# Economics 101A (Lecture 23) 

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## Outline

## 1. Second-price Auction

2. Auctions: eBay Evidence
3. Dynamic Games
4. Oligopoly: Stackelberg

## 1 Second-price Auction

- Nicholson, Ch. 18, pp. 659-66 [Not in old book]
- Sealed-bid auction
- Highest bidder wins object
- Price paid is second highest price
- Two individuals: $I=2$
- Strategy $s_{i}$ is bid $b_{i}$
- Each individual knows value $v_{i}$
- Payoff for individual $i$ is

$$
u_{i}\left(b_{i}, b_{-i}\right)=\left\{\begin{array}{cll}
v_{i}-b_{-i} & \text { if } & b_{i}>b_{-i} \\
\left(v_{i}-b_{-i}\right) / 2 & \text { if } & b_{i}=b_{-i} \\
0 & \text { if } & b_{i}<b_{-i}
\end{array}\right.
$$

- Show: weakly dominant to set $b_{i}^{*}=v_{i}$
- To show:

$$
u_{i}\left(v_{i}, b_{-i}\right) \geq u_{i}\left(b_{i}, b_{-i}\right)
$$

for all $b_{i}$, for all $b_{-i}$, and for $i=1,2$.

1. Assume $b_{-i}>v_{i}$

- $u_{i}\left(v_{i}, b_{-i}\right)=0=u_{i}\left(b_{i}, b_{-i}\right)$ for any $b_{i}<b_{-i}$
- $u_{i}\left(b_{-i}, b_{-i}\right)=\left(v_{i}-b_{-i}\right) / 2<0$
- $u_{i}\left(b_{i}, b_{-i}\right)=\left(v_{i}-b_{-i}\right)<0$ for any $b_{i}>b_{-i}$

2. Assume now $b_{-i}=v_{i}$

## 3. Assume now $b_{-i}<v_{i}$

# 2 Auctions: Evidence from eBay 

- In second-price auction, optimal strategy is to bid one's own value
- Is this true?
- eBay has proxy system: If you have highest bid, you pay bid of second-highest bidder
- eBay is essentially a second-price auction
- Two deviations:

1. People bid multiple times - they should not in this theory
2. People may overbid

## An example: eBay Bidding for a Board Game

- Bidding environment with clear boundary for rational willingness to pay ("buy-it-now price").
- Empirical environment unaffected by common-value arguments (presumably bidding for private use; in addition "buy-it-now" price).
- Still non-negligible amount (\$100-\$200).
$\rightarrow$ Is there evidence of overbidding?
$\rightarrow$ If so, can we detect determinants of overbidding?


## The Object



## The Data

- Cashflow 101: board game with the purpose of finance/accounting education.
- Retail price : \$195 plus shipping cost (\$10.75) from manufacturer (www.richdad.com).
- Two ways to purchase Cashflow 101 on eBay
- Auction (quasi-second price proxy bidding)
- Buy-it-now
- Hand-collected data of all auctions and Buy-itnow transactions of Cashflow 101 on eBay from 2/19/2004 to 9/6/2004.


## Sample

- Listings
- 206 by individuals (187 auctions only, 19 auctions with buy-it-now option)
- 493 by two retailers (only buy-it-now)
- Remove non-US\$, terminated, unsold items and items without simultaneous professional buy-it-now listing. $\rightarrow$ 169 auctions
- Buy-it-now offers of the two retailers
- Continuously present for all but six days. (Often individual buy-itnow offers present as well; they are often lower.)
- $100 \%$ and $99.9 \%$ positive feedback scores.
- Same prices $\$ 129.95$ until 07/31/2004; \$139.95 since 08/01/2004.
- Shipping cost $\$ 9.95$; other retailer $\$ 10.95$.
- New items (with bonus tapes/video).


