1 + \omega_2^1\right)^2 \\ -4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1\end{array}}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

- Some complicated solution!

- Problem set has solution that is much easier to compute (and interpret)

3 Existence and Welfare Theorems

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex

- Is Walrasian Equilibrium always unique? Not necessarily

- Is Walrasian Equilibrium efficient? Yes.

- **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

- Figure

- **Second Fundamental Welfare theorem.** Given convex preferences, for every Pareto efficient allocation $\Big((x_1^1, x_1^1), (x_1^2, x_2^2)\Big)$ there exists some endowment (ω_1, ω_2) such that $\Big((x_1^1, x_1^1), (x_1^2, x_2^2)\Big)$ is a Walrasian Equilibrium for endowment (ω_1, ω_2).

- Figure

- Significance of these results:
  - First Theorem: Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
  - BUT: problems with externalities and public good
  - BUT: what about distribution?

  - Second Theorem: Can redistribute endowments to achieve any efficient outcome as a WE.
  - But redistribution is hard to implement, and distortive.

4 Next lecture

- Asymmetric Information

- Moral Hazard