Economics 101A
(Lecture 10)

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Outline

1. Complements and substitutes
2. Do utility functions exist?
3. Application 1: Labor Supply
4. Application 2: Intertemporal choice

1 Complements and substitutes

- Nicholson, Ch. 6, pp. 182-187 (161–166, 9th)

- How about if price of another good changes?

- Generalize Slutsky equation

- Slutsky Equation:

$$\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_j} = \frac{\partial h_i(\mathbf{p},v(\mathbf{p},M))}{\partial p_j} - x_j^*(p_1,p_2,M)\frac{\partial x_i^*(\mathbf{p},M)}{\partial M}$$

- Substitution effect

$$\frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} > 0$$

for $n = 2$ (two goods). Ambiguous for $n > 2$.

- Income effect:

$$-x_j^*(p_1, p_2, M)\frac{\partial x_i^*(\mathbf{p}, M)}{\partial M}$$

  – negative if good i is normal

  – positive if good i is inferior

- How do we define complements and substitutes?

- Def. 1. Goods i and j are **gross substitutes** at price $\mathbf{p}$ and income M if

$$\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_j} > 0$$

- Def. 2. Goods i and j are **gross complements** at price $\mathbf{p}$ and income M if

$$\frac{\partial x_i^* (\mathbf{p}, M)}{\partial p_j} < 0$$

- Example 1 (ctd.): $x_1^* = \alpha M / p_1$, $x_2^* = \beta M / p_2$.

- Gross complements or gross substitutes? Neither!

- Notice: $\frac{\partial x_i^*(\mathbf{p},M)}{\partial p_j}$ is usually different from $\frac{\partial x_j^*(\mathbf{p},M)}{\partial p_i}$

- Better definition.

- Def. 3. Goods i and j are **net substitutes** at price $\mathbf{p}$ and income M if

$$\frac{\partial h_i^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} > 0$$

- Def. 4. Goods i and j are **net complements** at price $\mathbf{p}$ and income M if

$$\frac{\partial h_i^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_j} = \frac{\partial h_j^* (\mathbf{p}, v(\mathbf{p}, M))}{\partial p_i} < 0$$

- Example 1 (ctd.): $h_1^* = \overline{u} \left(\frac{\alpha}{1-\alpha} \frac{p_2}{p_1} \right)^{1-\alpha}$

- Net complements or net substitutes? Net substitutes!

2 Do utility functions exist?

- Preferences and utilities are theoretical objects

- Many different ways to write them

- How do we tie them to the world?

- Use actual choices – revealed preferences approach

- Typical economists' approach. Compromise of:
  - realism
  - simplicity

- Assume a class of utility functions (CES, Cobb-Douglas...) with free parameters

- Estimate the parameters using the data

3 Labor Supply

- Nicholson (Ch. 16, pp. 477–484, 9th)

- Labor supply decision: how much to work in a day.

- Goods: consumption good c, hours worked h

- Price of good p, hourly wage w

- Consumer spends $24 - h = l$ hours in units of leisure

- Utilify function: $u(c, l)$

- Budget constraint?

- Income of consumer: $M + wh = M + w(24 - l)$

- Budget constraint: $pc \leq M + w(24 - l)$ or

$$pc + wl \leq M + 24w$$

- Notice: leisure l is a consumption good with price w. Why?

- General category: **opportunity cost**

- Instead of enjoying one hour of TV, I could have worked one hour and gained wage w.

- You should value the marginal hour of TV w!

- Opportunity costs are very important!

- Example 2. CostCo has a warehouse in SoMa

- SoMa used to have low cost land, adequate for warehouses

- Price of land in SoMa triples in 10 years.

- Should firm relocate the warehouse?

- Did costs of staying in SoMa go up?

- No.

- Did the opportunity cost of staying in SoMa go up?

- Yes!

- Firm can sell at high price and purchase land in cheaper area.