## Listing Example (02/12/2004)

| Rich Dad's Cashflow Quadrant, Rich dad ... © | \$12.50 | 4 | 1 d 00 h 14 m |
| :---: | :---: | :---: | :---: |
| Rich Dad's Cashflow Quadrant by Robert T. ... | \$9.00 | 9 | 1d 00h 43m |
| Real Estate Investment Cashflow Software \$\$\$! ®(V) | \$10.49 | 2 | 1 d 04 h 36 m |
| CASHFLOW(8) 101202 Robert Kivosaki Best Pak \$ @(V) | \$207.96 | FBuy/ How | 1d 06 h 47 m |
| TRY IT TODAY, WITH ABSOLUTELY NO RISK, |  |  |  |
| CASHFLOW@ 101 Robert Kivosaki Plus Bonuses! P(V) | \$129.95 | $=$ Buy H Now | 1d08h 02m |
| Your satisfation is GUARANTEED, $100 \%$ \$ back |  |  |  |
| MLNT Cashflow 101 *Robert Kiyosaki Game NR! ®(V) | \$140.00 | 13 | 1d 08h 04m |
| It's easy to be rich. Brand New. Still sealed |  |  |  |
| cashflow Hard Money Funding 101 real estate ®® | \$14.99 | FBuy HNow | 1d 09h 28 m |
| BRANDNEW RICHDAD CASHFLOW FOR KIDS EGAME B | \$20.00 | 1 | 1d 13 h 54 m |
| CASHFLOW@101 Robert Kiyosaki Plus Bonuses! ®() | \$129.95 | $=$ Buy HNow | 1d 14h 17 m |
| Your satisfaction is GUARANTEED, $100 \%$ b back |  |  |  |
| CASHFLOW@101202 Robert Kiyosaki Best Pak \$ ©® | \$207.96 | $=$ Buy/ How | 1d 15h 47 m |

## Listing Example - Magnified



## Bidding history of an item



## Hypotheses

Given the information on the listing website:

- (H1) An auction should never end at a price above the concurrently available purchase price.
- (H2) Mentioning of higher outside prices should not affect bidding behavior.

Figure 1. Starting Price (startprice)
$\rightarrow 45 \%$ below $\$ 20$; mean $=\$ 46$; SD $=43.88$
$\rightarrow$ only 6 auctions with first bid (not price) above buy-it-now


Figure 2. Final Price (finalprice)
$\rightarrow 41 \%$ are above "buy-it-now" (mean \$132; SD 16.83)


Figure 4. Total Price (incl. shipping cost)
$\rightarrow 51 \%$ are above "buy-it-now" plus its shipping cost (mean=\$144.20; SD=15.00)


## The Other Lesson?

Some unsolicited eBay advice.

- Can make money by selling "Cashflow 101" to those who aspire to become financially smart, and overpay for the board game!
- Sellers : add exaggerated retail price, pay 20 cents extra (now 40 cents) for 10 day listing!
- Buyers : check out the "buy-it-now" price before you bid!


## 3 Dynamic Games

- Nicholson, Ch. 8, pp. 255-266 (better than Ch. 15, pp. 449-454, 9th)
- Dynamic games: one player plays after the other
- Decision trees
- Decision nodes
- Strategy is a plan of action at each decision node
- Example: battle of the sexes game

$$
\begin{array}{ccc}
\text { She } \backslash \text { He } & \text { Ballet } & \text { Football } \\
\text { Ballet } & 2,1 & 0,0 \\
\text { Football } & 0,0 & 1,2
\end{array}
$$

- Dynamic version: she plays first
- Subgame-perfect equilibrium. At each node of the tree, the player chooses the strategy with the highest payoff, given the other players' strategy
- Backward induction. Find optimal action in last period and then work backward
- Solution
- Example 2: Entry Game

$$
\begin{array}{ccc}
1 \backslash 2 & \text { Enter } & \text { Do not Enter } \\
\text { Enter } & -1,-1 & 10,0 \\
\text { Do not Enter } & 0,5 & 0,0
\end{array}
$$

- Exercise. Dynamic version.
- Coordination games solved if one player plays first
- Can use this to study finitely repeated games
- Suppose we play the prisoner's dilemma game ten times.

$$
\begin{array}{ccc}
1 \backslash 2 & D & N D \\
D & -4,-4 & -1,-5 \\
N D & -5,-1 & -2,-2
\end{array}
$$