- Let's go back to labor supply

- Maximization problem is

$$\max u(c, l)$$
$$s.t.\ pc + wl \leq M + 24w$$

- Standard problem (except for $24w$)

- First order conditions

- Assume utility function Cobb-Douglas:

$$u(c, l) = c^{\alpha} l^{1-\alpha}$$

- Solution is

$$c^* = \alpha \frac{M + 24w}{p}$$

$$l^* = (1-\alpha)\left(24 + \frac{M}{w}\right)$$

- Both c and l are normal goods

- Unlike in standard Cobb-Douglas problems, c^* depends on price of other good w

- Why? Agents are endowed with M AND 24 hours of l in this economy

- Normally, agents are only endowed with M

4 Intertemporal choice

- Nicholson (Ch. 17, pp. 502–506, 9th)

- So far, we assumed people live for one period only

- Now assume that people live for two periods:
  - $t = 0$ – people are young
  - $t = 1$ – people are old

- $t = 0$: income M_0, consumption c_0 at price $p_0 = 1$

- $t = 1$: income $M_1 > M_0$, consumption c_1 at price $p_1 = 1$

- Credit market available: can lend or borrow at interest rate r

- Budget constraint in period 1?

- Sources of income:

  - M_1

  - $(M_0 - c_0) * (1 + r)$ (this can be negative)

- Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) * (1 + r)$$

or

$$c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1$$

- Utility function?

- Assume

$$u(c_0, c_1) = U(c_0) + \frac{1}{1+\delta} U(c_1)$$

- $U' > 0$, $U'' < 0$

- δ is the discount rate

- Higher δ means higher impatience

- Elicitation of δ through hypothetical questions

- Person is indifferent between 1 hour of TV today and $1 + \delta$ hours of TV next period

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta} U(c_1)$$

$$s.t.\ c_0 + \frac{1}{1+r} c_1 \leq M_0 + \frac{1}{1+r} M_1$$

- Lagrangean

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_0)}{U'(c_1)} = \frac{1+r}{1+\delta}$$

- Case $r = \delta$
  - $c_0^* \quad\quad c_1^*$?
  - Substitute into budget constraint using $c_0^* = c_1^* = c^*$:

$$\frac{2+r}{1+r} c^* = \left[M_0 + \frac{1}{1+r} M_1 \right]$$

    or

$$c^* = \frac{1+r}{2+r} M_0 + \frac{1}{2+r} M_1$$

  - We solved problem virtually without any assumption on U!
  - Notice: $M_0 < c^* < M_1$

- Case $r > \delta$
  - $c_0^* \quad\quad c_1^*$?

- Comparative statics with respect to income M_0

- Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1+r}{1+\delta} U'(c_1) = 0$$

- Substitute c_1 in using $c_1 = M_1 + (M_0 - c_0)(1+r)$ to get

$$U'(c_0) - \frac{1+r}{1+\delta} U'(M_1 + (M_0 - c_0)(1+r)) = 0$$

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta} U''(c_1)(1+r)}{U''(c_0) - \frac{1+r}{1+\delta} U''(c_1) * (-(1+r))}$$

- Denominator is always negative

- Numerator is positive

- $\partial c_0^* (r, \mathbf{M}) / \partial M_0 > 0$ — consumption at time 0 is a normal good.

- Can also show $\partial c_0^* (r, \mathbf{M}) / \partial M_1 > 0$

- Comparative statics with respect to interest rate r

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial r} = -\frac{-\frac{1}{1+\delta}U'(c_1)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))} - \frac{-\frac{1+r}{1+\delta}U''(c_1) * (M_0 - c_0)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative

- Numerator: First term negative (substitution eff.)

- Numerator: Second term (income effect:)
  - positive if $M_0 > c_0$
  - negative if $M_0 < c_0$.

5 Next Lectures

- Application III: Altruism and Charitable Donations
- Introduction to Probability
- Risk Aversion
- Coefficient of risk aversion