- What is the subgame perfect equilibrium?
- The result differs if infinite repetition with a probability of terminating
- Can have cooperation
- Strategy of repeated game:
- Cooperate (ND) as long as opponent always cooperate
- Defect (D) forever after first defection
- Theory of repeated games: Econ. 104


# 4 Oligopoly: Stackelberg 

- Nicholson, Ch. 15, pp. 543-545 (better than Ch. 14, pp. 423-424, 9th)
- Setting as in problem set
- 2 Firms
- Cost: $c(y)=c y$, with $c>0$
- Demand: $p(Y)=a-b Y$, with $a>c>0$ and $b>0$
- Difference: Firm 1 makes the quantity decision first
- Use subgame perfect equilibrium
- Solution:
- Solve first for Firm 2 decision as function of Firm 1 decision:

$$
\max _{y_{2}}\left(a-b y_{2}-b y_{1}^{*}\right) y_{2}-c y_{2}
$$

- F.o.c.: $a-2 b y_{2}^{*}-b y_{1}^{*}-c=0$
- Firm 2 best response function:

$$
y_{2}^{*}=\frac{a-c}{2 b}-\frac{y_{1}^{*}}{2}
$$

- Firm 1 takes this response into account in the maximization:

$$
\max _{y_{1}}\left(a-b y_{1}-b y_{2}^{*}\left(y_{1}\right)\right) y_{1}-c y_{1}
$$

or

$$
\max _{y_{1}}\left(a-b y_{1}-b\left(\frac{a-c}{2 b}-\frac{y_{1}}{2}\right)\right) y_{1}-c y_{1}
$$

- F.o.c.:

$$
a-2 b y_{1}-\frac{(a-c)}{2}+b y_{1}-c=0
$$

or

$$
y_{1}^{*}=\frac{a-c}{2 b}
$$

and

$$
y_{2}^{*}=\frac{a-c}{2 b}-\frac{y_{1}^{*}}{2}=\frac{a-c}{2 b}-\frac{a-c}{4 b}=\frac{a-c}{4 b}
$$

- Total production:

$$
Y_{D}^{*}=y_{1}^{*}+y_{2}^{*}=3 \frac{a-c}{4 b}
$$

- Price equals

$$
p^{*}=a-b\left(\frac{3}{4} \frac{a-c}{b}\right)=\frac{1}{4} a+\frac{3}{4} c
$$

- Compare to monopoly:

$$
y_{M}^{*}=\frac{a-c}{2 b}
$$

and

$$
p_{M}^{*}=\frac{a+c}{2}
$$

- Compare to Cournot:

$$
Y_{D}^{*}=y_{1}^{*}+y_{2}^{*}=2 \frac{a-c}{3 b}
$$

and

$$
p_{D}^{*}=\frac{1}{3} a+\frac{2}{3} c .
$$

- Compare with Cournot outcome
- Firm 2 best response function:

$$
y_{2}^{*}=\frac{a-c}{2 b}-\frac{y_{1}^{*}}{2}
$$

- Firm 1 best response function:

$$
y_{1}^{*}=\frac{a-c}{2 b}-\frac{y_{2}^{*}}{2}
$$

- Intersection gives Cournot
- Stackelberg: Equilibrium is point on Best Response of Firm 2 that maximizes profits of Firm 1
- Plot iso-profit curve of Firm 1:

$$
\bar{\Pi}_{1}=(a-c) y_{1}-b y_{1} y_{2}-b y_{1}^{2}
$$

- Solve for $y_{2}$ along iso-profit:

$$
y_{2}=\frac{a-c}{b}-y_{1}-\frac{\bar{\Pi}_{1}}{b y_{1}}
$$

- Iso-profit curve is flat for

$$
\frac{d y_{2}}{d y_{1}}=-1+\frac{\bar{\Pi}}{b\left(y_{1}\right)^{2}}=0
$$

or

$$
y_{1}=
$$

Figure

## 5 Next lecture

- General Equilibrium
- Edgeworth Box

